Pak. J. Statist.
2024 Vol. 40(2), 243-258

IMPROVED JOINT MONITORING OF MEAN AND VARIANCE WITH AN APPLICATION TO PRODUCTION DATA

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ABSTRACT

This study focuses on the application of Statistical Process Control (SPC) techniques to detect shifts in the production process. The use of control charts as an effective means for monitoring these shifts, we introduce a novel control chart based on the Max Extended Exponentially Weighted Moving Average (MxEEWMA) statistic. The proposed chart is designed for simultaneous monitoring of mean and variance shifts in the production process. An extensive numerical evaluation of the chart's performance is conducted, employing Average Run Lengths (ARLs) and Standard Deviation of Run Lengths (SDRLs) through Monte Carlo simulation. The comparative analyses are carried out between the Max EEWMA control chart and existing Max EWMA control charts. Furthermore, the practical applicability of the proposed control chart is illustrated using a real-world dataset.

KEYWORDS

EWMA, EEWMA Statistics, Max-EEWMA Control Chart, Joint Monitoring.

1. INTRODUCTION

Statistical quality control describes the application of statistical techniques in observing and maintaining the quality of goods and services. Statistical quality control, commonly referred as SPC, is a procedure for monitoring and evaluating the process through statistical study. It uses a statistical technique to evaluate and investigate the differences in process production. Most often, it is used in manufacturing processes to monitor process quality and maintain processes. Regardless of the quality of the design, it can guarantee that the product or service is produced as planned and designed. SPC may be used to sustain the reliability of how goods is produced, however, it will not improve consistency of poorly designed products. Hence, products are manufactured according to designed reliability.

In the 1920s, Walter A. Shewhart, renowned inventor of control charts, presented the idea of control charts to sustain the product's quality. He developed many control charts like R and S, which are based on information from the last observation and samples collected from the production process rather than past data. Later, (Roberts, 1959) and (Page, 1954) established the exponentially weighted moving average (EWMA) chart and cumulative sum (CUSUM) chart, which are more sensitive to monitoring small and moderate shifts in comparison to the preceding charts. Xie (1999) and Chen et al. (2001)

proposed the idea of combined supervising of both parameters mean shift as well as variance shift with a single control chart and named it the Maximum EWMA (Max-EWMA) control chart. Ostadsharif Memar and Niaki (2011) presented a new scheme of EWMA control charts to monitor both mean and variance concurrently by using a single statistic to overcome the difficulties in existing charts of monitoring both measurements by using two distinct charts. The simulation study of the proposed chart with other present different combinations of charts shows significantly better performance in monitoring both measurements simultaneously. Khoo et al. (2010) proposed a double EWMA control chart (Max-DEWMA) as an expansion of single Max-EWMA charts that significantly perform better in detecting both moderate and smaller shifts simultaneously in mean and variance. Naveed et al. (2018) established a generalized memory type control chart known as the Extended EWMA control chart to detect the small shifts, which is based on both past and current information from variables of interest with their past information and weights of EEWMA statistics. The proposed control chart efficiently detects the smaller shift as compared to other charts. (Noor-ul-Amin et al., 2019) suggested a new control chart for joint observing under pair ranked set sampling with likelihood ratio test. It was concluded that the recommended chart is efficient in identifying variance and mean shifts individually as well as jointly. Javaid et al. (2020) recommended a Max-EWMA control chart to analyze the influence of measurement errors on joint examining of mean and variance. The Covariate model technique, along with Max-EWMA, was worked to analyze the influence of measurement error and multiple measurements on the monitoring process. Measurement errors have a negative impact on Max-EWMA control chart efficiency, and it can be enhanced by taking multiple measurements of quality characteristics to identify smaller shifts during the monitoring process. (Karoon et al., 2022) presented a new idea of using the EEWMA control chart for exponential white noise with a first-order autoregressive (AR (1)) process. The implementation of the developed control chart was estimated through the application of real-life data of 24-hour absorption of PM10 (particulate matter) in polluted air, which is a major cause of the environmental problem of air pollution. Real life applications reveal that the suggested control chart outperforms the EWMA chart under several circumstances.(Chatterjee et al., 2022) developed a new TEWMA-Max (Triple EWMA-Max) control chart for combined examining of procedure mean and variability. (Engmann & Han, 2022) proposed a new method of multi-chart with EWMA and CUSUM for simultaneous detection of mean and variability during procedure.(Riaz et al., 2022) evaluate the measurement error effect on the implementation of combined monitoring of mean and coefficient of variation. The rest of the article is structured as, In Section 2, we evaluate the existing Max-EWMA chart for joint monitoring. In section 3, we explained the proposed Max-EEWMA control chart for simultaneously monitoring. To evaluate the performance of the proposed chart, a simulation study is explained in section 4. In section 5, a real-life case is presented to illustrate the suggested chart in a real-life application. The main findings are given in section 6, and the conclusion is presented in the concluding section.

