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USING LINEAR WAVELETS IN ANALYZING THE GARCH MODEL WITH THE SIMULATION

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ABSTRACT

Noise and outliers greatly affect the estimation of parameters in the GARCH model, often resulting in large residuals. To address this problem, the present paper proposes to introduce linear wavelet shrinkage methods to denoise the data and reduce the impact of outliers. In particular, several first-order linear wavelets—Daubechies, Coiflets and Dmey—were used to remain in the same order as the classical GARCH model to be able to match the dataset. The thresholding level was determined using the Minimax method, and a soft thresholding rule was applied to obtain denoised data. These processed data were then used for estimating the GARCH model parameters, which were subsequently compared to classical GARCH models based on the original, unaltered data, without introducing any artificial contamination in either simulation studies or real-world datasets. The performance of the estimated models was evaluated using the Akaike and Bayesian information criteria, implemented through a MATLAB program developed for this purpose. The results indicate that GARCH models estimated with the proposed wavelet-based approach outperform classical GARCH models, with the Dmey wavelet demonstrating superior efficiency compared to the other linear wavelets considered.

KEYWORDS

Time Series, GARCH Model, Linear Wavelet, Threshold, Soft Rule.

1. INTRODUCTION

In recent times, there has been a great fascination with wave transforms as a modern filtering method, as their importance increases day after day through the increase in the number of applications and new fields that they introduce, which have been widely utilized in the field of time series in recent years, including autoregressive models conditional on the heterogeneity of variance (Al-Hashimi et al., 2025).

Studies have proven that they lead to significantly improved predictions instead of using raw data without processing, and it is not an exaggeration to say that wavelet transforms have brought about a real revolution in signal analysis because they have

overcome many of the problems that researchers encounter and provided practical solutions to them (Hayawi et al., 2025).

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is a statistical model commonly employed in finance and econometrics to examine and predict fluctuations in time series data. The model was developed by Tim Bollerslev in 1986 and has subsequently become a widely used instrument for modelling and forecasting monetary market volatility (Hayawi et al., 2025).

Wavelets are often used in signal processing, where signals are analyzed and decomposed into multiple frequencies.. The term "linear wavelet" may not have a precisely defined meaning on its own without additional context this technique is based on wavelet transformations and involves the transformation of a linear shrinkage function to the wavelet coefficients of a signal (Hayawi, 2020).

Many researchers have used the process of processing noisy time series data using the wavelet shrinkage method before estimating the parameters of various linear models such as AR, MA, ARMA, etc. In 2019, researchers (Taha and Mardin) proposed using wavelet shrinkage to reduce the noise of the time series dynamic model data. It was applied to the time series data of OPEC oil prices as an external variable and the quantity of oil supplied in the markets as an internal variable.

In 2025, (Ali et al.) he proposed a way to effectively de-noise, but there comes a point when performance deteriorates due to noise. Savitsky gulag and an MR filter were used for the correction method

In this research, it was proposed to employ linear wavelet shrinkage (Daubechies, Coiflets, and Dmey wavelet with minimax method and soft rule) to reduce the noise in the data of the GARCH model while maintaining the same model order (because of linear wavelet) and the conditions of the GARCH model and estimating the best model fitting the data based on Akaike and Bayesian Information Criteria with less noise, knowing that no noise is added to the simulated and real data.

2. THEORETICAL ASPECT

The theoretical aspect presented some basic concepts on the research subject from the statistical side, as shown in the following paragraphs.

2.1 Time Series

Time series data consists of a set of observations usually arranged with time. Time series analysis seeks to identify, model and predict data points using methodologies that consider the temporal correlation in the data. A trend, known as a trend cycle, represents the extended-term general rise or fall in the observed period. On the other hand, seasonality denotes variations in values over time linked to the calendar (Haydier et al. 2023).

Establishing whether a time series is stationary or non-stationary is crucial as an initial step in exploration before model selection, as certain models, like Autoregressive Moving Average (ARMA) models, assume stationary data. A time series is considered stationary when its joint probability distributions remain unchanged despite shifts in time dependency **(**Hayawi & Alsharabi, 2022).

