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R-OPTIMAL DESIGNS FOR POISSON REGRESSION MODEL IN TWO PARAMETERS

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ABSTRACT

In the generalized linear model (GLM) setup, the information matrix depends on the unknown parameters of the model. In such a situation, an experimenter has to adopt the approach of finding local optimal designs i.e. first guess the best value for the parameters and then calculate the optimal designs [see Chernoff (1953)]. In this paper, locally R-optimal designs for the Poisson regression model with two parameters including intercept are discussed. The R-optimality criterion has been proposed in the literature as an alternative to the most frequently used D-optimality criterion when the experimenter wishes to minimize the volume of the confidence region for unknown parameters based on Bonferroni t-intervals [see Dette (1997)]. The necessary and sufficient conditions of this optimality criterion are confirmed through the equivalence theorem.

KEYWORDS

R-optimal design, Information matrix, Poisson regression model, Bonferroni t-intervals, Equivalence theorem.

1. INTRODUCTION

The Poisson regression model is a particular form of Generalized Linear Models (GLMs) where the responses are count data. Count data plays a vital role in marketing, medical and pharmaceutical development, psychological research, etc. For instance, (I) Rasch Poisson counts model forecasts the person's ability in an item response group [see Rasch, 1960], (II) count data in psychological applied research [see Vives et al., 2006], (III) for analysis of count data, there is a variety of literature [see e.g. Cameron and Trivedi (2013)]. In these situations, standard linear models are not appropriate instead the Poisson regression model is more suitable to describe such data.

The inspiration behind finding an optimal design is to discuss statistical conclusions about the quantities of interest by selecting the control variable properly. The values of the control variables are chosen to minimize the variability of the estimators of the unknown parameters involved with the regression model. The foundation work on optimal design was laid by Kiefer (1959) and Kiefer and Wolfowitz (1959). The task of finding the optimal design becomes quite challenging as the information matrix depends upon the unknown parameters i.e., to find the best design to estimate the unknown parameters, and yet one has to know the parameters to obtain the best design. A simple approach to this problem is to make a best initial guess of the parameters and then choose the design that optimizes the design optimality criterion function evaluated at the guess point. This approach leads to a locally optimal design, introduced by Chernoff (1953).

In literature for the Poisson regression model, Ford et al. (1992) used canonical transformations to obtain optimal designs for the one-dimensional setup. For this model with one or two variables, Wang et al. (2006a) investigated the dependence problem of locally D- optimal designs on functions of the parameter values, and Wang et al. (2006b) developed a sequential design. Rodriguez-Torreblanca and Rodriguez-Diaz (2007) discussed D- and C- optimal designs for Poisson and negative binomial regression models. Russel et al. (2009) derived an analytic solution for D-optimal design for the main effects of Poisson models. The optimal designs for this model with random intercept were discussed by Niaparast (2010). Ying Zhang & Keying Ye (2014) verified the equivalence theorem for Bayesian D-optimal design for Poisson regression models. They also established the canonical form of the problem as well as the general equivalence theorem. Lal et al. (2018) adopted an algorithmic approach to obtain D-optimal designs for the Poisson model with count data as a response. Olamide et al. (2021) discussed E-optimal experimental designs for Poisson regression models two and three variables.

The organization of this article is as follows. Section 2 provides the preliminaries. In Section 3, R-optimal designs for the Poisson regression model with two parameters are discussed. Finally, the article is concluded with conclusions in Section 4.

2. PRELIMINARIES

Let us consider an experiment where the ith observation on a response variable, y_i , has a Poisson distribution, with a rate λ_i dependent on q-independent covariates $x_1, x_2, ..., x_q$ through the log-linear model i.e.

$$\ln(\lambda_i) = \eta_i = h(x_i)'\beta, \; i = 1, 2, .., n \tag{1}$$

where $x_i = (x_1, x_2, ..., x_q)$, $h(x_i) = (1, x_i')'$, $\beta = \beta_1, \beta_2, ..., \beta_q$ are q-dimensional vectors of the unknown parameters. For the model Equation (1) and log link, the Fisher information matrix is $q \times q$ dimension at x and β can be defined as

$$M(x, \beta) = \kappa\, h(x) h'(x) \tag{2}$$

where $\kappa = e^{\eta_i}$ is the log-link function or intensity function. For more details on the log-link function [see Freise et al., 2021].

To obtain the R-optimal design for the model Equation (2), consider the approximate design $\xi \in \Xi$ (Ξ the set of all approximate designs) of the form

$$\xi = \begin{Bmatrix} x_1 & \dots & x_q \\ w_1 & \dots & w_q \end{Bmatrix}, \; w_i(>0) \text{ and } \sum_{i=1}^{q} w_i = 1 \tag{3}$$

where $x_1, x_2, ..., x_q \in \Delta\,(\Delta \subset \Re^q)$ are the 'q' distinct points and w_i is the weight associated with the point x_i for $i = 1, 2, ..., q$. For the model Equation (1), the Fisher information matrix of a design ξ at parameter β is defined as

$$M(\xi, \beta) = \sum_{i=1}^{q} w_i \, M(x_i, \beta) \tag{4}$$

One can refer to pg 152, (Russel, 2018) for more details.

