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# A HYBRID SLIDING-WINDOW DISCRETE GREY-ARIMA MODEL FOR SHORT-TERM FORECASTING OF IRAQI GOLD PRICES

**Nadia Ali Ayyed**[1] and **Mohammed H. AL-Sharoot**[2]
Faculty of Administration and Economics
University of Al-Qadisiyah, Iraq
Email: [1]nidea.iead@uobasrah.edu.iq
[2]mohammed.alsharoot@qu.edu.iq

## ABSTRACT

This study tests the forecasting performance of grey-based models and residual correction based on ARIMA for short-term Iraqi gold price forecast. However, due to the volatility and uncertainty of gold price movement, the short-term prediction of gold price movement is still a challenge for traditional time series models. Two distinct grey forecasting models, Recursive Discrete Grey Model RLS-DGM(1,1) and Sliding-Window Discrete Grey Model SW-DGM(1,1), were estimated and compared with out-of-sample accuracy measures and grey-system evaluation criteria. The results show that the RLS-DGM(1,1) has the best forecasting accuracy in the baseline, with the minimum error values and excellent grey-system indicators. SW-DGM(1,1) also achieved Level 1 accuracy but with larger forecasting errors, and was thus chosen for residual correction. However, the performance of the ARIMA based residual correction was not improved, suggesting that the remaining residual structure was not well characterized by a linear process. Overall, the grey-based models performed better than the benchmark models of ARIMA-only and ETS-only.

## KEYWORDS

Grey forecasting models; Discrete Grey Model (DGM); Sliding-Window Discrete Grey Model (SW-DGM); Recursive Least Squares (RLS); Exponential Smoothing (ETS).

## 1. INTRODUCTION

Due to the strategic importance of gold as a commodity and financial asset, gold price forecasting has received much research attention. Gold price series are generally nonlinear, non-stationary, volatile and subject to sudden structural changes. These make conventional forecasting methods less reliable in the context of unstable market conditions. Grey system theory is a suitable modeling framework for such environments as it can cope with limited and uncertain information effectively. The Discrete Grey Models (DGMs) have been widely used to extract dominant trends from uncertain time series. However, classical DGMs with fixed parameters may lose forecasting accuracy in the long and highly volatile datasets of finance. To overcome this limitation, adaptive grey models such as recursive and sliding-window DGM have been proposed. Recursive DGM updates the parameters sequentially as new observations become available. Sliding-window DGM re-estimates the parameters using only the most recent observations, which allows the model to respond to

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local changes in the data. However, a systematic empirical comparison between recursive and sliding-window grey models is still required, especially for the volatile gold price series. Moreover, the possible advantage of residual correction after grey modeling is still a significant issue, for some systematic patterns may not be accounted for by the baseline grey model. Thus, in this study we assess the short-term forecasting of Iraqi gold prices using RLS-DGM(1,1), SW-DGM(1,1) and an ARIMA based residual correction framework. Moreover, ARIMA-only and ETS-only models are considered as non-grey benchmark models for external comparison. The contribution of this study is to offer a structured empirical evaluation of grey-based forecasting models in terms of forecasting accuracy and parameter stability. The study also explores whether the ARIMA-based residual correction can improve the forecasting performance of the selected grey model. The rest of this paper is organized as follows. Section 2 shows the aim and objectives of the paper. The theoretical background is presented in Section 3. Section 4 is about the application part, which includes data description, model estimation, residual diagnostics, residual correction, benchmark models, and comparison. Section 5 concludes the paper, while Section 6 presents future research directions.

## 2. THE AIM OF THE PAPER

This study intends to improve the short-term forecasting performance of the Iraqi gold prices. The estimated and compared grey and non-grey forecasting models, and the grey model with lower parameter stability and lower forecasting accuracy are identified. The performance of the grey model is improved by ARIMA based residual correction.

## 3. THEORETICAL SIDE

### 3.1 Grey System Theory (GST)

Grey System Theory (GST), introduced by Deng (1982), is a forecasting and modelling approach designed for systems with limited, uncertain, or incomplete information. Unlike many conventional statistical methods, grey theory does not require large samples or strict distributional assumptions. Its main idea is to reduce randomness in the original data and reveal the underlying development trend through information-generation techniques. Therefore, grey forecasting models are suitable for short and imperfect time series. Based on this theoretical foundation, discrete, adaptive, and residual-corrected grey models are used in this study to improve the forecasting performance of Iraqi gold prices.

### 3.2 Discrete Grey Model DGM(1,1)

The Discrete Grey Model DGM(1,1) was proposed by Xie and Liu (2005) as a discrete-time extension of the classical GM(1,1) model. Its main contribution was the introduction of a direct discrete formulation that is more consistent with time series observed at discrete intervals, thereby reducing the mismatch between the continuous whitening equation and discretely recorded data. Similar to the classical GM(1,1) model, DGM(1,1) is particularly suitable for small-sample and poor-information systems, and in practice it can be established with very limited observations, with four observations generally regarded as the minimum requirement. However, the principal difference between DGM(1,1) and GM(1,1) does not lie in the minimum sample size, but rather in the model structure itself, since DGM(1,1) provides a discrete representation that is often more appropriate for

discretely observed time series. The building stages of the DGM(1,1) can be defined as follows:

**Step 1:** Defining the raw data sequence.

Let the original non-negative data sequence be defined as:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}. \tag{3.1}$$

where $x^{(0)}(k)$ denotes the original observation at time $k$, and $n$ is the number of observations.

**Step 2:** Applying the Accumulated Generating Operation (AGO).

The original sequence is transformed into a monotonically increasing accumulated sequence $X^{(1)}$ in order to reduce randomness and enhance the underlying trend:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}. \tag{3.2}$$

Each accumulated value is calculated as:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1,2, \dots, n. \tag{3.3}$$

**Step 3:** Formulating the DGM(1,1) difference equation.

In the DGM(1,1) model, the relationship between two successive accumulated values is expressed as:

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2, \quad k = 1,2, \dots, n-1. \tag{3.4}$$

where $\beta_1$ is the development coefficient governing the dynamic evolution of the accumulated sequence, and $\beta_2$ is the grey input term representing the overall external influence on the system. Unlike the continuous GM(1,1) model, DGM(1,1) is formulated directly through a discrete difference equation.

**Step 4:** Parameter estimation using least squares.

The parameter vector is defined as:

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}. \tag{3.5}$$

The known response vector $Y$ and the design matrix $B$ are constructed as follows:

$$Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, \qquad B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{bmatrix}. \tag{3.6}$$

Accordingly, the least-squares estimate of the parameter vector is obtained by:

$$\hat{\beta} = (B^T B)^{-1} B^T Y, \qquad \hat{\beta} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}. \tag{3.7}$$

**Step 5:** Time-response formula.

After estimating $\hat{\beta}_1$ and $\hat{\beta}_2$, the fitted accumulated sequence is obtained from the following time-response formula:

$$\hat{x}^{(1)}(k) = \left[x^{(0)}(1) - \frac{\hat{\beta}_2}{1-\hat{\beta}_1}\right]\hat{\beta}_1^{k-1} + \frac{\hat{\beta}_2}{1-\hat{\beta}_1}, \quad k = 1,2,\dots,n. \tag{3.8}$$

This formulation satisfies the initial condition $\hat{x}^{(1)}(1) = x^{(0)}(1)$ and keeps the indexing consistent with the accumulated sequence.