2.1.1 ARCH Model

Homogeneity and no autocorrelation is usually an advantage of models using cross-sectional data, while autocorrelation of errors is an advantage of models using time series. The appearance of this dispersion, in some cases, reflect irregular fluctuation over time of the dependent variables. The Autoregressive Conditional Heteroscedasticity (ARCH) model gains its practical importance from the fact that the state of uncertainty, specifically the state of uncertainty associated with a particular investment, varies with time, and thus the uncertainty of prediction changes with changing periods and not only with changing forecast horizons. Researcher Robert Engle is a prominent American economist and a professor at the Stern School of Business at New York University is considered the first to propose ARCH models.

It stands for Autoregressive Conditional Heteroscedasticity. The first part, 'AR' (Autoregressive), refers to the modelling of the mean. In contrast, the other part is CH (Conditional Heteroscedasticity), which is the non-stationarity of the variance, meaning that the variance depends on the series' past and changes with time. This model means that the non-stationarity of the variance in the present period is linked to its instability in prior periods and can be written with the following mathematical model (Pemberton, 2011):

$$\sigma_t^2 = a_t + a_1 a_{t-1} + a_2 a_{t-2} + \cdots + a_p a_{t-p}, a_i > 0, i = 0,1,..,p \quad (1)$$

when $p = 1$ Based on the above equation, we obtain the mathematical equation for the ARCH model, which is as follows:

$$\sigma_t^2 = a_0 + a_1 a_{t-1} \quad (2)$$

It has the property of conditional on the heterogeneity defined by the following formula:

$$\sigma_y^2 = \frac{a_0}{1 - a_1} \quad (3)$$

2.1.2 GARCH Model

It is possible to expand the ARCH model, as researcher Bollerslev expanded the ARCH models with a more general model called the GARCH model, as the conditional variance in this model in period *t* does not depend only on the error squared in the preceding period additionally relies on the conditional variance in the previous period (Kichgaessner and Wolters, 2008):

$$\sigma_t^2 = a_t + a_1 a_{t-1} + a_2 a_{t-2} + \cdots + a_p a_{t-p} + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \cdots + \beta_q \sigma_{t-q}^2 \quad (4)$$

Equation (4) can be reformulated as follows:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i a_{t-1} + \sum_{j-1}^{q} \beta_i \sigma_{t-j}^2 \quad (5)$$

$$\sum_{i=1}^{p} a_i + \sum_{j=1}^{q} \beta_i \quad < 1 \quad (6)$$

2.1.3 Akaike Information Criterion (AIC)

In 1974, the researcher Akaike introduced an information standard (AIC). When time series models are fitted with k parameters to the time series data under study, the suitability of these models to the data is evaluated according to the AIC standard for each model, and the model that gives the lowest value for the standard is chosen. It can be written as follows (Yadkar and Muhammad, 2015; Ali et al., 2022):

$$AIC = nln\sigma_e^2 + 2k \tag{7}$$

where: n: Sample size, k: number of parameters, σ^2: Variance of the residuals of the fitted model.

2.1.4 Bayesian Information Criterion (BIC)

The model testing criterion is limited to a specific set of models. preferably the one with the lowest code for a function. The standard is defined by the following formula (Akaike, 1981):

$$BIC = nlog\sigma_e^2 - (n-k)ln\left(1 - \frac{k}{n}\right) + kln\, n + k\, ln\left(\left(\frac{\sigma_y^2}{\sigma_a^2} - 1k\right)\right) \tag{8}$$

where: σ_y^2 is time series variance.

2.2 Wavelets

Wavelet contraction is a method to remove noise and reduce noise in signals that can be reduced using wavelet shrinkage, a technique that involves thresholding wavelet coefficients. The wavelet deflation method was introduced for generalizing curve estimation problems. There are multiple compelling reasons to employ wavelet contraction in the estimation function. (Al-Hashimi et al., 2023).

2.2.1 Linear Wavelet

During the past years, the wavelet has been applied in many different fields, such as physics, statistics, and engineering. The wavelet works on signal processing as well as image processing. The wavelet is a function type used to decompose functions into components with varying frequencies, each of which is then analyzed separately. Mathematically, they are defined as real-valued functions on a complete real axis, oscillating regularly around zero.

A. The initial component is the scaling function, also referred to as the father function, which can be derived using the following formula:

$$\varphi(y) = \sum_{k=0}^{n} c(k)\theta(2y - k) \tag{9}$$

where $c(k)$: represents the Low-Pass Filter coefficients and equation (9) is called the Dilation equation.