R-optimal Design:

A design $\xi \in \Xi$ with a non-singular information matrix $M(\xi)$ is called R-optimal for the model Equation (2) if it minimizes

$$\psi(\xi) = \prod_{i=1}^{q} (M^{-1}(\xi)_{ii} \tag{5}$$

for all $\xi \in \Xi$. The necessary and sufficient conditions for the R-optimality will be verified through the following equivalence theorem. For more details, [see Dette (1997)].

Equivalence Theorem:

For model Equation (1), let

$$\varphi(x, \xi) = h'(x) M^{-1}(\xi) \left(\sum_{i=1}^{q} \frac{e_i e_i'}{e_i' M^{-1}(\xi^*) e_i} \right) M^{-1}(\xi) h(x) \, . \tag{6}$$

A design $\xi^* \in \Xi$ is R-optimal if and only if

$$\sup_{x \in \Delta} \varphi(x, \xi^*) = q$$

with equality attained at the support points ξ^*.

3. R-OPTIMAL DESIGNS FOR TWO PARAMETERS

In this section, locally R-optimal designs for the model Equation (1) i.e. $h'(x)\beta = \beta_0 + \beta_1 x > 0$ for all $x \in R$. Here, the restriction of search to two-, and three-support points design by considering discrete values of β_0, β_1 in the randomly chosen intervals [1, 15].

3.1 Designs based on Two Support Points

Let us consider a 2-point design ξ of the form

$$\xi = \begin{Bmatrix} a & b \\ w & 1-w \end{Bmatrix} \text{ where } 0 < w < 1. \tag{7}$$

Theorem 3.1.1

The design ξ^* that assigns a weight w^* to the point a^* and 1- w^* to the point b^* in Δ is an R-optimal design where a^*, b^*, and w^* are given in Table 1 (Appendix).

Proof:

The information matrix for the model Equation (7) at the two-point design ξ defined in Equation (4) is given by

$$M(\xi) = \begin{bmatrix} e^{\beta_0+b\beta_1}(1-w) + e^{\beta_0+a\beta_1}w & be^{\beta_0+b\beta_1}(1-w) + ae^{\beta_0+a\beta_1}w \\ be^{\beta_0+b\beta_1}(1-w) + ae^{\beta_0+a\beta_1}w & b^2e^{\beta_0+b\beta_1}(1-w) + a^2e^{\beta_0+a\beta_1}w \end{bmatrix} \tag{8}$$

The inverse of the above information matrix is given by

$$M^{-1}(\xi) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{9}$$

with

$$M_{11} = \frac{e^{-\beta_0-(a+b)\beta_1}(b^2e^{b\beta_1}(-1+w) - a^2e^{a\beta_1}w)}{(a-b)^2(-1+w)w}$$

$$M_{12} = M_{21} = \frac{e^{-\beta_0-(a+b)\beta_1}(-be^{b\beta_1}(-1+w) + ae^{a\beta_1}w)}{(a-b)^2(-1+w)w}$$

$$M_{22} = \frac{e^{-\beta_0-(a+b)\beta_1}(e^{b\beta_1}(-1+w) - e^{a\beta_1}w)}{(a-b)^2(-1+w)w}$$

Using Equation (5), I obtain the function

$$\psi(\xi) = \frac{\left\{e^{-2(\beta_0+(a+b)\beta_1)}(e^{b\beta_1}(-1+w) - e^{a\beta_1}w)(b^2e^{b\beta_1}(-1+w) - a^2e^{a\beta_1}w)\right\}}{(a-b)^4(-1+w)^2w^2} \tag{10}$$

Now, the problem is to minimize the function $\psi(\xi)$ w.r.t a, b, and w for given values of β_0 and β_1. This is done using the “Nminimize” function of *Wolfram*

Mathematica 7.0 software and getting the optimal values a^*, b^* and w^*. The numerical values of a^*, b^* and w^* are given in Table 1 (Appendix).

Next, by using Equation (9), the quadratic form as specified in Equation (6) which is as follows:

$$\varphi(x,\xi^*) = e^{(\beta_0+\beta_1 x)}\left\{M_{11}+M_{12}x+\left(\frac{(-be^{b\beta_1}(-1+w)+ae^{a\beta_1}w)(M_{12}+M_{22}x)}{(e^{b\beta_1}(-1+w)-e^{a\beta_1}w)}\right)\right.$$
$$\left.+x\left(M_{12}+M_{22}x+\frac{(-be^{b\beta_1}(-1+w)+ae^{a\beta_1}w)(M_{11}+M_{12}x)}{(b^2(e^{b\beta_1}(-1+w)-a^2e^{a\beta_1}w)}\right)\right\}. \quad (11)$$

Replacing the numerical values of a^*, b^* and w^* in Equation (11) and using the "Nminimize" function of *Wolfram Mathematica 7.0* software I find $\sup_{x\in\Delta}\varphi(x,\xi^*)=2$. Thus the necessary and sufficient condition of the equivalence theorem is established. This proves Theorem 3.1.1. □

3.2 Designs based on Three Support Points

Let us consider a 3-point design ξ of the form

$$\xi = \begin{Bmatrix} a & b & c \\ w/2 & 1-w & w/2 \end{Bmatrix} \text{ where } 0<w<1. \quad (12)$$

Theorem 3.2.1

The design ξ^* that assigns a weight $w^*/2$ to the point a^*, $1\text{-}w^*$ to the point b^*, and $w^*/2$ to the point c^* in Δ is an R-optimal design where a^*, b^*, c^* and w^* are given in Table 2 (Appendix).

