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EMPLOYING THE MINIMUM AVERAGE VARIANCE ESTIMATOR VIA RECIPROCAL ADAPTIVE BRIDGE FOR EFFICIENT FEATURE SELECTION

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ABSTRACT

This paper introduces a novel method for feature selection in statistical models, combining the Minimum Average Variance Estimator (MAVE) with the Reciprocal Adaptive Bridge (RAB) penalty. The primary goal is to enhance variable selection in problems involving a large number of predictors while maintaining variance reduction. The proposed method leverages the benefits of Sufficient Dimension Reduction (SDR) along with the flexibility of the RAB penalty, allowing for dimension reduction with improved selectivity and accuracy in feature selection. Through a series of simulation studies and real-world application to medical data (Diabetes Dataset), the proposed method (SMAVE-RAB) is compared with other penalty methods such as Lasso and ALasso. Results from both simulation and application show that SMAVE-RAB outperforms the alternative methods in terms of Mean of Mean Squared Error (MMSE) and Standard Deviation (SD), highlighting its efficiency, accuracy, and stability when handling large datasets, particularly in high-dimensional medical settings.

KEYWORDS

Feature Selection, Sufficient Dimension Reduction, Reciprocal Adaptive Bridge, Minimum Average Variance Estimator.

1. INTRODUCTION

In high-dimensional statistical models, variable selection and model estimation present significant challenges due to the complexity and interdependence among predictors. Efficient variable selection is crucial in ensuring that models are both interpretable and accurate, especially when working with large datasets that contain numerous predictors. To address these challenges, various penalty-based methods, such as Lasso and ALasso, have been developed, aiming to reduce variance and select only the most important predictors. However, these methods often encounter limitations in

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balancing selective accuracy, model stability, and computational efficiency, particularly in real-world applications.

In this context, this paper proposes a novel approach that integrates the Minimum Average Variance Estimator (MAVE) with the Reciprocal Adaptive Bridge (RAB) penalty. This method combines MAVE's efficiency in dimension reduction with RAB's flexible penalty, enabling the retention of only the most relevant predictors while promoting sparsity. The proposed method, SMAVE-RAB, is tested not only through comprehensive simulation studies but also through a real-world application on medical data (Diabetes Dataset), where the challenges of high-dimensional feature selection are critical for accurate disease progression predictions.

The effectiveness of SMAVE-RAB is evaluated through a series of simulations and practical implementation, where it is tested against SMAVE-Lasso and SMAVE-ALasso on datasets with varying sample sizes. The results consistently demonstrate the superiority of SMAVE-RAB in terms of both accuracy and stability, making it a promising tool for high-dimensional feature selection, especially in fields like healthcare where precision and reliability are vital.

2. METHODOLOGY

2.1 Reciprocal Adaptive Bridge

The Bridge Method (Frank & Friedman 1993) is a general form of regularized regression that introduces flexibility in how the coefficients are penalized, providing a smooth transition between different types of penalization methods (such as Lasso and Ridge) (Schott, 2005).

The general optimization problem for the Bridge method can be written as:

$$(\hat{\beta}) = \min_{\beta} \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \sum_{j=1}^{p} |\beta_j|^{\alpha} \quad (1)$$

where $\hat{\beta}$ are the estimated regression coefficients. $(y_i - x_i'\beta)^2$ is the usual squared loss function measuring the error between observed values y_i and the predicted values $x_i'\beta$. $\lambda > 0$ is the regularization parameter that controls the level of penalty applied to the model. Higher values of λ will lead to more regularization and smaller estimated coefficients. $\alpha > 0$ is the concavity parameter, determining the form of the regularization applied to the coefficients.

The Adaptive Bridge method is an extension of the Bridge method, incorporating adaptive weights into the penalty to further enhance flexibility in selecting important predictors (Huang & Zhang, 2008). The goal is to improve model sparsity by allowing different levels of penalization for different coefficients based on their importance.

The optimization problem for the Adaptive Bridge method is typically written as:

$$(\hat{\beta}) = \min_{\beta} \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \sum_{j=1}^{p} w_j |\beta_j|^{\alpha} \quad (2)$$

w_j are the adaptive weights, applied to each coefficient $\hat{\beta}$, which vary based on the importance of the predictors.