B. The next part is the Wavelet Function, also referred to as the mather Function, which can be obtained from the following formula:

$$W(y) = \sum_{k=0}^{n} d(k)\theta(2y - k) \tag{10}$$

where $d(k)$: represent the coefficients of the High-Pass Filter, and the above equation is called the Wavelet equation (Ali & Saleh, 2021).

There are types of Wavelets transforms that are suitable for various practical applications, including:

A. Daubechies Wavelet

In the year 1992, researcher Ingrid Daubechies (Db), who is famous for her work with wavelets, named this wavelet after her. It is generated from a group of wavelets to improve the properties of a frequency field. One of the features of this wavelet is the smoothness is achieved by employing the minimum possible number of parameters, as ensured by the DN criterion.

Let the measurement function$f(y)$ is (Daubechies, 1992; Hayawi et al., 2022):

$$f(y) = \sum_{k=-\infty}^{\infty} c(k)f(2y - k) \tag{11}$$

where $c(k)$ is Low-pass filter coefficients (Burrus et al. 1998).

B. Coiflets Wavelet

The Coiflets Wavelet (Coif) are discrete wavelets designed by Ingrid Daubechies, at the request of Ronald Coifman, to have Scaling functions characterized by vanishing moments. The wavelet is closely symmetric, and their wavelet functions have $\frac{N}{3}$ vanishing moments and scaling functions $\frac{N}{3-1}$ and has been used in many applications using Calderón–Zygmund operators (Ali, 2025)

C. Dmey Wavelet

The Dmey wavelet is part of the Daubechies wavelet family and is commonly used in signal processing compression. Daubechies wavelets, like Dmey, are known to provide good balance of time and frequency localization. (Ali & Mardin, 2019).

The Dmey wavelet has a special linear phase that is useful when phase preservation is required. It is often used in situations where linear phase characteristics are essential, for example, when filter banks are needed to process signals.

2.2.2 Threshold Method:

Different threshold methods can be used to estimate threshold at the first level of the Maximal overlap discrete wavelet transform (MODWT) coefficients., where (Mahmood et al., 2025):

$$\tilde{R}(F) = inf_{\tilde{f}}\, sup_{f \in \tilde{R}(F)} R\left(\tilde{f}, f\right) \tag{12}$$

where:

$$R(\tilde{f},f)=\frac{1}{n}\sum_{i=1}^{n}E[\tilde{f},f]^2 \tag{13}$$

where $f=f(x_i)$ and $\tilde{f}=\tilde{f}(x_i)$, where represents the vectors of true and estimated sample values. This method relies on minimizing the mean square error to the greatest extent possible. without excessive homogeneity.

MODWT is a type of wavelet transform in which the data is decomposed into various levels of detail through wavelet filters. These coefficients represent information from different frequency bands of a signal. The MODWT coefficients at the first level represent finer details, where the most important features of the signal are usually retained.

Threshold $\tilde{f}=\tilde{f}(x_i)$ is the value we want to estimate. This threshold is used to select which wavelet coefficients are "important" and which wavelet coefficients are "noise or outliers". Lower bound coefficients are known to be noise, and are either zero or modified. The goal is to find the threshold that minimizes error without over-smoothing the data.

The MODWT thresholding method is used to remove noise or outliers in data by estimating a optimal threshold which minimizes the difference between the true and denoised signal. It achieves the minimization of outliers or noise, while still retaining important features while maintaining data preservation through a risk function.

2.2.3 Thresholding Rule (Taha et al., 2025):

The soft threshold is:

$$W(n)^{(s)}=sign\{Wn\}\{|Wn|-\delta\}_{+} \tag{14}$$

where:

$$Sign\{W_n\}=\begin{cases}+1 & if \quad Wn>0\\ -1 & if \quad Wn<0\end{cases}$$

and

$$|W_n|-\delta=\begin{cases}-\delta & if \quad (|Wn|-\delta)\geq 0\\ 0 & if \quad (|Wn|-\delta)<0\end{cases}$$

MODWT coefficients are represented using the retain or discard rule. If they are larger than the threshold, they are zeroed out, and if not, they are kept. The soft thresholding approach cuts the impact of small wavelet coefficients by shrinking them and excluding those below the threshold.