Proof:

Using Equation (12), the information matrix for the model Equation (4) at the three-point design ξ will be

$$M(\xi) = \begin{bmatrix} e^{\beta_0+b\beta_1}(1\text{-}w)+\frac{1}{2}\left(e^{\beta_0+a\beta_1}w\right)+\frac{1}{2}\left(e^{\beta_0+c\beta_1}w\right) & be^{\beta_0+b\beta_1}(1\text{-}w)+\frac{1}{2}\left(ae^{\beta_0+a\beta_1}w\right)+\frac{1}{2}\left(ce^{\beta_0+c\beta_1}w\right) \\ be^{\beta_0+b\beta_1}(1\text{-}w)+\frac{1}{2}\left(ae^{\beta_0+a\beta_1}w\right)+\frac{1}{2}\left(ce^{\beta_0+c\beta_1}w\right) & b^2e^{\beta_0+b\beta_1}(1\text{-}w)+\frac{1}{2}\left(a^2e^{\beta_0+a\beta_1}w\right)+\frac{1}{2}\left(c^2e^{\beta_0+c\beta_1}w\right) \end{bmatrix}$$

The inverse of the above information matrix is given by

$$M^{-1}(\xi) = \begin{bmatrix} M_{11}^+ & M_{12}^+ \\ M_{21}^+ & M_{22}^+ \end{bmatrix} \quad (13)$$

with

$$M_{11}^{+}=\frac{e^{-\beta_0}(4b^2e^{b\beta_1}(w-1)-2(a^2e^{a\beta_1}+c^2e^{c\beta_1})w)}{w(-4abe^{(a+b)\beta_1}(w-1)+2e^{b\beta_1}(b^2e^{a\beta_1}+(b-c)^2e^{c\beta_1})(w-1)+2ace^{\beta_1(a+c)}w-c^2e^{\beta_1(a+c)}w+a^2e^{a\beta_1}(2e^{b\beta_1}(w-1)-e^{c\beta_1}w))}$$

$$M_{12}^{+}=M_{21}^{+}=\frac{e^{-\beta_0}(-4be^{b\beta_1}(w-1)+2(ae^{a\beta_1}+ce^{c\beta_1})w)}{w(-4abe^{(a+b)\beta_1}(w-1)+2e^{b\beta_1}(b^2e^{a\beta_1}+(b-c)^2e^{c\beta_1})(w-1)+2ace^{\beta_1(a+c)}w-c^2e^{\beta_1(a+c)}w+a^2e^{a\beta_1}(2e^{b\beta_1}(w-1)-e^{c\beta_1}w))}$$

$$M_{22}^{+}=\frac{e^{-\beta_0}(4e^{b\beta_1}(w-1)-2e^{a\beta_1}w-2e^{c\beta_1}w)}{w(-4abe^{(a+b)\beta_1}(w-1)+2e^{b\beta_1}(b^2e^{a\beta_1}+(b-c)^2e^{c\beta_1})(w-1)+2ace^{\beta_1(a+c)}w-c^2e^{\beta_1(a+c)}w+a^2e^{a\beta_1}(2e^{b\beta_1}(w-1)-e^{c\beta_1}w))}$$

Using Equation (6), I obtain the function

$$\psi(\xi)=\frac{\left(e^{-2\beta_0}\left(4e^{b\beta_1}(w-1)-2e^{a\beta_1}w-2e^{c\beta_1}w\right)\left(4b^2e^{b\beta_1}(w-1)-2(a^2e^{a\beta_1}+c^2e^{c\beta_1})w\right)\right)}{\left(w^2\left((-4abe^{(a+b)\beta_1}\left(w-1\right)+2e^{b\beta_1}\left(b^2e^{a\beta_1}+(b-c)^2e^{c\beta_1}\right)(w-1)+2ace^{\beta_1(a+c)}w-c^2e^{\beta_1(a+c)}w+a^2e^{a\beta_1}\left(2e^{b\beta_1}(w-1)-e^{c\beta_1}w\right)\right)^2\right)} \tag{14}$$

Next, the problem is need to minimize the function $\psi(\xi)$ w.r.t a, b, c and w for given values of β_0 and β_1. This is achieved by using the "Nminimize" function of *Wolfram Mathematica 7.0* software and getting the optimal values a^*, b^*, c^* and w^*. The numerical values of a^*, b^*, c^* and w^* are given in Table 2 (Appendix).

Next, by using Equation (13), the quadratic form as specified in Equation (6) which is as follows:

$$\varphi(x,\xi^*)=e^{(\beta_0+\beta_1x)}\left\{M_{11}^{+}+M_{12}^{+}x+\left(\frac{(-4be^{b\beta_1}(w-1)+2(ae^{a\beta_1}+ce^{c\beta_1})w)(M_{12}^{+}+M_{22}^{+}x)}{(4e^{b\beta_1}(w-1)-2e^{a\beta_1}w-2e^{c\beta_1}w)}\right)+x\left(M_{12}^{+}+M_{22}^{+}x+\frac{(-4be^{b\beta_1}(w-1)+2(ae^{a\beta_1}+ce^{c\beta_1})w)(M_{11}^{+}+M_{12}^{+}x)}{(4b^2e^{b\beta_1}(w-1)-2(a^2e^{a\beta_1}+c^2e^{c\beta_1})w)}\right)\right\} \tag{15}$$

Replacing the numerical values of a^*, b^*, c^* and w^* in Equation (15) using the "Nminimize" function of *Wolfram Mathematica 7.0* software I find $\sup_{x\in\Delta}\varphi(x,\xi^*)=2$.