The Reciprocal Adaptive Bridge method is an extension of the Adaptive Bridge method that introduces reciprocal weights into the penalty term. These reciprocal weights provide an additional layer of adaptiveness, making it even more effective in variable selection and shrinkage by giving higher penalties to smaller coefficients and lower penalties to larger ones. This method enhances the ability to retain important predictors while penalizing less important ones more aggressively.

The optimization problem for the Reciprocal Adaptive Bridge method is written as:

$$(\hat{\beta}) = \min_{\beta} \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \sum_{j=1}^{p} \frac{\mathrm{w}_j}{|\beta_j|^{\alpha}} \tag{3}$$

where λ is the regularization parameter. α controls the degree of shrinkage for large versus small coefficients. w_j are adaptive weights that adjust the penalty for each coefficient β_j, allowing more flexibility in how different predictors are regularized.

2.2 Sufficient Dimension Reduction and Minimum Average Variance Estimator

In the given problem, the focus is on Sufficient Dimension Reduction (SDR) using the Minimum Average Variance Estimator (MAVE). This approach aims to reduce the dimensionality of the predictor space x, while preserving the important information regarding the relationship between x and the response variable y.

The model is defined as follows:

$$y = f(\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_p) + \varepsilon$$

where y is the outcome variable, $x \in \mathrm{R}^p$ is a $p \times 1$ vector of predictors, ε is the error term.

Assumptions:

- $E(y|x) = f(\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_p)$ represents the conditional mean of y given x.
- $E(\varepsilon|x) = 0$, meaning that the error has a zero mean given x.

Sufficient Dimension Reduction (SDR) seeks to find a subspace $S \subseteq \mathrm{R}^p$ such that the response variable y is conditionally independent of $E(y|x)$, given the projection of x onto this subspace [Cook (1998) Buja & Eyuboglu (1992)]:

$$y \perp\!\!\!\perp E(y|x) | P_s x \tag{4}$$

where P_s is a projection operator onto the subspace S.

Let $d = \dim(S)$ and $\mathrm{B} = (\beta_1, \beta_2, \dots, \beta_d)$ be the basis for the subspace S. The idea is that the predictors x can be replaced by the lower-dimensional projections $x^T\beta_1, x^T\beta_2, \dots, x^T\beta_d$ where $d \leq p$. This reduces the dimensionality of the predictor space without losing essential information about y.

It has been shown that $S_{E(y|x)}$ is the intersection of all subspaces that satisfy the conditional independence in Equation (4).

Minimum Average Variance Estimator (MAVE) is an efficient method for estimating $S_{E(y|\boldsymbol{x})}$, the subspace that satisfies the SDR condition. The goal of MAVE is to find the matrix B that minimizes the following criterion:

$$\min_{\boldsymbol{B}}\{E[y - E(y|x^T B)]^2\},$$

subject to the constraint $B^T B = I_d$, where I_d is the identity matrix of size d.

The conditional variance given $x^T B$ is defined as:

$$\sigma_B^2(x^T B) = E[\{y - E(y|x^T B)\}^2 \,|x^T B].$$

The main objective is then to minimize the expected variance across all x:

$$\min_B E[y - E(y|x^T B)]^2 = \min_B E\{\sigma_B^2(x^T B)\}.$$

For any given point $\boldsymbol{x}_0$, the conditional variance $\sigma_B^2(x^T B)$ can be locally approximated by the following expression:

$$\sigma_B^2(x_0^T B) \approx \sum_{i=1}^{n}\{y_i - E(y_i|x_i^T B)\}^2\, \omega_{i0}$$

or equivalently,

$$\sigma_B^2(x_0^T B) \approx \sum_{i=1}^{n}[y_i - \{a_0 + (x_i - x_0)^T B\, b_0\}]^2\, \omega_{i0},$$

where $\omega_{i0} \geq 0$ are the kernel weights, and $\sum_{i=1}^{n}\omega_{i0} = 1$. a_0 and b_0 are local linear approximations.