This thresholding procedure is particularly useful when using data compression and denoising because it helps minimize noise or outliers while retaining important data characteristics.

2.3 Proposed Method:

- The proposed method consists of denoising the GARCH model data in order to estimate parameters using wavelet shrinking, such as Using one of the linear wavelets (Db (1), Coif (1), and (Dmey (1)), with time series data to obtain maximal overlap discrete wavelet transformation coefficients.

- Estimate the thresholding level (γ) using the Minimax thresholding from the first level for Maximal overlap discrete wavelet transformation coefficients.
- Use the soft threshold rule on the maximal overlap discrete wavelet transformation coefficients at the estimated threshold level (γ) by keeping or killing (converting them to zero) to obtain modified coefficients with little noise.
  Finding the inverse of the maximal overlap discrete wavelet transformation coefficients to denoise the data while preserving 99.97% of the data energy.
- Using de-noise time series data to estimate GARCH model parameters.

3. THE APPLIED ASPECT

The applied aspect dealt with two types of data: the first type represents simulated data, while the second type represents real data.

3.1 Simulation Study

Data generation was repeated (1000) times in simulating the GARCH models for several different sample sizes (100, 300, 500, and 1000), and for several different parameter values of GARCH (1:3) and ARCH {1:3}, and the GARCH models were estimated using the classical method and the proposed method using three linear wavelets, specifically (Daubechies, Coiflets, and Dmey) estimated the threshold level using the Minimax method and applied the Soft threshold rule to obtain de-noise data that will be used in estimating GARCH models. Calculate the AIC and BIC criterion average for all models and summaries the results in the following tables for several cases (The values of the model parameters are chosen arbitrarily, with the condition that the model is stable less than one or their sum is less than one):

Case 1. When Constant = 5, GARCH = 0.4, ARCH = 0.5.

Table 1
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (1, 1))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	614.5369	431.8633	413.5603	**402.8534**
	BIC	622.3524	439.6789	421.3758	**410.6689**
300	AIC	1836.0179	1061.7826	946.3853	**911.2647**
	BIC	1847.1292	1072.8940	957.4966	**922.3761**
500	AIC	3054.6912	1633.0494	1392.3367	**1328.2714**
	BIC	3067.3350	1645.6932	1404.9805	**1340.9152**
1000	AIC	6105.8594	3026.9512	2348.6226	**2278.5140**
	BIC	6120.5826	3041.6744	2363.3459	**2293.2373**

Case 2. When Constant = 5, GARCH = 0.4, and ARCH = {0.25 0.25}

Table 2
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (1, 2))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	624.21	428.18	413.89	**403.82**
	BIC	634.63	438.60	424.31	**414.24**
300	AIC	1863.3	1008.8	914.37	**882.63**
	BIC	1878.1	1023.6	929.19	**897.44**
500	AIC	3098.1	1520.8	1317.0	**1257.8**
	BIC	3115.0	1537.6	1333.8	**1274.6**
1000	AIC	6192.2	2750.5	2182.5	**2098.1**
	BIC	6211.8	2770.1	2202.2	**2117.7**

Case 3. When Constant = 5, GARCH = {0.25 0.25}, and ARCH = 0.4

Table 3
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (2, 1))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	638.27	443.65	426.53	**412.80**
	BIC	648.69	454.07	436.95	**423.22**
300	AIC	1906.1	1056.5	948.14	**904.29**
	BIC	1920.9	1071.3	962.96	**919.10**
500	AIC	3170.6	1601.3	1361.9	**1283.6**
	BIC	3187.4	1618.2	1378.8	**1300.5**
1000	AIC	6338.6	2912.3	2251.7	**2124.6**
	BIC	6358.2	2931.9	2271.4	**2144.2**

Case 4. When Constant = 5, GARCH = {0.20 0.20}, and ARCH = {0.20 0.20}

Table 4
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (2, 2))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	590.91	384.35	371.10	**355.45**
	BIC	603.94	397.37	384.13	**368.48**
300	AIC	1763.2	830.87	741.31	**689.53**
	BIC	1781.7	849.39	759.83	**708.05**
500	AIC	2932.8	1180.4	987.13	**892.64**
	BIC	2953.9	1201.4	1008.2	**913.71**
1000	AIC	5861.4	1977.6	1425.9	**1262.6**
	BIC	5885.9	2002.1	1450.4	**1287.2**