Thus the necessary and sufficient condition of the equivalence theorem is established. This proves Theorem 3.2.1. □

4. CONCLUSION

In the literature on the construction of optimal designs, the widely used optimality criterion is the D-optimality criterion. An experimenter decides to consider the D-optimality criterion when he/she is interested in the confidence ellipsoid of the estimators of the unknown parameters However, if the experimenter wishes to construct a rectangular confidence region then he/she should prefer an R-optimal design instead of a D- optimal design.

This article obtains locally R-optimal designs for two parameters Poisson regression model when the model is associated with a log link function based on two- and three-support point designs. The support points of the optimal designs and the weights assigned to these points are calculated numerically using the "Nminimize" function of *Wolfram Mathematica 7.0* software whereas the necessary and sufficient condition of R-optimality i.e. the equivalence theorem is also verified at the support points. A catalog of support points and the weight assigned to each of the support points corresponding to R-optimal designs are listed in Table 1 and Table 2 (Appendix). These Tables provide the solutions for only those values of β_0 and β_1 when the equivalence theorem is satisfied.

From Table 1: two-support points design, I find that the support points lie in the first quadrant of the two-dimensional space. The values of the first coordinate and second coordinate of the support points are different i.e. 1 and less than 1. Further, I observe that the weight associated with the coordinate value 1 always has less weight as compared to the other one. In the case of Table 2: three-support points design, among design points and weights, two are same always. Also, I examine that, from both Table 1 and Table 2, the design points and weights are the same, as the intercept parameter β_0 increased and the slope parameter β_1 constant. For example, $(\beta_0 = 2, \beta_1 = 1)$, $(\beta_0 = 3, \beta_1 = 1)$ and so on. More research work is required in this direction as the equivalence theorem does not hold for many discrete values of the unknown parameters which indicates that the proposed designs are sensitive towards the R-optimality criterion with the varying parameter choices.

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APPENDIX
R-Optimal Designs for Poisson regression Model with Two Parameters

Table 1 and Table 2 provides locally R-optimal designs is for $\beta = (\beta_0, \beta_1)'$ with $\beta_0, \beta_1 \in [1,15]$.

Table 1
Two Support Points Design

β	$\beta_0=1,\beta_1=1$	$\beta_0=1,\beta_1=2$	$\beta_0=1,\beta_1=3$	$\beta_0=1,\beta_1=4$
x	1, 0	1, 0	1, 0.1797	0.3718, 1
w	0.2574, 0.7246	0.1859, 0.8141	0.161, 0.839	0.8311, 0.1689
β	$\beta_0=1,\beta_1=5$	$\beta_0=1,\beta_1=6$	$\beta_0=1,\beta_1=7$	$\beta_0=1,\beta_1=8$
x	0.4935, 1	0.5764, 1	0.6362, 1	0.6813, 1
w	0.8233, 0.1767	0.8173, 0.1827	0.8126, 0.1874	0.809, 0.191
β	$\beta_0=1,\beta_1=9$	$\beta_0=1,\beta_1=10$	$\beta_0=1,\beta_1=11$	$\beta_0=1,\beta_1=12$
x	0.7165, 1	0.7447, 1	0.7678, 1	0.7871, 1
w	0.8061, 0.1939	0.8038, 0.1962	0.8018, 0.1982	0.8002, 0.1998
β	$\beta_0=1,\beta_1=13$	$\beta_0=1,\beta_1=14$	$\beta_0=2,\beta_1=1$	$\beta_0=2,\beta_1=2$
x	0.8035, 1	0.8175, 1	1, 0	1, 0
w	0.7988, 0.2012	0.7977, 0.2023	0.2574, 0.7426	0.1859, 0.8141
β	$\beta_0=2,\beta_1=3$	$\beta_0=2,\beta_1=4$	$\beta_0=2,\beta_1=5$	$\beta_0=2,\beta_1=6$
x	1, 0.1797	0.3718, 1	0.4935, 1	0.5764, 1
w	0.1610, 0.839	0.8311, 0.1689	0.8233, 0.1767	0.8173, 0.1827
β	$\beta_0=2,\beta_1=7$	$\beta_0=2,\beta_1=8$	$\beta_0=2,\beta_1=9$	$\beta_0=2,\beta_1=10$
x	0.6362, 1	0.6813, 1	0.7165, 1	0.7447, 1
w	0.8126, 0.1874	0.809, 0.191	0.8061, 0.1939	0.8038, 0.1962
β	$\beta_0=2,\beta_1=11$	$\beta_0=2,\beta_1=12$	$\beta_0=2,\beta_1=13$	$\beta_0=2,\beta_1=14$
x	0.7678, 1	0.7871, 1	0.8035, 1	0.8175, 1
w	0.8018, 0.1982	0.8002, 0.1998	0.7988, 0.2012	0.7977, 0.2023
β	$\beta_0=3,\beta_1=1$	$\beta_0=3,\beta_1=2$	$\beta_0=3,\beta_1=3$	$\beta_0=3,\beta_1=4$
x	1, 0	1, 0	1, 0.1797	0.3718, 1
w	0.2574, 0.7426	0.1859, 0.8141	0.1610, 0.839	0.8311, 0.1689