To estimate *B*, the following optimization problem is solved:

$$\min_{B:\, B^T B = I_d}\left(\sum_{j=1}^{n}\sum_{i=1}^{n}\left[y_i - \left\{a_j + \left(x_i - x_j\right)^T B\, b_j\right\}\right]^2 \omega_{ij}\right). \tag{5}$$

Here a_j and b_j are coefficients estimated for each observation j, ω_{ij} are the kernel weights for the i-th observation relative to the j-th observation.

This iterative procedure updates the basis matrix B to minimize the conditional variance while preserving the dimensionality reduction constraint $\mathrm{B}^T\mathrm{B} = \mathrm{I}_d$.

2.3 The Sparse MAVE with Reciprocal Adaptive Bridge Penalty

The Sparse MAVE (SMAVE) method is an extension of the Minimum Average Variance Estimator (MAVE), introduced to achieve sufficient dimension reduction (SDR) while simultaneously selecting relevant predictors. The Reciprocal Adaptive Bridge (RAB) penalty adds further refinement by allowing differential penalization on coefficients, encouraging sparsity in high-dimensional data. Combining these two methods creates a powerful framework for dimension reduction and variable selection in nonlinear regression problems, particularly in settings where the number of predictors is large.

The combined optimization problem for Sparse MAVE with Reciprocal Adaptive Bridge Penalty (SMAVE-RAB) can be formulated as:

$$\min_{B: B^T B = I_d} \sum_{j=1}^{n}\sum_{i=1}^{n}\left[y_i - \left\{a_j + \left(x_i - x_j\right)^T B\, b_j\right\}\right]^2 \omega_{ij} + \lambda \sum_{j=1}^{p} \frac{w_j}{|\beta_j|^{\alpha}}. \qquad (6)$$

where $\sum_{j=1}^{n}\sum_{i=1}^{n}\left[y_i - \left\{a_j + \left(\mathrm{x}_i - \mathrm{x}_j\right)^T \mathrm{B}\, b_j\right\}\right]^2 \omega_{ij}$ is the MAVE loss function that minimizes the average variance.

$\lambda \sum_{j=1}^{p} \frac{w_j}{|\beta_j|^{\alpha}}$ is the **Reciprocal Adaptive Bridge Penalty**.

Although the current work does not provide a full theoretical proof for the convergence or oracle properties of the proposed SMAVE-RAB method, its construction builds upon well-established regularization frameworks. The Reciprocal Adaptive Bridge (RAB) penalty extends the Adaptive Bridge approach, which has been shown to enjoy oracle properties and variable selection consistency under suitable conditions, as discussed by Fan and Li (2001) and Huang et al. (2008). These properties imply that the estimator can correctly identify the true non-zero coefficients with high probability and achieve asymptotic normality for large samples. Moreover, by integrating RAB into the MAVE framework, the proposed SMAVE-RAB maintains the dimension reduction structure while benefiting from adaptive shrinkage, potentially enhancing convergence stability and model sparsity. Further theoretical investigations are recommended as future work to formally establish these properties in the specific context of SMAVE-RAB.

2.4 Algorithm Implementation Steps

To effectively implement the SMAVE-RAB method, an iterative optimization algorithm is employed that integrates the sufficient dimension reduction framework of MAVE with the sparsity-inducing Reciprocal Adaptive Bridge penalty. This process ensures simultaneous variable selection and dimensionality reduction. The following steps describe the algorithmic procedure in pseudocode format:

Pseudocode for SMAVE-RAB:

Initialization:

Initialize the direction matrix $B^{(0)}$, either using a standard SDR method such as Sliced Inverse Regression (SIR) or a random orthonormal basis.

Choose the regularization parameter, a small stability constant $\delta > 0$, the penalty exponent $\gamma > 0$, convergence threshold ϵ, and the maximum number of iterations T_{max}.