Case 5. When Constant = 5, GARCH = {0.20 0.20}, and ARCH = {0.15 0.15 0.15}

Table 5
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (2, 3))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	614.58	405.52	393.11	**377.77**
	BIC	630.21	421.15	408.74	**393.41**
300	AIC	1829.2	887.73	805.49	**751.47**
	BIC	1851.5	909.96	827.71	**773.69**
500	AIC	3041.6	1275.8	1103.6	**998.52**
	BIC	3066.8	1301.1	1128.9	**1023.8**
1000	AIC	6077.3	2170.2	1655.9	**1511.3**
	BIC	6106.8	2199.6	1685.3	**1540.8**

Case 6. When Constant = 5, GARCH = {0.15 0.15 0.15}, and ARCH = {0.20 0.20}

Table 6
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (3, 2))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	618.93	411.77	396.77	**380.87**
	BIC	634.56	427.40	412.40	**396.50**
300	AIC	1842.1	907.52	819.15	**726.82**
	BIC	1864.3	929.74	841.37	**785.04**
500	AIC	3062.5	1311.7	1122.4	**1007.3**
	BIC	3087.8	1337.0	1147.7	**1032.6**
1000	AIC	6119.4	2228.2	1684.3	**1518.9**
	BIC	6148.8	2257.6	1713.7	**1548.3**

Case 7. When Constant = 5, GARCH = {0.15 0.15 0.15}, and ARCH = {0.15 0.15 0.15}

Table 7
The Criteria Average of the Estimated Models for the Classical and Proposed Method (GARCH (3, 3))

Sample Size	Criterion	Classical	Db1	Coif	Dmey
100	AIC	649.99	440.49	427.23	**411.97**
	BIC	668.23	458.73	445.47	**430.20**
300	AIC	1929.8	984.42	904.53	**851.20**
	BIC	1955.8	1010.4	930.46	**877.13**
500	AIC	3206.5	1440.3	1274.2	**1166.6**
	BIC	3236.0	1469.8	1303.7	**1196.1**
1000	AIC	6404.7	2511.8	2011.4	**1861.0**
	BIC	6439.0	2546.1	2045.7	**1895.4**

Tables (1-7) show that the proposed method was better than the classical method for all simulation cases because the model criteria values (AIC and BIC) for the proposed method are less than the classical method. It was also shown that the Dmey linear wavelet was better compared to other linear wavelets (Daubechies and Coiflets). This means that the proposed method for reducing data noise was highly efficient and had better parameter values than the traditional method (without wavelet shrinkage), even without adding noise to the data due to wavelet estimation with the shrinkage used to treat the noise.

3.2 Real Data

This example shows how to specify and fit classical and proposed GARCH model to foreign exchange rate returns (daily Deutschmark/British pound foreign exchange rates) observed from 31-December-1979 to 24-Jan-1993 (4774 observations) in Figure 1.

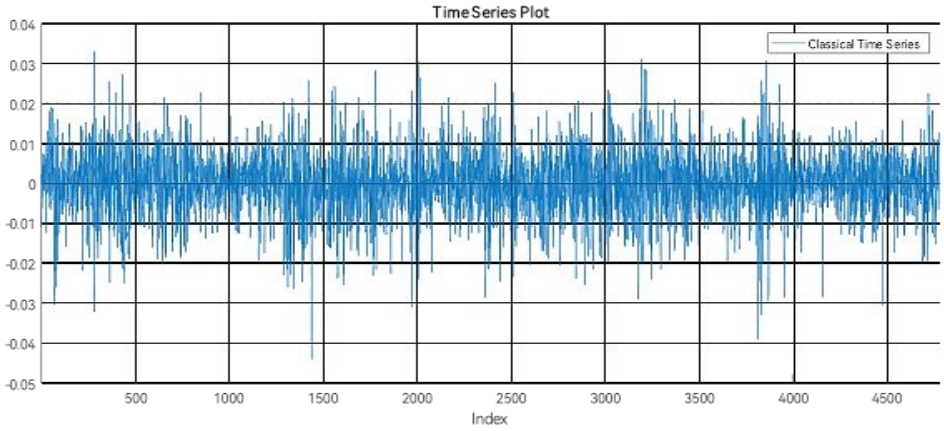

Figure 1: Time Series for Real Data

The estimated GARCH (1,1) model with the test results of the estimated parameters and some criteria for the efficiency of the estimated model are summarized in Tables 8 and 9.