**Step 6:** Inverse Accumulated Generating Operation (IAGO).

The predicted values of the original sequence are obtained by applying the inverse accumulated generating operation:

$$\hat{x}^{(0)}(1) = x^{(0)}(1), \tag{3.9}$$

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \quad k = 2,3,\dots,n. \tag{3.10}$$

The DGM(1,1) model provides a simple and interpretable discrete grey forecasting structure. However, because its parameters are estimated from a fixed historical sample, the model may be less responsive to structural changes and non-stationary dynamics in financial time series. This limitation motivates the use of adaptive grey models, such as the Sliding-Window Discrete Grey Model and the Recursive Least Squares Discrete Grey Model.

$$\hat{x}^{(0)}(k+1) = \left(\hat{\beta}_1 - 1\right) \quad \left[x^{(0)}(1) - \frac{\hat{\beta}_2}{1-\hat{\beta}_1}\right]\hat{\beta}_1^{k-1} \qquad k = 1,2,3,\dots,n-1 \tag{3.11}$$

where $\hat{\beta}_1$and$\hat{\beta}_2$ are the estimated parameters of the DGM(1,1) model, and $\hat{x}^{(0)}(k+1)$ is the restored forecast value obtained after applying IAGO. The DGM(1,1) model provides a simple and interpretable discrete grey forecasting structure. However, because its parameters are estimated from a fixed historical sample, its performance may deteriorate when the underlying system exhibits structural changes or non-stationary behaviour. This limitation motivates the use of adaptive grey models, such as SW-DGM(1,1) and RLS-DGM(1,1), which update model parameters over time.

### 3.3 Adaptive Grey Models

The adaptive grey models were developed to overcome the static property of the conventional DGM(1,1) whose parameters are estimated from a fixed historical sample. Observations that are old in non-stationary economic and financial time series may be irrelevant for the current behavior of the system. Thus, adaptive models incrementally update their parameters as new information becomes available. In this paper, SW-DGM(1,1) and RLS-DGM(1,1) are used as adaptive extensions of DGM(1,1) to improve the forecasting flexibility of structurally evolving time series.

### 3.3.1 Sliding-Window Discrete Grey Model (SW-DGM(1,1))

The Sliding-Window Discrete Grey Model (SW-DGM(1,1)) is an adaptive extension of the traditional DGM(1,1) model. It re-estimates model parameters in a fixed-length moving window, instead of over the entire historical sample, which reduces the influence of outdated observations and improves the responsiveness to recent changes in non-stationary time series. Therefore, SW-DGM(1,1) can be applied for short-term forecasting under structural changes and meanwhile it still retains the small-sample advantages of the grey system theory (Wang et al., 2018). Let's say the original time series is non-negative:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad x^{(0)}(k) \geq 0.$$

For a sliding window ending at time $t$, the local raw sequence is defined as:

$$X_t^{(0)} = \{x^{(0)}(t-m+1), x^{(0)}(t-m+2), \dots, x^{(0)}(t)\}, \quad t = m, m+1, \dots, n. \tag{3.12}$$

For convenience, the observations inside the window are re-indexed locally as:

$$x_t^{(0)}(i) = x^{(0)}(t-m+i), \quad i = 1,2, \dots, m. \tag{3.13}$$

The accumulated generating operation within the window is computed as:

$$x_t^{(1)}(i) = \sum_{j=1}^{i} x_t^{(0)}(j), \quad i = 1,2, \dots, m. \tag{3.14}$$

Thus, the accumulated sequence in window $t$ is:

$$X_t^{(1)} = \{x_t^{(1)}(1), x_t^{(1)}(2), \dots, x_t^{(1)}(m)\}. \tag{3.15}$$

Within each sliding window, the DGM(1,1) difference equation is written as:

$$x_t^{(1)}(i) = \beta_{1,t} x_t^{(1)}(i-1) + \beta_{2,t}, \quad i = 2,3, \dots, m. \tag{3.16}$$

where $\beta_{1,t}$ is the development coefficient in window $t$, $\beta_{2,t}$ is the grey input term, $m$ is the window length, and $t$ denotes the window position.

The parameters are estimated by least squares. Define:

$$Y_t = \begin{bmatrix} x_t^{(1)}(2) \\ x_t^{(1)}(3) \\ \vdots \\ x_t^{(1)}(m) \end{bmatrix}, \quad B_t = \begin{bmatrix} x_t^{(1)}(1) & 1 \\ x_t^{(1)}(2) & 1 \\ \vdots & \vdots \\ x_t^{(1)}(m-1) & 1 \end{bmatrix}. \tag{3.17}$$

Then the parameter vector is obtained as:

$$\hat{\beta}_t = \begin{bmatrix} \hat{\beta}_{1,t} \\ \hat{\beta}_{2,t} \end{bmatrix} = (B_t^T B_t)^{-1} B_t^T Y_t. \tag{3.18}$$

Given the initial condition:

$$\hat{x}_t^{(1)}(1) = x_t^{(1)}(1), \tag{3.19}$$

the fitted accumulated response inside window $t$ is:

$$\hat{x}_t^{(1)}(i) = \hat{\beta}_{1,t}^{\,i-1} x_t^{(1)}(1) + \hat{\beta}_{2,t} \frac{\hat{\beta}_{1,t}^{\,i-1} - 1}{\hat{\beta}_{1,t} - 1}, \quad i = 2,3,\dots,m, \quad \hat{\beta}_{1,t} \neq 1. \tag{3.20}$$

The fitted values in the original scale are obtained using the inverse accumulated generating operation:

$$\hat{x}_t^{(0)}(i) = \hat{x}_t^{(1)}(i) - \hat{x}_t^{(1)}(i-1), \quad i = 2,3,\dots,m. \tag{3.21}$$

The one-step-ahead forecast is obtained by extending the local response to $i = m + 1$ and then applying IAGO:

$$\hat{x}^{(0)}(t+1) = \hat{x}_t^{(1)}(m+1) - \hat{x}_t^{(1)}(m). \tag{3.22}$$

This procedure is repeated as the window moves forward through the series. Therefore, the SW-DGM(1,1) model produces time-varying parameter estimates and forecasts that are more responsive to recent observations than the fixed-sample DGM(1,1) model. For forecasting, the one-step-ahead accumulated forecast is obtained by extending the local difference equation one step beyond the current window:

$$\hat{x}_t^{(1)}(m+1) = \hat{\beta}_{1,t} x_t^{(1)}(m) + \hat{\beta}_{2,t}. \tag{3.23}$$

Therefore, the one-step-ahead forecast in the original scale is:

$$\hat{x}^{(0)}(t+1) = \hat{x}_t^{(1)}(m+1) - x_t^{(1)}(m) = \left(\hat{\beta}_{1,t} - 1\right) x_t^{(1)}(m) + \hat{\beta}_{2,t}. \tag{3.24}$$

For a multi-step-ahead forecasting horizon (h), the accumulated-domain forecast can be written as:

$$\hat{x}_t^{(1)}(m+h) = \hat{\beta}_{1,t}^{\,h} x_t^{(1)}(m) + \hat{\beta}_{2,t} \frac{\hat{\beta}_{1,t}^{\,h} - 1}{\hat{\beta}_{1,t} - 1}, \qquad h \geq 1, \quad \hat{\beta}_{1,t} \neq 1. \tag{3.25}$$

The corresponding multi-step forecast in the original domain is obtained as:

$$\hat{x}^{(0)}(t+h) = \hat{x}_t^{(1)}(m+h) - \hat{x}_t^{(1)}(m+h-1), \qquad h \geq 1 \tag{3.26}$$

As the window advances through the series, this process is repeated. Consequently, compared to the fixed-sample DGM(1,1) model, the SW-DGM(1,1) model generates time-varying parameter estimates and forecasts that are more sensitive to recent observations.