β	$\beta_0 = 3, \beta_1 = 5$	$\beta_0 = 3, \beta_1 = 6$	$\beta_0 = 3, \beta_1 = 7$	$\beta_0 = 3, \beta_1 = 8$
x w	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8126 & 0.1874 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.809 & 0.191 \end{Bmatrix}$
β	$\beta_0 = 3, \beta_1 = 9$	$\beta_0 = 3, \beta_1 = 10$	$\beta_0 = 3, \beta_1 = 11$	$\beta_0 = 3, \beta_1 = 12$
x w	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8061 & 0.1939 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8038 & 0.1962 \end{Bmatrix}$	$\begin{Bmatrix} 0.7678 & 1 \\ 0.8018 & 0.1982 \end{Bmatrix}$	$\begin{Bmatrix} 0.7871 & 1 \\ 0.8002 & 0.1998 \end{Bmatrix}$
β	$\beta_0 = 4, \beta_1 = 1$	$\beta_0 = 4, \beta_1 = 2$	$\beta_0 = 4, \beta_1 = 3$	$\beta_0 = 4, \beta_1 = 4$
x w	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7246 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1797 \\ 0.161 & 0.839 \end{Bmatrix}$	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$
β	$\beta_0 = 4, \beta_1 = 5$	$\beta_0 = 4, \beta_1 = 6$	$\beta_0 = 4, \beta_1 = 7$	$\beta_0 = 4, \beta_1 = 8$
x w	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8126 & 0.1874 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.809 & 0.191 \end{Bmatrix}$
β	$\beta_0 = 4, \beta_1 = 9$	$\beta_0 = 4, \beta_1 = 10$	$\beta_0 = 4, \beta_1 = 11$	$\beta_0 = 4, \beta_1 = 12$
x w	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8061 & 0.1939 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8038 & 0.1962 \end{Bmatrix}$	$\begin{Bmatrix} 0.7678 & 1 \\ 0.8018 & 0.1982 \end{Bmatrix}$	$\begin{Bmatrix} 0.7871 & 1 \\ 0.8002 & 0.1998 \end{Bmatrix}$
β	$\beta_0 = 4, \beta_1 = 13$	$\beta_0 = 4, \beta_1 = 14$	$\beta_0 = 5, \beta_1 = 1$	$\beta_0 = 5, \beta_1 = 2$
x w	$\begin{Bmatrix} 0.8035 & 1 \\ 0.7988 & 0.2012 \end{Bmatrix}$	$\begin{Bmatrix} 0.8175 & 1 \\ 0.7977 & 0.2023 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7426 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$
β	$\beta_0 = 5, \beta_1 = 3$	$\beta_0 = 5, \beta_1 = 4$	$\beta_0 = 5, \beta_1 = 5$	$\beta_0 = 5, \beta_1 = 6$
x w	$\begin{Bmatrix} 1 & 0.1797 \\ 0.1610 & 0.839 \end{Bmatrix}$	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$
β	$\beta_0 = 5, \beta_1 = 7$	$\beta_0 = 5, \beta_1 = 8$	$\beta_0 = 5, \beta_1 = 9$	$\beta_0 = 5, \beta_1 = 10$
x w	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8126 & 0.1874 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.809 & 0.191 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8061 & 0.1939 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8038 & 0.1962 \end{Bmatrix}$
β	$\beta_0 = 5, \beta_1 = 11$	$\beta_0 = 5, \beta_1 = 12$	$\beta_0 = 6, \beta_1 = 1$	$\beta_0 = 6, \beta_1 = 2$
x w	$\begin{Bmatrix} 0.7678 & 1 \\ 0.8018 & 0.1982 \end{Bmatrix}$	$\begin{Bmatrix} 0.7871 & 1 \\ 0.8002 & 0.1998 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7426 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$
β	$\beta_0 = 6, \beta_1 = 3$	$\beta_0 = 6, \beta_1 = 4$	$\beta_0 = 6, \beta_1 = 5$	$\beta_0 = 6, \beta_1 = 6$
x w	$\begin{Bmatrix} 1 & 0.1797 \\ 0.1610 & 0.839 \end{Bmatrix}$	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$