Table 8
GARCH (1, 1) Conditional Variance Model (Real Data)

Parameter	Value	Standard Error	Z-Statistic	p-value
Constant	0.0000016	$4.3753e^{(-0.07)}$	3.7241	0.0002
GARCH {1}	0.9137300	0.0068843	132.7263	0.0000
ARCH {1}	0.0585120	0.0050003	11.7018	0.0000

Table 9
The Model Criteria (Real Data)

The Criteria	The Value
AIC	-33328.6682
BIC	-33309.2566

The basic GARCH (1, 1) results are given in Table 8. The coefficients sum up to a number less than one, which is required to have a mean reverting variance process. Standard Errors, Z-statistics, and p-values complete the table, which indicates the significance of the estimated model parameters because the p-values are less than the significance level (005). While Table 9 shows the values of the estimated model efficiency criteria. The conditional variances and standard errors for the estimated GARCH (1, 1) model are shown in Figure 2.

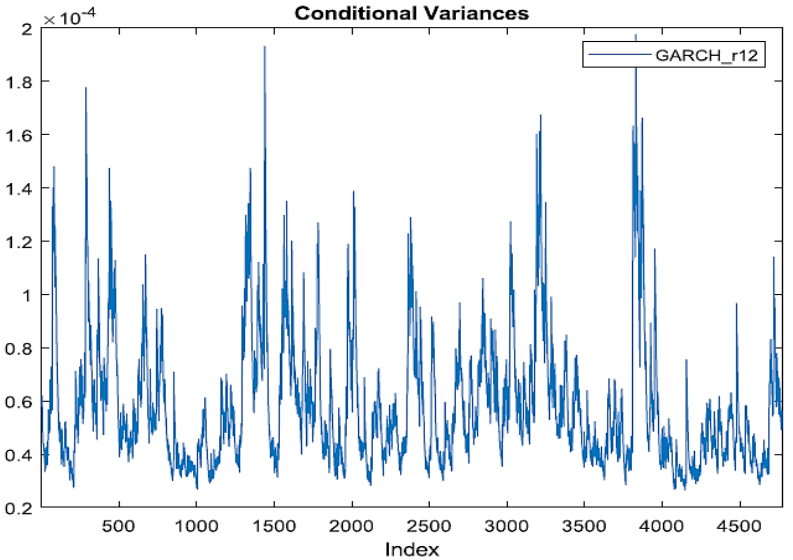

Figure 2: Plot the Fit of Model GARCH (1, 1) for Real Data

The square residuals ACF (Figure 3) show non-significant autocorrelation in the squared residual series.

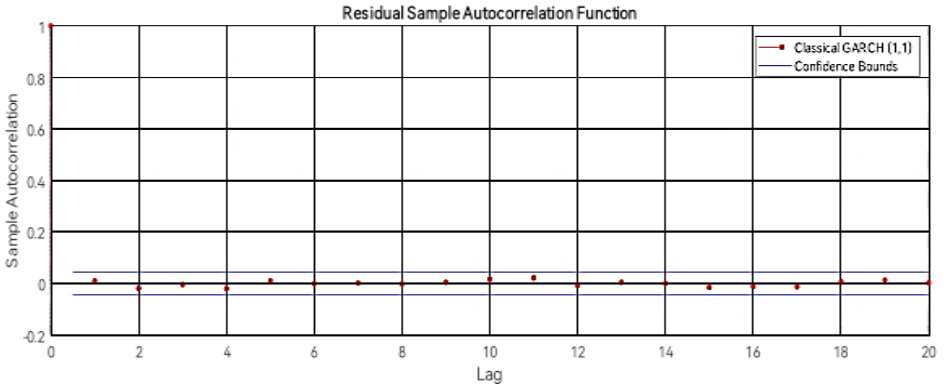

Figure 3: The Plot of the Square Residuals ACF of Model GARCH (1, 1) for Real Data

This indicates good fitness for the estimated model GARCH (1, 1) for the real data. The discrete wavelet transformation of the first-order (linear) Dmey wavelet was used to transform the data generated for the real data, estimate the threshold level using the minimax method, then apply the soft threshold rule, and finally calculate the inverse of the discrete wavelet transformation to obtain de-noise data Y-w, as in Figure 4 which shows that heterogeneity of random error variance still exists for de-noise data, which indicates a decrease in variance over time.