### 3.3.2 Recursive Discrete Grey Model (RLS-DGM(1,1))

The Recursive Discrete Grey Model (RLS-DGM(1,1)) is an adaptive extension of the classical DGM(1,1) model. Unlike the fixed-sample DGM(1,1), which estimates its parameters once using a historical sample, RLS-DGM(1,1) updates its parameters sequentially as new observations become available. This recursive updating mechanism allows the model to adapt gradually to time-varying dynamics and structural changes in the series, while retaining past information through the recursive least squares procedure (Wang et al., 2016). To express the RLS-DGM(1,1) model in regression form define:

$$y_k = x^{(1)}(k), \quad \varphi_k = \begin{bmatrix} x^{(1)}(k-1) \\ 1 \end{bmatrix}, \quad \theta_k = \begin{bmatrix} \beta_1(k) \\ \beta_2(k) \end{bmatrix}, \quad k = 2,3,\dots,n. \tag{3.27}$$

Then, the recursive DGM(1,1) model can be written compactly as:

$$y_k = \varphi_k^T \theta_k + \varepsilon_k, \qquad k = 2,3,\dots,n. \tag{3.28}$$

In this formulation, the symbols in Equation (3.23) define the accumulated response variable, the regression vector, and the time-varying parameter vector, while Equation (3.24) includes the one-step prediction error. The recursive gain vector is computed as:

$$K_k = \frac{P_{k-1}\varphi_k}{\lambda + \varphi_k^T P_{k-1}\varphi_k}, \quad 0 < \lambda \leq 1. \tag{3.29}$$

Equation (3.29) shows how the recursive gain is calculated using the covariance matrix from the previous updating step and the forgetting factor. A forgetting factor value close to one emphasizes the past observations more, whereas a smaller value allows for faster adaptation to recent changes. The parameter vector is updated as follows:

$$\theta_k = \theta_{k-1} + K_k(y_k - \varphi_k^T \theta_{k-1}). \tag{3.30}$$

$$P_k = \frac{1}{\lambda}(P_{k-1} - K_k \varphi_k^T P_{k-1}). \tag{3.31}$$

After updating the parameter estimates at time k, the one-step-ahead forecast in the accumulated domain is obtained as:

$$\hat{x}^{(1)}(k+1) = \hat{\beta}_1(k)x^{(1)}(k) + \hat{\beta}_2(k). \tag{3.32}$$

Finally, the forecast in the original data domain is recovered using the inverse accumulated generating operation (IAGO):

$$\hat{x}^{(0)}_{RLS-DGM}(k+1) = \hat{x}^{(1)}(k+1) - x^{(1)}(k). \tag{3.33}$$

As new observations come in, the RLS-DGM(1,1) model continuously updates its parameters to produce forecasts. In contrast to SW-DGM(1,1), which adapts by recurrent re-estimation inside a fixed moving window, this makes it more appropriate for time series that are progressively changing.

### 3.4 Correction of Residuals the Grey Model Based on Arima

Grey-ARIMA based residual correction approach is a hybrid logic consistent with the general role of ARIMA models which are designed to represent auto correlation structures in time-series data (Hyndman & Athanasopoulos, 2021). In a hybrid Grey-ARIMA forecasting, the ARIMA model was fitted to the residual series and its forecasts are then used as a correction term to improve the original grey prediction (Li et al., 2017). Let the original observed series is the Grey–ARIMA residual-correction approach is used when the selected grey model captures the main trend of the series, but its residuals still contain an autocorrelation structure that can be further modeled. In this study, ARIMA is not used as an independent forecasting model in this subsection; rather, it is fitted to the residual sequence generated by the selected grey model and its residual forecasts are added back as correction terms. Let the original observed series be defined as:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(N)\}.$$

where $x^{(0)}(k)$ denotes the observed value of the original series at time $k$, and $N$ is the total number of observations. Let the initial forecasts obtained from the selected grey model be:

$$\hat{X}_G^{(0)} = \{\hat{x}_G^{(0)}(1), \hat{x}_G^{(0)}(2), \dots, \hat{x}_G^{(0)}(N)\}. \tag{3.34}$$

where $\hat{x}_G^{(0)}(k)$ denotes the forecast produced by the selected grey model at time $k$.

The residual sequence of the grey model is then calculated as:

$$e(k) = x^{(0)}(k) - \hat{x}_G^{(0)}(k), \qquad k = 1,2, \dots, N. \tag{3.35}$$

where $e(k)$ represents the forecast residual at time $k$. A positive residual means that the observed value is greater than the grey-model forecast, whereas a negative residual means that the grey-model forecast is greater than the observed value. To model the remaining linear dependence in the residual sequence, an ARIMA model is fitted to $e(k)$. The general ARIMA$(p, d, q)$ model for the residual series can be written as:

$$\phi(B)(1 - B)^d e_t = \theta(B)\varepsilon_t. \tag{3.36}$$

where $e_t$ denotes the residual series indexed over time, $B$ is the backshift operator, $d$ is the differencing order, $\phi(B)$ is the autoregressive polynomial of order $p$, $\theta(B)$ is the moving-average polynomial of order $q$, and $\varepsilon_t$ is a white-noise error term. The ARIMA forecast of the residual is used as a correction component:

$$C_A(k) = \hat{e}_{ARIMA}(k), \qquad k = 1,2, \dots, N. \tag{3.37}$$

where $C_A(k)$ denotes the ARIMA-based residual correction term at time $k$, and $\hat{e}_{ARIMA}(k)$ denotes the residual forecast obtained from the ARIMA model. Accordingly, the final Grey–ARIMA corrected forecast is obtained by adding the ARIMA-based residual correction term to the original grey-model forecast:

$$\hat{x}_{GA}^{(0)}(k) = \hat{x}_G^{(0)}(k) + C_A(k), \qquad k = 1,2, \dots, N. \tag{3.38}$$

where $\hat{x}_{GA}^{(0)}(k)$ denotes the final corrected forecast produced by the Grey–ARIMA model. Equivalently, by substituting Equation (3.38) into Equation (3.39), the corrected forecast can be expressed as:

$$\hat{x}_{GA}^{(0)}(k) = \hat{x}_G^{(0)}(k) + \hat{e}_{ARIMA}(k), \qquad k = 1,2, \dots, N. \tag{3.39}$$

This equation shows that the final forecast consists of two components: the main trend forecast generated by the selected grey model and the short-term residual adjustment generated by the ARIMA model. This structure is appropriate when the grey model captures the dominant trend, while the residual series still contains stable linear autocorrelation that can be exploited through ARIMA modeling.

### 3.5 Forecasting Accuracy Evaluation

The forecasting performance of the proposed models was evaluated using both grey-system diagnostic measures and conventional out-of-sample accuracy measures. For grey models, the posterior error ratio $C$ and the small error probability $P$ were used as

supplementary indicators of residual dispersion and model reliability. Let $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ denote the observed and predicted values, respectively. The residual error is defined as:

$$e(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), \quad k = 1,2, \dots, n. \tag{3.40}$$

The posterior error ratio is calculated as:

$$C = \frac{S_2}{S_1}. \tag{3.41}$$

where $S_1$ and $S_2$ are the standard deviations of the original series and the residual series, respectively. The small error probability is computed as:

$$N_P = \text{card}\{k: |e(k) - \bar{e}| < 0.6745 S_1\}, \qquad P = \frac{N_P}{n}. \tag{3.42}$$

A lower value of $C$ and a higher value of $P$ indicate better residual stability. In addition, the models were compared using standard out-of-sample forecasting accuracy measures, namely RMSE, MAE, and MAPE:

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x^{(0)}(k) - \hat{x}^{(0)}(k)\right)^2}. \tag{3.43}$$

$$MAE = \frac{1}{n}\sum_{k=1}^{n}\left|x^{(0)}(k) - \hat{x}^{(0)}(k)\right|. \tag{3.44}$$

$$MAPE = \frac{100}{n}\sum_{k=1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|. \tag{3.45}$$

Finally, parameter stability was assessed using the coefficient of variation:

$$CV_\theta(\%) = \frac{S_\theta}{|\bar{\theta}|} \times 100 \tag{3.46}$$

where $S_\theta$ and $\bar{\theta}$ are the standard deviation and mean of the estimated parameter $\theta$, respectively. Lower values of $CV_\theta(\%)$ indicate more stable parameter estimates. These measures were calculated on the out-of-sample testing period to avoid overfitting and to ensure a fair comparison among the proposed grey models and the non-grey benchmark models.