β	$\beta_0=6,\beta_1=7$	$\beta_0=6,\beta_1=8$	$\beta_0=6,\beta_1=9$	$\beta_0=6,\beta_1=10$
x w	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8126 & 0.1874 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.809 & 0.191 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8061 & 0.1939 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8038 & 0.1962 \end{Bmatrix}$
β	$\beta_0=6,\beta_1=11$	$\beta_0=7,\beta_1=1$	$\beta_0=7,\beta_1=2$	$\beta_0=7,\beta_1=3$
x w	$\begin{Bmatrix} 0.7678 & 1 \\ 0.8018 & 0.1982 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7426 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1797 \\ 0.1610 & 0.839 \end{Bmatrix}$
β	$\beta_0=7,\beta_1=4$	$\beta_0=7,\beta_1=5$	$\beta_0=7,\beta_1=6$	$\beta_0=7,\beta_1=7$
x w	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.6735 & 1 \\ 0.8179 & 0.1821 \end{Bmatrix}$
β	$\beta_0=8,\beta_1=1$	$\beta_0=8,\beta_1=2$	$\beta_0=8,\beta_1=3$	$\beta_0=8,\beta_1=4$
x w	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7426 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1797 \\ 0.1610 & 0.839 \end{Bmatrix}$	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$
β	$\beta_0=8,\beta_1=5$	$\beta_0=8,\beta_1=6$	$\beta_0=8,\beta_1=8$	$\beta_0=9,\beta_1=1$
x w	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.6735 & 1 \\ 0.8179 & 0.1821 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2574 & 0.7426 \end{Bmatrix}$
β	$\beta_0=9,\beta_1=2$	$\beta_0=9,\beta_1=3$	$\beta_0=9,\beta_1=4$	$\beta_0=9,\beta_1=5$
x w	$\begin{Bmatrix} 1 & 0 \\ 0.1859 & 0.8141 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1797 \\ 0.1610 & 0.839 \end{Bmatrix}$	$\begin{Bmatrix} 0.3718 & 1 \\ 0.8311 & 0.1689 \end{Bmatrix}$	$\begin{Bmatrix} 0.4935 & 1 \\ 0.8233 & 0.1767 \end{Bmatrix}$
β	$\beta_0=9,\beta_1=6$	$\beta_0=10,\beta_1=6$	$\beta_0=11,\beta_1=6$	$\beta_0=11,\beta_1=14$
x w	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.8175 & 1 \\ 0.7977 & 0.2023 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=15$	$\beta_0=12,\beta_1=6$	$\beta_0=13,\beta_1=6$	$\beta_0=14,\beta_1=6$
x w	$\begin{Bmatrix} 0.8296 & 1 \\ 0.7966 & 0.2034 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$
β	$\beta_0=15,\beta_1=6$	-	-	-
x w	$\begin{Bmatrix} 0.5764 & 1 \\ 0.8173 & 0.1827 \end{Bmatrix}$	-	-	-

Table 2
Three Support Points Design

β	$\beta_0=1,\beta_1=1$	$\beta_0=1,\beta_1=2$	$\beta_0=1,\beta_1=3$
x	$1 \quad 0 \quad 1$	$1 \quad 0 \quad 1$	$1 \quad 0.1797 \quad 0.1797$
w	$0.1287 \quad 0.7426 \quad 0.1287$	$0.0929 \quad 0.8141 \quad 0.0929$	$0.161 \quad 0.678 \quad 0.161$
β	$\beta_0=1,\beta_1=4$	$\beta_0=1,\beta_1=5$	$\beta_0=1,\beta_1=6$
x	$1 \quad 0.3718 \quad 0.3718$	$1 \quad 0.4935 \quad 0.4935$	$1 \quad 0.5764 \quad 0.5764$
w	$0.1688 \quad 0.6624 \quad 0.1688$	$0.1766 \quad 0.6468 \quad 0.1766$	$0.1826 \quad 0.6347 \quad 0.1826$
β	$\beta_0=1,\beta_1=7$	$\beta_0=1,\beta_1=8$	$\beta_0=1,\beta_1=9$
x	$0.6362 \quad 0.6362 \quad 1$	$1 \quad 0.6813 \quad 1$	$1 \quad 0.7165 \quad 0.7165$
w	$0.1873 \quad 0.6253 \quad 0.1873$	$0.0954 \quad 0.8091 \quad 0.0954$	$0.1938 \quad 0.6123 \quad 0.1938$
β	$\beta_0=1,\beta_1=10$	$\beta_0=1,\beta_1=11$	$\beta_0=1,\beta_1=12$
x	$1 \quad 0.7447 \quad 1$	$0.7678 \quad 0.7679 \quad 1$	$1 \quad 0.7871 \quad 0.7871$
w	$0.0980 \quad 0.8039 \quad 0.0980$	$0.1981 \quad 0.6037 \quad 0.1981$	$0.1997 \quad 0.6006 \quad 0.1997$
β	$\beta_0=1,\beta_1=13$	$\beta_0=1,\beta_1=14$	$\beta_0=1,\beta_1=15$
x	$0.8035 \quad 0.8035 \quad 1$	$0.8175 \quad 1 \quad 0.8175$	$1 \quad 0.8296 \quad 0.8296$
w	$0.2011 \quad 0.5978 \quad 0.2011$	$0.3988 \quad 0.2023 \quad 0.3988$	$0.2033 \quad 0.5934 \quad 0.2033$
β	$\beta_0=2,\beta_1=1$	$\beta_0=2,\beta_1=2$	$\beta_0=2,\beta_1=3$
x	$1 \quad 0 \quad 1$	$1 \quad 0 \quad 1$	$1 \quad 0.1797 \quad 0.1797$
w	$0.1287 \quad 0.7426 \quad 0.1287$	$0.0929 \quad 0.8141 \quad 0.0929$	$0.161 \quad 0.678 \quad 0.161$
β	$\beta_0=2,\beta_1=4$	$\beta_0=2,\beta_1=5$	$\beta_0=2,\beta_1=6$
x	$1 \quad 0.3718 \quad 0.3718$	$1 \quad 0.4935 \quad 0.4935$	$1 \quad 0.5764 \quad 0.5764$
w	$0.1688 \quad 0.6624 \quad 0.1688$	$0.1766 \quad 0.6468 \quad 0.1766$	$0.1826 \quad 0.6347 \quad 0.1826$
β	$\beta_0=2,\beta_1=7$	$\beta_0=2,\beta_1=8$	$\beta_0=2,\beta_1=9$
x	$0.6362 \quad 0.6362 \quad 1$	$1 \quad 0.6813 \quad 1$	$1 \quad 0.7165 \quad 0.7165$
w	$0.1873 \quad 0.6253 \quad 0.1873$	$0.0954 \quad 0.8091 \quad 0.0954$	$0.1938 \quad 0.6123 \quad 0.1938$
β	$\beta_0=2,\beta_1=10$	$\beta_0=2,\beta_1=11$	$\beta_0=2,\beta_1=12$
x	$1 \quad 0.7447 \quad 1$	$0.7678 \quad 0.7679 \quad 1$	$1 \quad 0.7871 \quad 0.7871$
w	$0.0980 \quad 0.8039 \quad 0.0980$	$0.1981 \quad 0.6037 \quad 0.1981$	$0.1997 \quad 0.6006 \quad 0.1997$
β	$\beta_0=2,\beta_1=13$	$\beta_0=2,\beta_1=14$	$\beta_0=3,\beta_1=1$
x	$0.8035 \quad 0.8035 \quad 1$	$0.8175 \quad 1 \quad 0.8175$	$1 \quad 0 \quad 1$
w	$0.2011 \quad 0.5978 \quad 0.2011$	$0.3988 \quad 0.2023 \quad 0.3988$	$0.1287 \quad 0.7426 \quad 0.1287$