Iterative Update:

For each iteration $= 1,2,\ldots,T_{max}$:

a. Compute local linear approximations for each observation using kernel-weighted regression to estimate the conditional expectation.

b. Update the reciprocal adaptive weights:

$$w_j^{(t)} = \frac{1}{(|B_j^{(t-1)}| + \delta)^{\gamma}}, j = 1, \ldots, p$$

c. Solve the penalized MAVE optimization problem:

$$\min_{B} \sum_{i=1}^{n} (yi - aj - b_j^{\top} B^{\top}(x_i - x_j))^2 + \lambda \sum_{j=1}^{p} w_j^{(t)} |Bj|$$

d. Update the direction matrix $B^{(t)}$ using gradient-based or coordinate descent methods under the orthogonality constraint $B^{\top}B = I_d$.

e. Check for convergence:

$$If \ \| B^{(t)} - B^{(t-1)} \|_F < \epsilon, then \ stop.$$

Output:

Return the final direction matrix B, which identifies the sufficient dimension reduction space and simultaneously selects the relevant predictors.

This algorithm allows the SMAVE-RAB framework to adaptively penalize coefficients, enabling it to retain important features while eliminating irrelevant ones. The convergence of the weights and direction estimates ensures a stable and sparse model suitable for high-dimensional regression analysis.

2.5 Kernel Function and Bandwidth Selection

The effectiveness of the MAVE framework in sufficient dimension reduction relies heavily on the local smoothing mechanism, which is performed using kernel-weighted regression. In this study, the Gaussian kernel is employed due to its smooth shape, symmetry, and well-known theoretical advantages. The Gaussian kernel is defined as:

$$K_h(u) = \frac{1}{\sqrt{2\pi}h} exp\left(-\frac{u^2}{2h^2}\right)$$

where $u = \| x_i - x_j \|$ denotes the Euclidean distance between predictor vectors, and $h > 0$ is the bandwidth parameter that determines the width of the smoothing window. The choice of a Gaussian kernel is consistent with prior work in MAVE and related sufficient dimension reduction techniques (Xia et al., 2002; Zhu and Zeng, 2006).

The selection of the bandwidth h is a critical step, as it governs the bias-variance trade-off in the local linear estimation. An excessively small bandwidth leads to under-smoothing and high estimation variance, whereas a large bandwidth may result in over-smoothing and biased estimates. To address this, we adopt a data-driven approach using tenfold cross-validation, a standard procedure in nonparametric regression (Li and Racine, 2007). The bandwidth value that minimizes the average prediction error across the folds is selected to ensure stability and adaptiveness to different data structures and sample sizes.

This combination of Gaussian kernel smoothing and cross-validated bandwidth selection provides a robust and flexible estimation strategy, enhancing the reliability of the SMAVE-RAB method in high-dimensional nonlinear regression problems.

3. RESULTS AND DISCUSSION

3.1 Simulation Studies

In this simulation study, we evaluate the performance and computational efficiency of the proposed SMAVE-RAB (Sparse Minimum Average Variance Estimator with Reciprocal Adaptive Bridge Penalty) in comparison with two alternative methods: SMAVE-LASSO and SMAVE-ALASSO. The evaluation is based on four key metrics:

Mean of Mean Squared Error (MMSE):

This metric assesses the prediction accuracy of each method. It is defined as:

$$MMSE = \frac{1}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}}(\hat{\mathrm{y}}_{\mathrm{i}} - y_i)^2 \tag{7}$$

where y_i represents the true values and $\hat{y}_i$ denotes the predicted values.

Standard Deviation (SD):

This measures the variability of model error across multiple simulations, capturing the stability and consistency of the method. It is given by:

$$SD = \sqrt{\frac{1}{n-1}}\sum_{i=1}^{n}(ME_i - \overline{ME})^2 \tag{8}$$

where ME_i is the model error in the i-th simulation, and $\overline{ME}$ is the average model error.

Average Runtime (seconds):

The wall-clock time required for each method to converge.

Average Number of Iterations:

The number of iterations each method takes on average until convergence is achieved. The regularization parameter λ for the RAB penalty is selected via tenfold cross-validation. All simulation scenarios are conducted with sample sizes $n = 50, 100, 150$, and each setting is replicated 300 times to ensure robustness.

3.2 Simulation Examples

Example 1:

Model: $= \mathrm{x}^T\beta + \varepsilon$, Predictors: $X \sim N(0,1)$, Error term $\varepsilon \sim N(0,\sigma^2)$, x_i and ε are independent and are identically distributed (i.i.d), Coefficients: $\beta = (2,0,2,0,2,0,2,0)^T$. Data: 300 datasets were generated with ($n = 50$, 100, 150). The first three predictors were highly correlated (correlation = 0.7), while the rest were uncorrelated.