**Table 3.1**
**Summarizes the Standard Grey-System Accuracy Levels based on $C$ and $P$**

| Accuracy Level | Condition for $C$ | Condition for $P$ |
|---|---|---|
| Grade I / Excellent | $C \leq 0.35$ | $P \geq 0.95$ |
| Grade II / Qualified | $0.35 < C \leq 0.50$ | $0.80 \leq P < 0.95$ |
| Grade III / Barely Qualified | $0.50 < C \leq 0.65$ | $0.70 \leq P < 0.80$ |
| Grade IV / Unqualified | $C > 0.65$ | $P < 0.70$ |

## 4. APPLICATION SIDE

### 4.1 Data Description and Descriptive Statistics

Daily gold prices denominated in USD per troy ounce were collected for the Iraqi gold market, yielding 367 consecutive observations spanning from May 9, 2024, to May 10, 2025. The Iraqi gold market represents a relevant case study due to its dual role as a store of value and a hedge against exchange-rate instability in economically vulnerable regions. All empirical analyses were conducted using Python 3.12.0 on an Ubuntu 24.04 Linux operating system, consistent with best practices in computational research.

**Figure 4.1(a): Time Series Plot of Gold Prices**

Figure 4.1(a) shows a clear upward trend in daily Iraqi gold prices, accompanied by short-term fluctuations and abrupt changes. The increasing variability toward the end indicates non-stationarity and supports the need for an adaptive forecasting approach.

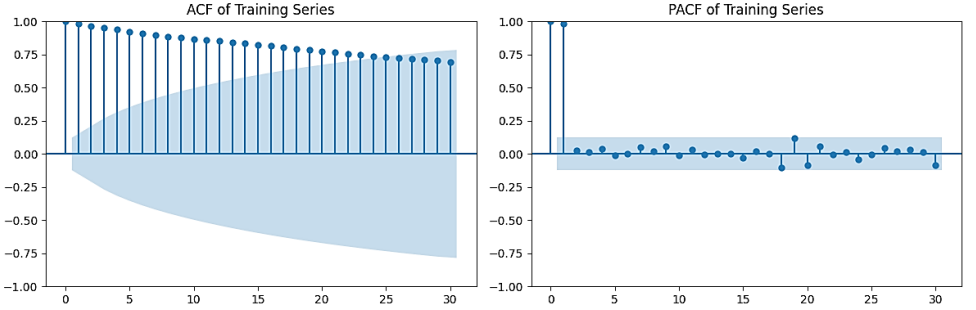


**Figure 4.1(b): ACF and PACF of Gold Price Series**

Figure 4.1(b) shows strong serial dependence and non-stationarity, as indicated by the slowly decaying ACF and the dominant first-lag PACF spike.

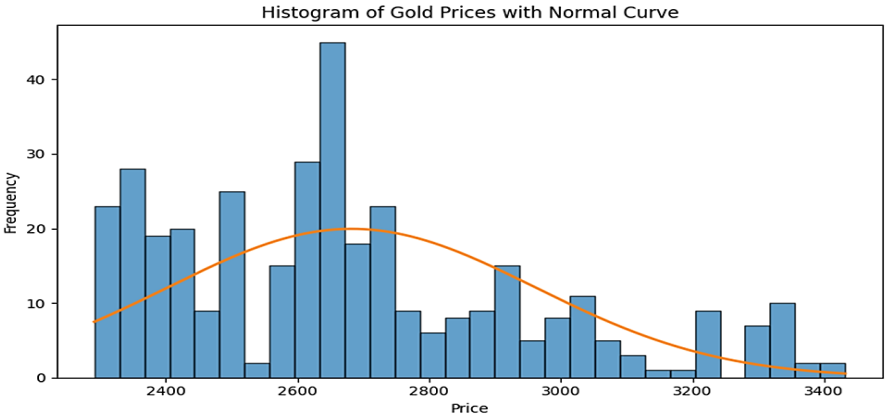


**Figure 4.1(c): Distribution of Gold Price (Histogram with Normal Curve)**

**Table 4.1**
**Descriptive Statistics for Gold Prices**

| Sample Size (N) | Minimum | Maximum | Mean | Median | ADF Statistic | Standard Deviation | Skewness | Kurtosis | Coefficient of Variation |
|---|---|---|---|---|---|---|---|---|---|
| 367 | 2291.40 | 3431.60 | 2682.13 | 2648.70 | 0.989 | 279.05 | 0.77 | -0.065 | 0.1048 |

The gold price series consists of 367 observations, which were divided into 256 training observations and 111 testing observations for evaluating forecasting performance. The descriptive statistics in Table 4.1 show that gold prices range from 2291.40 to 3431.60, with a mean of 2682.13 and a standard deviation of 279.05, indicating noticeable variability in the series. The ADF statistic suggests non-stationarity, while the positive skewness value 0.77 indicates a slight right-skewed distribution.

### 4.2 Estimation of the Study Models

Before estimating the grey forecasting models, the Iraqi gold price series was examined and found suitable for grey modelling. The series has sufficient observations, shows a clear trend with short-term fluctuations, and does not require strict stationarity or normality assumptions.

#### 4.2.1 Recursive Discrete Grey Model (RLS-DGM)

In this section, the Recursive Discrete Grey Model (RLS-DGM(1,1)) is applied to the Iraqi gold price series as an adaptive grey forecasting model. Unlike the fixed-parameter DGM(1,1), the RLS-DGM updates its parameters sequentially as new observations become available. The model was estimated using the training sample, and its forecasting performance was evaluated on the out-of-sample testing period. The dataset was divided into 256 observations for training and 111 observations for testing. The recursive estimation was performed using different forgetting-factor values $\lambda$=0.95, 0.97, 0.99 in order to examine the sensitivity of the RLS-DGM model and select the most stable parameter setting.

**Table 4.2**
**Parameter Statistics for RLS-DGM**

| Model | Parameter | CV (%) | Decision |
|---|---|---|---|
| RLS-DGM (λ = 0.99) | β1 | 0.1972% | **Most Stable** |
| | β2 | 5.600% | |
| RLS-DGM (λ = 0.97) | β1 | 3.03% | **Less Stable** |
| | β2 | 20.11% | |
| RLS-DGM (λ = 0.95) | β1 | 3.83% | **Least Stable** |
| | β2 | 23.41% | |

Table 4.2 presents the stability statistics of the estimated RLS-DGM(1,1) parameters under different forgetting factors. The coefficient of variation, CV (%), was used as a stability measure, where a lower CV value indicates a more stable parameter estimate. The results show that the model with λ = 0.99 achieved the lowest CV values for both $\beta_1$ and $\beta_2$, with 0.1972% and 5.600%, respectively. Therefore, λ = 0.99 was selected as the most stable forgetting factor for the RLS-DGM(1,1) model.