β	$\beta_0=3,\beta_1=2$	$\beta_0=3,\beta_1=3$	$\beta_0=3,\beta_1=4$
x	1 0 1	1 0.1797 0.1797	1 0.3718 0.3718
w	0.0929 0.8141 0.0929	0.161 0.678 0.161	0.1688 0.6624 0.1688
β	$\beta_0=3,\beta_1=5$	$\beta_0=3,\beta_1=6$	$\beta_0=3,\beta_1=7$
x	1 0.4935 0.4935	1 0.5764 0.5764	0.6362 0.6362 1
w	0.1766 0.6468 0.1766	0.1826 0.6347 0.1826	0.1873 0.6253 0.1873
β	$\beta_0=3,\beta_1=8$	$\beta_0=3,\beta_1=9$	$\beta_0=3,\beta_1=10$
x	1 0.6813 1	1 0.7165 0.7165	1 0.7447 1
w	0.0954 0.8091 0.0954	0.1938 0.6123 0.1938	0.0980 0.8039 0.0980
β	$\beta_0=3,\beta_1=11$	$\beta_0=3,\beta_1=12$	$\beta_0=3,\beta_1=13$
x	0.7678 0.7679 1	1 0.7871 0.7871	0.8175 1 0.8175
w	0.1981 0.6037 0.1981	0.1997 0.6006 0.1997	0.3988 0.2023 0.3988
β	$\beta_0=4,\beta_1=1$	$\beta_0=4,\beta_1=2$	$\beta_0=4,\beta_1=3$
x	1 0 1	1 0 1	1 0.1797 0.1797
w	0.1287 0.7426 0.1287	0.0929 0.8141 0.0929	0.161 0.678 0.161
β	$\beta_0=4,\beta_1=4$	$\beta_0=4,\beta_1=5$	$\beta_0=4,\beta_1=6$
x	1 0.3718 0.3718	1 0.4935 0.4936	1 0.5764 0.5764
w	0.1688 0.6624 0.1688	0.1766 0.6468 0.1766	0.1826 0.6347 0.1826
β	$\beta_0=4,\beta_1=7$	$\beta_0=4,\beta_1=8$	$\beta_0=4,\beta_1=9$
x	0.6362 0.6362 1	1 0.6813 1	1 0.7165 0.7165
w	0.1873 0.6253 0.1873	0.0954 0.8091 0.0954	0.1938 0.6123 0.1938
β	$\beta_0=4,\beta_1=10$	$\beta_0=4,\beta_1=11$	$\beta_0=4,\beta_1=12$
x	1 0.7447 1	1 0.7678 1	1 0.7871 0.7871
w	0.0980 0.8039 0.0980	0.099 0.8019 0.099	0.1997 0.6006 0.1997
β	$\beta_0=4,\beta_1=13$	$\beta_0=5,\beta_1=1$	$\beta_0=5,\beta_1=2$
x	0.8035 0.8035 1	1 0 1	1 0 1
w	0.2011 0.5978 0.2011	0.1287 0.7426 0.1287	0.0929 0.8141 0.0929
β	$\beta_0=5,\beta_1=3$	$\beta_0=5,\beta_1=4$	$\beta_0=5,\beta_1=5$
x	1 0.1797 0.1797	1 0.3718 0.3718	1 0.4935 0.4935
w	0.161 0.678 0.161	0.1688 0.6624 0.1688	0.1766 0.6468 0.1766