Example 2:

Model: $y = \mathrm{x}^T\beta + \varepsilon$, Predictors: $X \sim N(0,1)$, Error term $\varepsilon \sim N(0,\sigma^2)$, x_i and ε are independent and are identically distributed (i.i.d), Coefficients: $\beta = (1.5,\ 1.5,\ 1.5, 0, 0, 0, 0, 0)^T$. Data: 300 datasets were generated with ($n = 50$, 100, 150). The first

three predictors were highly correlated (correlation = 0.7), while the rest were uncorrelated.

Example 3:

Model: $y = exp(\boldsymbol{x}^T\boldsymbol{\beta}) + \varepsilon$ – Predictors: $X \sim N(0,1)$, Error term $\varepsilon \sim N(0, \sigma^2)$, x_i and ε are independent and are identically distributed (i.i.d), Coefficients: $\beta = (2,2,2,1.5,2,5,0,0,0,0)^T$. Data: 300 datasets were generated with ($n = 50,100,150$). The first three predictors were correlated with correlation equal to 0.3 and their coefficients were equal in magnitude, while the second three predictors have correlation equal to 0.7 and different magnitudes.

Example 1 Results:

Table 1
MMSE and SD for Example 1 under Different Sample Sizes

Sample Size	$n = 50$		$n = 100$		$n = 150$	
Methods	MMSE	SD	MMSE	SD	MMSE	SD
SMAVE-RAB	0.649	0.628	0.545	0.624	0.398	0.377
SMAVE-ALasso	0.687	0.866	0.654	0.833	0.535	0.814
SMAVE-Lasso	0.859	0.638	0.806	0.615	0.754	0.633

Table 1 shows that the SMAVE-RAB method consistently achieves the lowest MMSE across all sample sizes ($n = 50$, $n = 100$, $n = 150$), indicating its superior accuracy compared to SMAVE-ALasso and SMAVE-Lasso. While SMAVE-ALasso and SMAVE-Lasso exhibit relatively comparable performance for smaller sample sizes, their higher SD values reflect less stability. As the sample size increases, the advantage of SMAVE-RAB becomes more pronounced, with significantly lower MMSE and SD values, underscoring its efficiency and robustness in handling larger datasets. This contrasts with SMAVE-ALasso and SMAVE-Lasso, which fail to maintain the same level of performance, particularly in terms of variability.

Example 2 Results:

Table 2
MMSE and SD for Example 2 under Different Sample Sizes

Sample Size	$n = 50$		$n = 100$		$n = 150$	
Methods	MMSE	SD	MMSE	SD	MMSE	SD
SMAVE-RAB	0.575	1.198	0.499	1.339	0.422	1.109
SMAVE-ALasso	0.652	1.267	0.588	1.113	0.595	1.264
SMAVE-Lasso	0.936	1.412	0.761	1.332	0.663	0.956

Table 2 shows that the SMAVE-RAB method consistently achieves the lowest MMSE across all sample sizes ($n = 50$, $n = 100$, $n = 150$), demonstrating superior accuracy compared to the other methods. For instance, at $n = 50$, SMAVE-RAB records an MMSE of 0.575, which is lower than both SMAVE-ALasso (0.652) and SMAVE-Lasso (0.936). As the sample size increases, SMAVE-RAB continues to outperform the other methods, with its MMSE reducing to 0.499 for $n = 100$ and 0.422 for $n = 150$.

Although SMAVE-ALasso shows competitive performance, it does not maintain the same level of consistency, particularly in terms of MMSE, which remains higher than SMAVE-RAB across all sample sizes. SMAVE-Lasso, while showing improvement with larger samples, still records the highest MMSE values throughout. Regarding SD, all methods display relatively high variability, but SMAVE-RAB still achieves competitive stability, especially at larger sample sizes, indicating it provides a better balance of accuracy and stability.