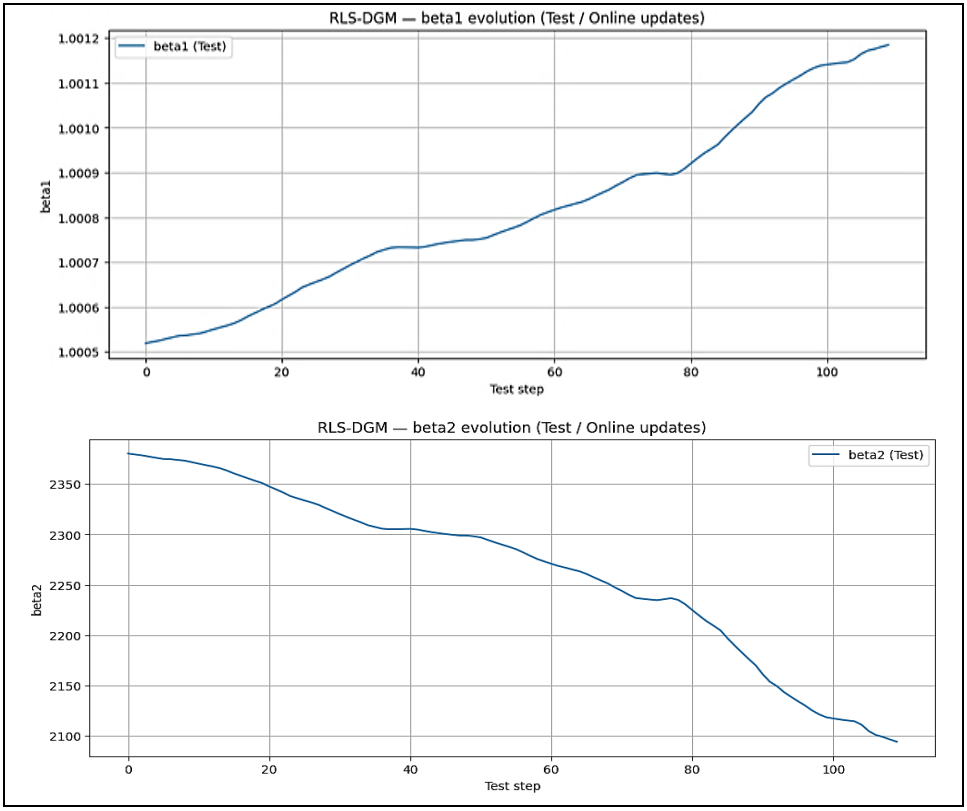


**Figure 4.2: Dynamic Evolution of $\beta_1$ and $\beta_2$ in the RLS-DGM(1,1) Model**

Figure 4.2 illustrate the dynamic evolution of the estimated parameters $\beta_1$ and $\beta_2$ during the out-of-sample recursive updating process. The parameter $\beta_1$ shows a smooth and gradual upward movement, indicating that the model updates the growth-related component in a stable manner. In contrast, $\beta_2$ shows a gradual downward trend, reflecting the adjustment of the intercept or grey input component as new observations are introduced. Although the two parameters change over time, their movements are systematic and not highly erratic, which supports the suitability of the RLS updating mechanism.

**Table 4.3**
**RLS-DGM Accuracy Evaluation Metrics**
**Based on Statistical and Grey System Measures**

| Metric | RMSE | C (Posterior Error Ratio) | P (Small Error Probability) | Accuracy Grade |
|---|---|---|---|---|
| **Value** | 34.6273 | 0.1806 | 1.0000 | Grade 1 |
| **Threshold** | - | $\leq 0.35$ | $\geq 0.95$ | — |
| **Assessment** | - | Excellent ✓ | Excellent ✓ | Excellent ✓ |

Table 4.3 reports the forecasting accuracy of the RLS-DGM(1,1) model based on both statistical and grey system evaluation measures. The RMSE value is 34.6273, indicating a relatively small forecasting error compared with the level of the gold price series. In addition, the posterior error ratio is C = 0.1806, which is lower than the threshold value of 0.35, while the small error probability is P = 1.0000, which exceeds the required threshold of 0.95. According to the grey system accuracy criteria, these results classify the model as Grade 1, indicating excellent forecasting performance. Therefore, the estimated RLS-DGM(1,1) model can be considered accurate and reliable for the out-of-sample forecasting period.

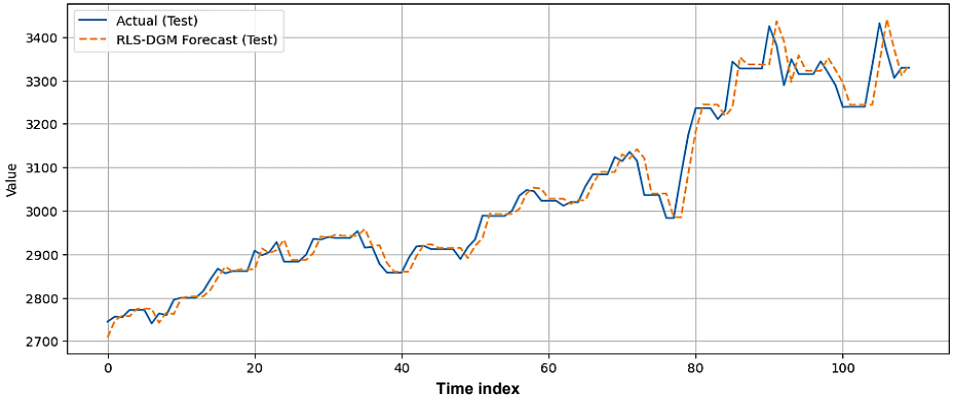


**Figure 4.3: Comparison between Actual and Forecasted Values using the RLS-DGM(1,1) Model**

Figure 4.3 shows the comparison between the actual gold prices and the one-step-ahead forecasts obtained from the RLS-DGM(1,1) model during the out-of-sample period. The forecasted series closely follows the general direction and fluctuations of the actual series, indicating that the model is able to capture the main dynamic behavior of the gold price data. Small deviations are observed at some turning points, which is expected in financial time series due to sudden market movements. Overall, the close alignment between the actual and forecasted values supports the strong predictive ability of the RLS-DGM(1,1) model.

### 4.2.2 Sliding-Window Discrete Grey Model (SW-DGM(1,1))

The Sliding-Window Discrete Grey Model (SW-DGM(1,1)) is an adaptive extension of the discrete grey model designed to handle non-stationary time series. It estimates the model parameters within a fixed-size moving window ($m$), using only the most recent observations and discarding older ones. This makes the model more responsive to short-term fluctuations and structural changes. In contrast, the rolling or recursive grey model updates parameters by gradually adding new observations while retaining past information. Thus, SW-DGM(1,1) focuses on recent data through a fixed window, whereas the rolling model relies on an expanding historical sample.

**Table 4.4**
**Performance Comparison of SW-DGM(1,1) Model under Different Window Sizes**

| Window Size ($m$) | RMSE | C (Posterior Error Ratio) | P (Small Error Probability) | Accuracy Grade | CV ($\beta_1$)% | CV ($\beta_2$)% |
|---|---|---|---|---|---|---|
| 10 | 53.5860 | 0.2772 | 0.955 | Level 1 | 2.83% | 17.41% |
| 15 | 54.942337 | 0.2998 | 0.865 | Level 2 | 3.2977 | 6.620 |
| 20 | 56.326095 | 0.4509 | 0.732 | Level 2 | 4.3001 | 7.670 |

Table 4.4 gives a comparative analysis of forecasting performance of SW-DGM(1,1) model with different sliding-window sizes. The results show that the window size $m$ = 10 is the best overall with the lowest RMSE value 53.5860, the lowest posterior error ratio C = 0.2772 and the highest small error probability P = 0.955. Moreover, this window size provides the lowest CV values for both $\beta_1$ and $\beta_2$, which means a better stability of parameters. Thus, $m$ = 10 was selected as the optimal sliding-window length for the SW-DGM(1,1) model.

**Table 4.5**
**The SW-DGM(1,1) Model Accuracy Evaluation Metrics Based on Statistical and Grey System Measures**

| Metric | RMSE | C (Posterior Error Ratio) | P (Small Error Probability) | Accuracy Grade |
|---|---|---|---|---|
| Value | 53.5860 | 0.2772 | 0.955 | Level 1 |
| Threshold | - | $\leq 0.35$ | $\geq 0.95$ | - |
| Assessment | - | Excellent ✓ | Excellent ✓ | Excellent ✓ |

Table 4.5 shows that the selected SW-DGM(1,1) model with window size $m$ =10 achieved RMSE = 53.5860, C = 0.2772, and P = 0.955. Since C is below 0.35 and P exceeds 0.95, the model is classified as Grade 1, indicating excellent forecasting performance according to grey-system accuracy criteria. Figure 4.4 shows that the forecasts follow the general upward trend of the actual gold prices, although deviations appear near sharp turning points, which supports the need for residual correction in the next stage.