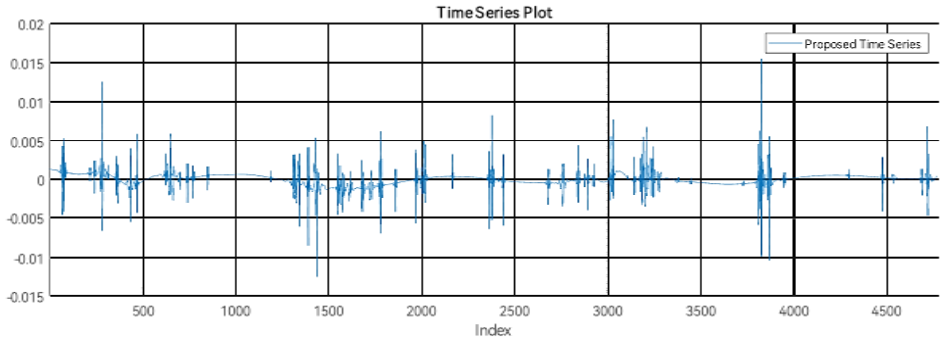

Figure 4: Time Series for Real Data after De-Noise

The estimated GARCH (1, 1) model with the test results of the estimated parameters and some criteria for the efficiency of the estimated model are summarised in Tables 10-11.

Table 10
GARCH (1, 1) Conditional Variance Model (De-Noise Real Data)

Parameter	Value	Standard Error	Z-Statistic	p-value
Constant	$4.3753e^{(-0.07)}$	$4.3753e^{(-0.07)}$	3.6773	0.00023575
GARCH {1}	0.15471	0.0052786	29.3097	$4.3753e^{(-189)}$
ARCH {1}	0.74407	0.027554	27.0039	$4.3753e^{(-160)}$

Table 11
The Model Criteria (De-Noise Real Data)

The Criteria	The Value
AIC	-56648.3835
BIC	-56628.9720

The basic GARCH (1, 1) results are given in Table 10. The coefficients sum up to a number less than one, which is required to have a mean reverting variance process (conditional on the heterogeneity of variance for the GARCH model). Standard Errors, Z-statistics, and p-values complete the table, which indicates the significance of the estimated model parameters. Since the p-values are less than the significance level (005), this means that the proposed method maintained the significance of the estimated parameters of the GARCH model. Table 11 shows the estimated model efficiency criteria (AIC and BIC), which are less than the criteria values in the classical model. The proposed method provides a more appropriate (goodness of fit) model for the data than the traditional method estimated model. Using AIC and BIC, the proposed GARCH (1, 1) model has slightly smaller (more negative) AIC and BIC values, therefore, the proposed GARCH (1, 1) model is the preferred model according to these criteria. The conditional variances and standard errors for the proposed estimated GARCH (1, 1) model are shown in Figure 5.