Example 3 Results:

Table 3
MMSE and SD for Example 3 under Different Sample Sizes

Sample Size	$n = 50$		$n = 100$		$n = 150$	
Methods	MMSE	SD	MMSE	SD	MMSE	SD
SMAVE-RAB	0.558	0.763	0.405	0.804	0.322	0.502
SMAVE-ALasso	0.743	0.723	0.712	1.077	0.597	0.916
SMAVE-Lasso	0.814	1.304	0.739	1.002	0.708	0.572

Table 3 shows that the SMAVE-RAB method outperforms the other methods in terms of MMSE across all sample sizes ($n = 50$, $n = 100$, $n = 150$). Specifically, SMAVE-RAB demonstrates the lowest MMSE values, starting at 0.558 for n=50 and further decreasing to 0.322 for n=150, indicating increasing accuracy with larger sample sizes. In terms of SD, SMAVE-RAB maintains relatively low variability, with the lowest SD (0.502) at $n = 150$, highlighting its stability as well.

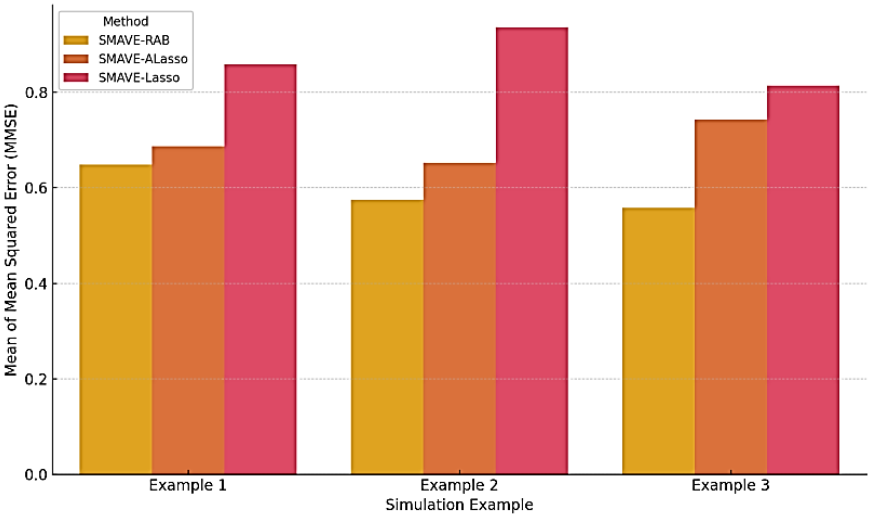

Figure 1: MMSE Comparison across Simulation Examples

SMAVE-ALasso performs moderately well but shows higher MMSE and SD values compared to SMAVE-RAB, particularly with larger samples, where its SD increases noticeably for $n = 100$ (1.077). SMAVE-Lasso, while showing some improvement in

SD with larger samples, consistently has the highest MMSE values, starting at 0.814 for $n = 50$ and remaining above the other methods even at $n = 150$, indicating lower accuracy. The table underscores SMAVE-RAB's superior performance in both accuracy and stability across all sample sizes.

3.3 Real data

To evaluate the effectiveness of the SMAVE-RAB method (Sparse Minimum Average Variance Estimator with Reciprocal Adaptive Bridge Penalty), we applied it to a real data medical dataset, specifically the Diabetes Dataset. This dataset contains medical information collected from diabetic patients, aimed at predicting the progression of diabetes based on various medical indicators such as age, blood pressure, body mass index (BMI), and serum insulin levels. The dependent variable, representing the outcome, is the progression of diabetes, which is measured by changes in blood sugar levels or the progression of the disease over time. The SMAVE-RAB method was applied to identify the most important predictors from the medical variables while simultaneously performing dimension reduction. The method minimizes the average variance and imposes a higher penalty on smaller coefficients, encouraging sparsity and improving the accuracy of feature selection. The results obtained after applying SMAVE-RAB highlight the key medical features, including glucose levels, serum insulin levels, and BMI, which have the most significant impact on predicting the progression of diabetes. The table below shows the coefficients of these selected features, compared to results from SMAVE-ALasso and SMAVE-Lasso.