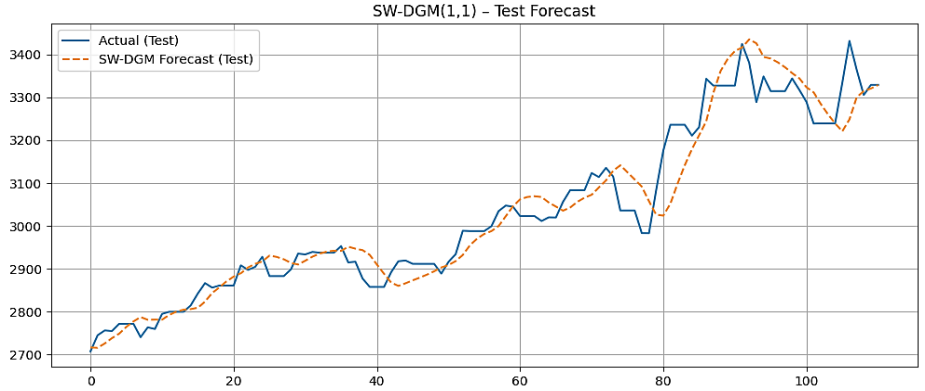


**Figure 4.4: Comparison between Actual and Forecasted Values using SW-DGM(1,1) Model**

The dynamic behavior of the calculated SW-DGM(1,1) parameters was investigated after the forecasting accuracy was assessed. The evolution of $\beta_1$ and $\beta_2$ provide valuable information on the model's stability and adaptability over time because the sliding-window technique re-estimates the model parameters at each window.

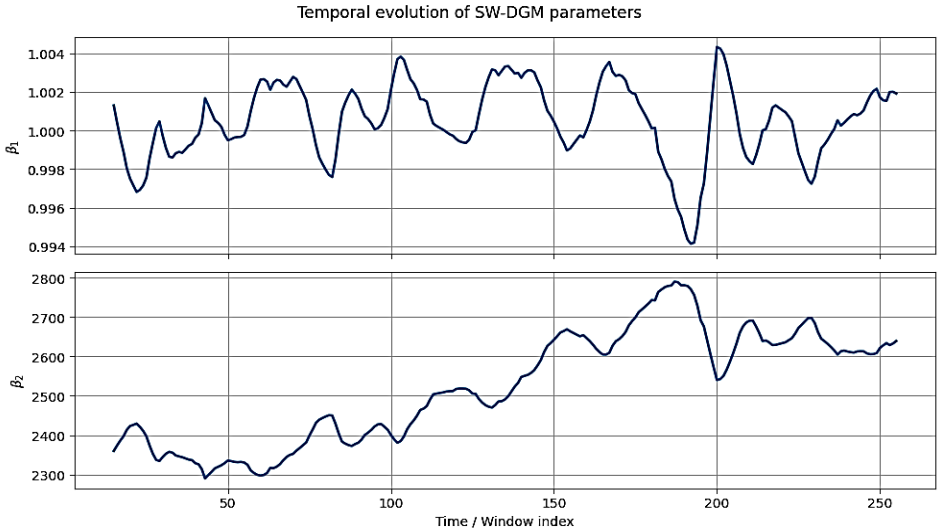


**Figure 4.5: Time-Varying Behavior of SW-DGM(1,1) Parameter Estimates (m=10)**

Figure 4.5 shows the time-varying behavior of the SW-DGM(1,1) parameter estimates using the selected window size of 10. As shown in Table 4.6, the parameter $\beta_1$ has a CV value of 2.83%, while $\beta_2$ has a CV value of 17.41%. These values indicate that the SW-DGM parameters are less stable than those of the RLS-DGM model, whose corresponding CV values are 0.1972% and 5.600%, respectively.

**Table 4.6**
**Descriptive Statistics of the Estimated SW-DGM(1,1) Parameters with a Sliding Window (10)**

| Parameter | Mean | Std | Min | Max | CV (%) |
|---|---|---|---|---|---|
| $\beta_1$ | 1.000480 | 0.001973 | 0.994120 | 1.004330 | 2.83% |
| $\beta_2$ | 2522.189062 | 141.254352 | 2290.008054 | 2789.840777 | 17.41% |

#### 4.2.2.1 Comparative Evaluation of SW-DGM and RLS-DGM Models

The RLS-DGM model has better forecasting accuracy and exhibits more stable parameters than the SW-DGM model, especially the near-constant behavior of β1 over sliding windows. This implies that the SW-DGM can capture the underlying trend of the series in a stable way. However, the relatively higher forecasting errors suggest that there is still systematic information in its residuals, which can include oscillations, irregularities or high frequency patterns. Residual correction is appropriate because it is designed to enhance the predictive accuracy of a structurally sound model without supplanting its original trend representation.

Table 4.7 reveals that in terms of forecasting accuracy, the RLS-DGM outperforms the SW-DGM with a smaller RMSE of 34.6273 in contrast to 53.5860 for the SW-DGM. It also obtained better grey-system indicators, $C = 0.1806$ and $P = 1.0000$, with both models still in Grade 1/Excellent classification. In terms of parameters stability, RLS-DGM is more stable because of lower CV values of β1 and β2. SW-DGM is selected for residual correction because its higher forecasting errors offer more scope for improvement with ARIMA-based residual modelling.

**Table 4.7**
**Comparative Summary of SW-DGM and RLS-DGM Performance Indicators**

| Criterion | SW-DGM | RLS-DGM | Preferred Model/ Interpretation |
|---|---|---|---|
| **RMSE** | 53.5860 | 34.6273 | RLS-DGM is more accurate |
| **Posterior Error Ratio (C)** | 0.2772 | 0.1806 | RLS-DGM is better |
| **Small Error Probability (P)** | 0.955 | 1.0000 | RLS-DGM is better |
| **Grey Accuracy Grade** | Level 1 / Excellent | Level 1/ Excellent | Both models are excellent |
| **CV($\beta_1$) %** | 2.83% | 0.1972% | RLS-DGM is more stable |
| **CV($\beta_2$) %** | 17.41% | 5.600% | RLS-DGM is more stable |

#### 4.2.2.2 Residual Diagnostics for the SW-DGM(1,1) Model

The residuals of the selected SW-DGM(1,1) model were diagnostically analyzed to determine their statistical characteristics prior to residual correction. The analysis contains stationarity, serial correlation and normality tests to check whether the residuals still contain systematic information that could be further modeled. The residual series is stationary as the ADF statistic (-4.67225) and p-value (9.485287 × 10^-5, less than 0.05) in table 4.8. So the null hypothesis of unit root is rejected, meaning that the residuals fluctuate around a constant mean and can be utilized for further residual modeling.

**Table 4.8**
**Diagnostic Tests of SW-DGM(1,1) Residuals**

| Test | Statistic | p-value | Decision | Interpretation |
|---|---|---|---|---|
| ADF | -4.67225 | 9.485287e-05 | Reject unit root | Stationary Residuals |
| Ljung–Box, lag 10 | 122.321987 | 1.708705e-21 | Reject no Autocorrelation | Serial Dependence Exists |
| Ljung–Box, lag 20 | 134.548647 | 5.435995e-19 | Reject no Autocorrelation | Serial Dependence Exists |
| Jarque–Bera | 27.1309 | 0.00000128 | Reject Normality | Non-Normal Residuals |

Figure 4.6 shows that several ACF and PACF coefficients are outside the confidence bounds and mostly at lower lags indicating the residuals are not purely random.