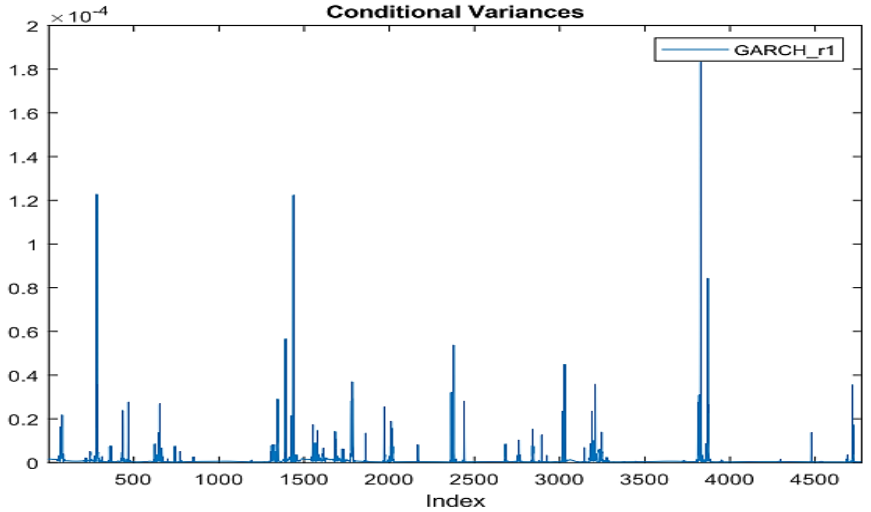

Figure 5: Plot the Fit of the Proposed Model GARCH (1, 1) for Real Data

The square residuals ACF (Figure 6) show non-significant autocorrelation in the squared residual series.

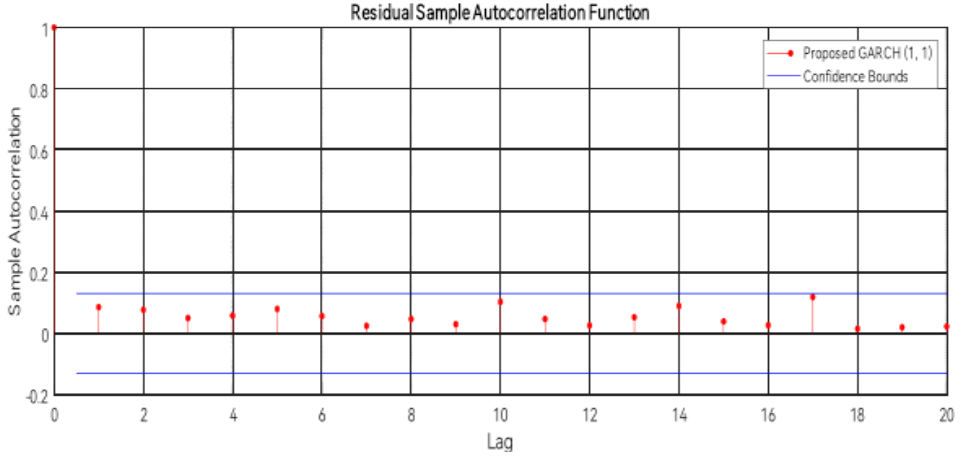

Figure 6: The Plot of the Square Residuals ACF of Model Proposed GARCH (1, 1) for Real Data

This result demonstrates the strong goodness of fit of the proposed GARCH (1,1) model when applied to both the original and denoised datasets, indicating its effectiveness in capturing the inherent volatility structure of the time series. The consistency of the model performance across both data forms suggests that the denoising procedure enhances parameter estimation without altering the fundamental volatility characteristics. Furthermore, the outcomes of the first simulation experiment confirm the presence of

conditional heteroscedasticity, as evidenced by the persistence of volatility clustering and the dependence of current conditional variance on past shocks and variances. These findings reinforce the validity of employing the GARCH (1,1) model and support its robustness as an appropriate tool for modeling heteroscedastic behavior in noisy financial time series.

4. CONCLUSION & RECOMMENDATIONS

4.1 Conclusions

1. The proposed methods consistently outperformed the classical method in all simulation cases. This was evidenced by better performance in model selection criteria (AIC and BIC), indicating that the proposed methods provided more accurate and efficient modeling, The simulation results show that the proposed methods consistently outperform the classical approach across all sample sizes and ARCH/GARCH parameter settings. This superiority is evidenced by lower AIC and BIC values, indicating a better balance between model fit and parsimony. The robustness of the proposed methods under different simulation conditions highlights their stability and effectiveness in reducing noise and improving parameter estimation, making them a reliable alternative to classical volatility modeling techniques.
2. The Linear Demy wavelet showed promising results when compared to other linear wavelets, such as Daubechies and Coiflets. This suggests that the Linear Demy wavelet could be a competitive or superior alternative for time series modeling and signal processing tasks.
3. The combination of the Linear Demy Wavelet and GARCH (1,1) model was the best-performing model for real data. Based on AIC and BIC, this model effectively captured the dynamics of real data and balanced accuracy with model complexity.

4.2 Recommendations

1. Use the proposed method even if there is no contamination in the data.
2. Conduct future studies using higher-order (non-linear) wavelets.
3. Comparison between GARCH and ARMA models after using wavelets.

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