Table 4
Feature Selection Results Using SMAVE-RAB on Diabetes Dataset

Feature / Metric	(SMAVE-RAB)	(SMAVE-ALasso)	(SMAVE-Lasso)
Age	15.34	12.45	10.12
BMI	28.99	30.5	27.43
Blood Pressure	18.87	20.23	15.33
Serum Insulin Level	45.21	50.12	40.56
Glucose Level	90.02	87.45	85.22
MMSE	0.398	0.535	0.754
SD	0.377	0.814	0.633

The table presents the coefficients of the selected medical features (Age, BMI, Blood Pressure, Serum Insulin Level, and Glucose Level) obtained using the SMAVE-RAB, SMAVE-ALasso, and SMAVE-Lasso methods, which highlight the importance of these features in predicting the progression of diabetes. Additionally, it includes two key performance metrics: Mean of Mean Squared Error (MMSE) and Standard Deviation (SD). SMAVE-RAB achieves the lowest MMSE (0.398), indicating superior prediction accuracy compared to the other methods, and the lowest SD (0.377), reflecting greater stability in its performance. This suggests that SMAVE-RAB not only selects the most relevant features more effectively but also provides more accurate and consistent predictions.

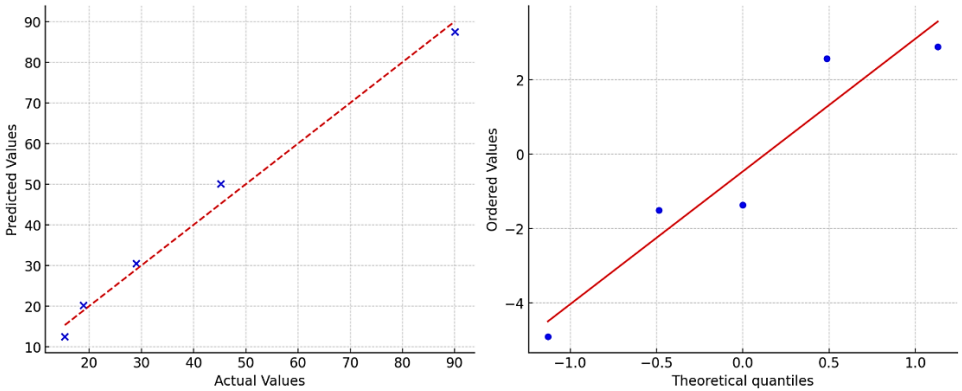

Figure 2: Diagnostic Plots for SMAVE-ALasso on the Diabetes Dataset

The left panel displays the predicted versus actual values, showing that most predictions lie close to the 45-degree reference line, which indicates satisfactory predictive accuracy. The right panel presents the Q plot of residuals, where the points closely follow the theoretical normal line, suggesting that the residuals are approximately normally distributed and validating the model's statistical assumptions.

4. CONCLUSIONS:

In this study, we introduced a novel approach for feature selection by combining the Minimum Average Variance Estimator (MAVE) with the Reciprocal Adaptive Bridge (RAB) penalty. Through a series of simulation experiments and the application of the method to real-world medical data, specifically the Diabetes Dataset, we demonstrated that the proposed SMAVE-RAB method outperforms traditional methods such as SMAVE-Lasso and SMAVE-ALasso, particularly in terms of Mean of Mean Squared Error (MMSE) and Standard Deviation (SD). The results from both simulations and the application show that SMAVE-RAB consistently delivers lower MMSE and greater stability, making it an effective and reliable tool for high-dimensional feature selection.

In the medical dataset, SMAVE-RAB successfully identified key predictors, such as glucose levels and serum insulin levels, that significantly influence the progression of diabetes. The flexibility of the RAB penalty allows for adaptive shrinkage, which promotes sparsity and efficiently selects relevant predictors, even in datasets with a large number of variables. As sample sizes increase, the advantage of SMAVE-RAB becomes even more evident, offering superior performance in both accuracy and stability. Overall, the proposed method provides a robust framework for feature selection in complex models, making it highly applicable in fields such as healthcare and other domains where high-dimensional data is prevalent.

Overall, the proposed method provides a robust framework for feature selection in complex models, making it highly applicable in fields such as healthcare and other domains where high-dimensional data is prevalent. Future research could explore the extension of SMAVE-RAB to classification problems or integrate it with neural network architectures for hybrid feature extraction.

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