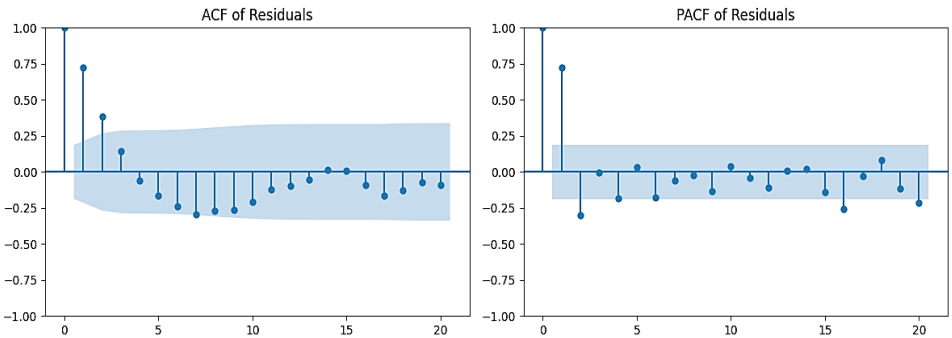


**Figure 4.6: ACF and PACF Plots of the Residual SW-DGM(1,1) Model**

This is confirmed by the Ljung-Box test in Table 4.8, where the p-values at lags 10 and 20 are very small. Therefore, the null hypothesis of no autocorrelation is rejected, indicating that the residuals exhibit a significant serial dependence.

**Table 4.9**
**Jarque–Bera Normality Test**

| Statistic | Skewness | Kurtosis | Jarque–Bera ($\chi^2$) | Significance (p-value) |
|---|---|---|---|---|
| Value | 0.7233 | 4.9561 | 27.1309 | 0.00000128 |

Table 4.9 shows the Jarque–Bera statistic of 27.1309 with a p-value of 0.00000128 which is less than 0.05. So the null hypothesis of normality is rejected and the residuals do not follow the normal distribution.

The Q-Q plot shown in Figure 4.7 visually supports this conclusion, since the residuals deviate from the reference line especially in the tails.

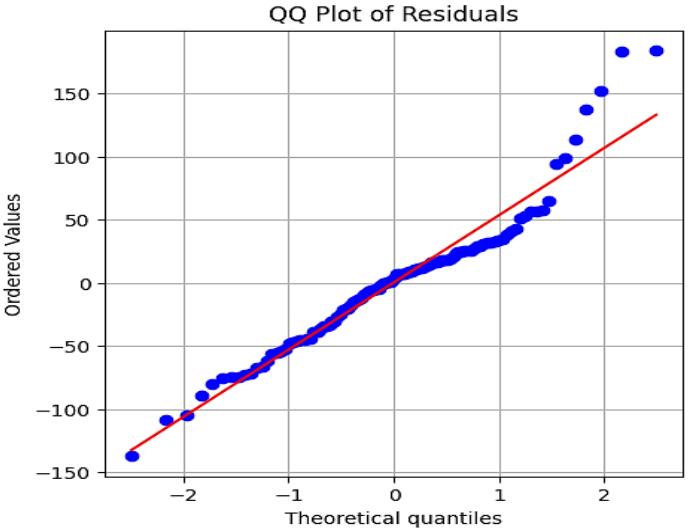


**Figure 4.7: Quantile–Quantile (Q–Q) Plot of Residuals for Normality Assessment**

**Table 4.10**
**Summary Statistics of Shock Characteristics in the Residual Series based on IQR Method**

| Shock Probability | Average Magnitude | Shock Maximum |
|---|---|---|
| 0.045456 | 158.6218 | 183.8453 |

In summary, the residual diagnostics show that the residuals of the SW-DGM(1,1) are stationary, but still exhibit significant autocorrelation and nonnormality. These results suggest that the residuals are not random noise and still contain systematic patterns. Therefore, it is statistically justified to use residual correction methods such as ARIMA.

#### 4.2.2.3 A Hybrid Sliding-Window Discrete Grey–ARIMA Model

The ARIMA-based correction modeled the linear temporal dependence that remained in the SW-DGM(1,1) residuals. Several candidate ARIMA models were estimated and the best specification was chosen according to the information criteria and the Ljung-Box diagnostic test. From Table 4.11 it is clear that the best residual correction model was selected as ARIMA(2,0,1). The model had the minimum AIC and SIC among the accepted candidates and successfully passed the Ljung-Box diagnostic test, indicating the adequacy of the model to represent the remaining linear dependence in the residual series.

**Table 4.11**
**ARIMA Model Selection Based on Information Criteria (AIC, SIC and Ljung–Box Test for Residual**

| Model | AIC | SIC | Ljung-Box (Q) | p-value | Decision |
|---|---|---|---|---|---|
| ARIMA(0,0,1) | 3160.086 | 3160.076 | 14.56 | 0.03 | Rejected |
| ARIMA(1,0,0) | 3000.086 | 3000.056 | 14.01 | 0.21 | Accepted |
| ARIMA(1,0 ,1) | 2320.077 | 2319.086 | 13.03 | 0.18 | Accepted |
| ARIMA(2, 0, 1) | 2160.086 | 2160.086 | 12.12 | 0.35 | Best |

The baseline SW-DGM(1,1) forecasts and the ARIMA adjusted forecasts are compared with the actual gold prices in Figure 4.8. Although there is no discernible visual improvement over the first SW-DGM forecasts, the corrected series exhibits the same general movement as the baseline model.

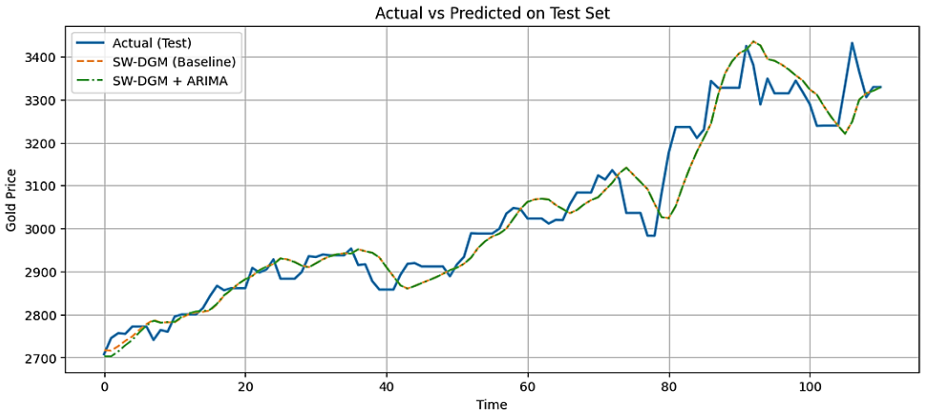


**Figure 4.8: The baseline SW-DGM(1,1) Forecasts and the ARIMA Adjusted Forecasts are compared with the Actual Gold Prices**

Table 4.12 provides visual evidence confirmation. Since MAPE, RMSE, and MAE slightly rise after correction, the ARIMA adjusted model does not outperform the baseline SW-DGM (1,1). Therefore, in this instance, the ARIMA-based adjustment is not effective.

**Table 4.12**
**Forecasting Performance Comparison between Baseline SW-DGM(1,1) and ARIMA-corrected SW-DGM(1,1)**

| Model | MAPE(%) | RMSE | MAE |
|---|---|---|---|
| SW-DGM(1,1) | 1.255440 | 53.586025 | 38.836676 |
| A Hybrid Sliding-Window Discrete Grey–ARIMA Model | 1.266730 | 53.770986 | 39.147447 |

This finding suggests that a linear ARIMA process may not adequately characterize the remaining residual structure. In order to capture periodic and multi-scale irregular patterns of the residual series.

### 4.3 Selected Non-Grey Benchmark Models

Two non-grey benchmark models an ARIMA-only model and an exponential smoothing model were estimated in order to provide a fair comparison with the grey forecasting models. In order to assess if the grey-based models perform better in forecasting, these models are compared to standard statistics.