β	$\beta_0=5,\beta_1=6$	$\beta_0=5,\beta_1=7$	$\beta_0=5,\beta_1=8$
x w	$\begin{Bmatrix}1 & 0.5764 & 0.5764\\ 0.1826 & 0.6347 & 0.1826\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.6362 & 1\\ 0.093 & 0.8127 & 0.093\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.6813 & 1\\ 0.0954 & 0.8091 & 0.0954\end{Bmatrix}$
β	$\beta_0=5,\beta_1=9$	$\beta_0=5,\beta_1=10$	$\beta_0=5,\beta_1=11$
x w	$\begin{Bmatrix}1 & 0.7165 & 0.7165\\ 0.1938 & 0.6123 & 0.1938\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.7447 & 1\\ 0.0980 & 0.8039 & 0.0980\end{Bmatrix}$	$\begin{Bmatrix}0.7678 & 0.7679 & 1\\ 0.1981 & 0.6037 & 0.1981\end{Bmatrix}$
β	$\beta_0=5,\beta_1=12$	$\beta_0=6,\beta_1=1$	$\beta_0=6,\beta_1=2$
x w	$\begin{Bmatrix}1 & 0.7871 & 0.7871\\ 0.1997 & 0.6006 & 0.1997\end{Bmatrix}$	$\begin{Bmatrix}1 & 0 & 1\\ 0.1287 & 0.7426 & 0.1287\end{Bmatrix}$	$\begin{Bmatrix}1 & 0 & 1\\ 0.0929 & 0.8141 & 0.0929\end{Bmatrix}$
β	$\beta_0=6,\beta_1=3$	$\beta_0=6,\beta_1=4$	$\beta_0=6,\beta_1=5$
x w	$\begin{Bmatrix}1 & 0.1797 & 0.1797\\ 0.161 & 0.678 & 0.161\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.3718 & 0.3718\\ 0.1688 & 0.6624 & 0.1688\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.4935 & 0.4935\\ 0.1766 & 0.6468 & 0.1766\end{Bmatrix}$
β	$\beta_0=6,\beta_1=6$	$\beta_0=6,\beta_1=7$	$\beta_0=6,\beta_1=8$
x w	$\begin{Bmatrix}1 & 0.5764 & 0.5764\\ 0.1826 & 0.6347 & 0.1826\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.6362 & 1\\ 0.093 & 0.8127 & 0.093\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.6813 & 1\\ 0.0954 & 0.8091 & 0.0954\end{Bmatrix}$
β	$\beta_0=6,\beta_1=9$	$\beta_0=6,\beta_1=10$	$\beta_0=7,\beta_1=1$
x w	$\begin{Bmatrix}1 & 0.7165 & 0.7165\\ 0.1938 & 0.6123 & 0.1938\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.7447 & 1\\ 0.0980 & 0.8039 & 0.0980\end{Bmatrix}$	$\begin{Bmatrix}1 & 0 & 1\\ 0.1287 & 0.7426 & 0.1287\end{Bmatrix}$
β	$\beta_0=7,\beta_1=2$	$\beta_0=7,\beta_1=3$	$\beta_0=7,\beta_1=4$
x w	$\begin{Bmatrix}1 & 0 & 1\\ 0.093 & 0.8136 & 0.093\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1797 & 0.1797\\ 0.161 & 0.678 & 0.161\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.3718 & 0.3718\\ 0.1688 & 0.6624 & 0.1688\end{Bmatrix}$
β	$\beta_0=7,\beta_1=5$	$\beta_0=7,\beta_1=6$	$\beta_0=7,\beta_1=7$
x w	$\begin{Bmatrix}1 & 0.4924 & 0.4851\\ 0.1737 & 0.6525 & 0.1737\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.5764 & 0.5764\\ 0.1826 & 0.6347 & 0.1826\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.6362 & 1\\ 0.0936 & 0.8127 & 0.0936\end{Bmatrix}$
β	$\beta_0=7,\beta_1=8$	$\beta_0=7,\beta_1=9$	$\beta_0=8,\beta_1=1$
x w	$\begin{Bmatrix}0.6813 & 0.6813 & 1\\ 0.1909 & 0.6181 & 0.1909\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.7165 & 0.7165\\ 0.1938 & 0.6123 & 0.1938\end{Bmatrix}$	$\begin{Bmatrix}1 & 0 & 1\\ 0.1287 & 0.7426 & 0.1287\end{Bmatrix}$
β	$\beta_0=8,\beta_1=2$	$\beta_0=8,\beta_1=3$	$\beta_0=8,\beta_1=5$
x w	$\begin{Bmatrix}1 & 0 & 1\\ 0.0929 & 0.8141 & 0.0929\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1797 & 0.1797\\ 0.161 & 0.678 & 0.161\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.4935 & 0.4935\\ 0.1766 & 0.6468 & 0.1766\end{Bmatrix}$

β	$\beta_0=8,\beta_1=5$	$\beta_0=9,\beta_1=1$	$\beta_0=9,\beta_1=2$
x w	$\begin{Bmatrix} 1 & 0.5764 & 0.5764 \\ 0.1826 & 0.6347 & 0.1826 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1287 & 0.7426 & 0.1287 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.0929 & 0.8141 & 0.0929 \end{Bmatrix}$
β	$\beta_0=9,\beta_1=4$	$\beta_0=9,\beta_1=5$	$\beta_0=10,\beta_1=1$
x w	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4935 & 0.4935 \\ 0.1766 & 0.6468 & 0.1766 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1287 & 0.7426 & 0.1287 \end{Bmatrix}$
β	$\beta_0=10,\beta_1=2$	$\beta_0=10,\beta_1=3$	$\beta_0=10,\beta_1=4$
x w	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.0929 & 0.8141 & 0.0929 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1797 & 0.1797 \\ 0.161 & 0.678 & 0.161 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=4$	$\beta_0=12,\beta_1=4$	$\beta_0=13,\beta_1=4$
x w	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$
β	$\beta_0=14,\beta_1=4$	$\beta_0=15,\beta_1=4$	-
x w	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3718 & 0.3718 \\ 0.1688 & 0.6624 & 0.1688 \end{Bmatrix}$	-