#### 4.3.1 ARIMA-only Benchmark Model

To compare the forecasting performance of the suggested grey-based models, a non-grey benchmark ARIMA-only model was computed. ARIMA(0,1,0) was chosen for the gold price series with 256 observations based on the automatic selection process.

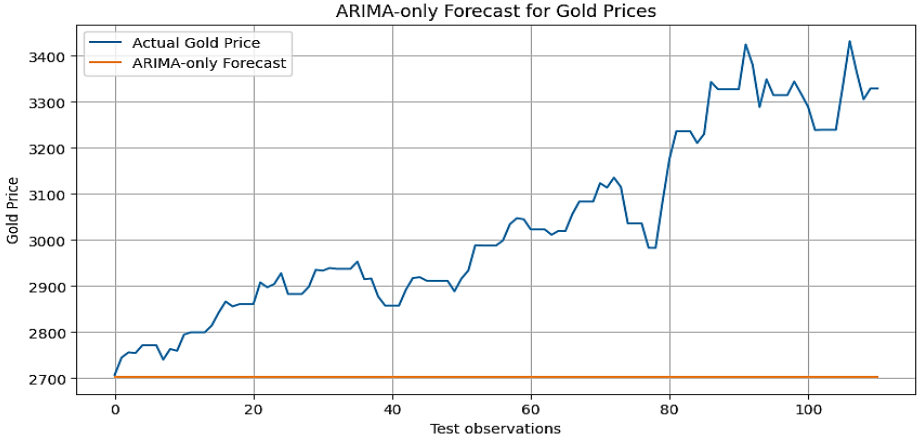


**Figure 4.9: Actual and Forecasted Gold Prices using the ARIMA-only Benchmark Model**

Figure 4.9 shows that the ARIMA-only benchmark model does not closely follow the actual fluctuations of the gold price series. This indicates its limited ability to capture the nonlinear and time-varying behavior of the series. Therefore, the ARIMA-only model is retained only as a non-grey benchmark for comparison with the proposed grey-based models.

### 4.3.2 Exponential Smoothing (ETS) Benchmark Model

For comparison with the grey-based forecasting models, the Exponential Smoothing model was estimated as a non-grey benchmark. The selected specification was ETS(AAN) with additive error, additive trend and no seasonality. Model was fitted on 256 observations and AIC = 2288.027, BIC = 2305.753 and HQIC = 2295.156. The model with only ETS obtained RMSE = 292.4161, MAE = 249.8799 and MAPE = 7.9676. These values indicate that the out-of-sample forecasting performance of the ETS benchmark is weaker in comparison to the grey-based models. Thus ETS is retained only as a non-grey benchmark for comparison. The limited forecasting performance of the ETS-only model, compared with the proposed grey-based models, can be visually confirmed from Figure 4.10 that it does not follow the actual fluctuations of the gold price series closely.

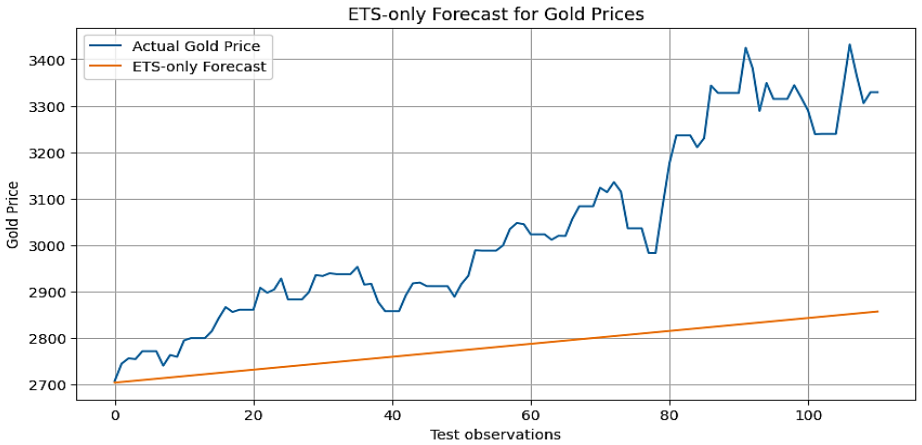


**Figure 4.10: Actual and Forecasted Gold Prices using the ETS-only Benchmark Model**

### 4.4 Comparative Evaluation of Forecasting Models

The grey-based models, residual-corrected models and non-grey benchmark models were estimated separately, and then an overall comparison was made using the same out-of-sample test period. The objective of this comparison is to assess the prediction accuracies of each model under the same conditions, and to observe whether the residual correction methods can improve the baseline SW-DGM(1,1) model. The comparison is mainly based on RMSE, MAE, and MAPE, with lower values indicating better forecasting performance.

**Table 4.13**
**Overall Forecasting Performance Comparison among the Estimated Models Based on the Out-of-Sample Test Period**

| Model | MAPE (%) | RMSE | MAE | C | P | Accuracy / Decision | Interpretation |
|---|---|---|---|---|---|---|---|
| RLS-DGM(1,1) | 0.953440 | 34.6273 | 27.536676 | 0.1806 | 1.0000 | Level 1 / Excellent | Best baseline grey accuracy according to RMSE, C, and P |
| SW-DGM(1,1) | 1.255440 | 53.586025 | 38.836676 | 0.2772 | 0.955 | Level 1 / Excellent | Selected baseline for residual correction |
| A Hybrid Sliding-Window Discrete Grey–ARIMA Model | 1.266730 | 53.770986 | 39.147447 | — | — | No improvement | ARIMA correction slightly increased the forecasting errors |
| ARIMA-only benchmark | 9.9676 | 372.4161 | 338.8799 | — | — | Weak benchmark | Used only as a non-grey benchmark for comparison |
| ETS-only benchmark | 7.9676 | 292.4161 | 249.8799 | — | — | Weak benchmark | Used only as a non-grey benchmark for comparison |

The comparison shows that RLS-DGM(1,1) has the best baseline grey forecasting performance with the lowest RMSE value of 34.6273 and excellent grey-system indicators. The SW-DGM(1,1) model also achieved Level 1 accuracy and was selected for residual correction, but the ARIMA-based correction did not improve its performance, with MAPE, RMSE and MAE slightly increasing. The ARIMA-only and ETS-only benchmark models resulted in much higher forecasting errors, confirming the superiority of the grey-based models in this empirical application.

## 5. CONCLUSIONS

The empirical results show that the grey-based forecasting models provide better predictive performance than the non-grey benchmark models. Among the baseline grey models, the RLS-DGM(1,1) achieved the highest forecasting accuracy, with the lowest RMSE value and excellent grey-system accuracy indicators. The SW-DGM(1,1) model also achieved Level 1 accuracy, but its errors were higher than those of RLS-DGM, which justified its selection for residual correction. However, the ARIMA-based residual correction did not improve the SW-DGM(1,1) performance, as the error measures slightly increased after correction. This failure may be attributed to the fact that the remaining residual structure was weak, irregular, and not adequately captured by a purely linear ARIMA process. Although the residuals showed some autocorrelation, they also exhibited non-normality, heavy-tailed behavior, and shock-like movements, indicating the presence of nonlinear or irregular components beyond the capability of ARIMA correction. In

contrast, the ARIMA-only and ETS-only benchmark models produced much larger forecasting errors, confirming the superiority of the grey-based models for forecasting gold prices in this study.

## 6. FUTURE RESEARCH

In future work, the proposed grey forecasting framework can be extended with nonlinear residual-correction methods such as machine learning and signal-decomposition techniques to better capture the short-term fluctuations in the gold prices. Future improvements may involve the selection of adaptive window-size, extension to other financial/commodity markets, inclusion of macroeconomic variables, and testing under multi-step-ahead forecasting horizons to evaluate generalizability, accuracy, efficiency, and interpretability.

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