2. MAX-EWMA CONTROL CHART FOR JOINT MONITORING

The existing Shewhart control chart utilizes entire available information in the present sample, but the EWMA control chart was developed in a way that most weights was provided to the latest subgroup and all preceding observations were given geometrically decreasing weights. The EWMA statistics are used to examine only the process production mean shift. Roberts (1959) proposed new EWMA statistics for the i^{th} sample of size 5 for monitoring only one parameter.

$$Z_i = \lambda \bar{Y}_i + (1-\lambda) Z_{i-1} \tag{1}$$

Xie (1999) and Chen et al. (2001) proposed an idea of combined observing of both parameters mean shift as well as variance shift with single control chart and known as it Maximum EWMA (Max-EWMA) control chart. If Y is normally distributed random variable of interest in process production with $\mu = \mu_0 + a\sigma_0$ mean and $\sigma^2 = b^2\sigma_0^2$ variance, where $\mu_0$and σ_0^2 are well-known parameters for variance and mean, respectively. The b and a are shifts in variance and mean, having 1 and 0 values for an in-control procedure, respectively. The Max-EWMA statistics can detect smaller shifts more efficiently. The transformed normally distributed statistics of mean and variance for in-control procedures with zero mean and unity variance for i^{th} sample are

$$U_i = \frac{\bar{Y}_i - \mu_0}{\sqrt{\sigma_0^2/n}}, \tag{2}$$

$$V_i = \emptyset^{-1}\left[H\left\{\frac{(n-1)S_i^2}{\sigma_0^2}, (n-1)\right\}\right], \tag{3}$$

where $i = 1,2,3, \dots, n$ is size of i^{th} sample, the mean $\bar{Y}_i = \frac{\sum_{i=1}^{n} Y_{ij}}{n}$ of i^{th} sample, and variance $S_i^2 = \frac{\sum_{i=1}^{n}\left(Y_{ij}-\bar{Y}_i\right)^2}{n-1}$ of i^{th} sample, $\emptyset^{-1}$ is the inverse function of standard normal distribution, while $H(\xi, v)$ follow χ^2 distribution with v degree of freedom.

By using equations (2) and (3), the two EWMA statistics are as follows

$$P_i = \lambda U_i + (1-\lambda) U_{i-1} \tag{4}$$

$$Q_i = \lambda V_i + (1-\lambda) V_{i-1}, \tag{5}$$

where $P_0 = Q_0 = 0$ for first sample and λ is smoothing constant, such that $0 < \lambda \leq 1$.

In proposed statistics the total of weights is unity. The quantities U_{i-1} and V_{i-1} denotes the variable's preceding value. We will use just one maximum absolute statistic for joint examining of mean and variance, instead of both statistics separately.

$$\text{Max} - \text{EWMA} = \text{Max}(|P_i|, |Q_i|) \tag{6}$$

Only one upper control limit (UCL) is adequate for plotting Max-EEWMA statistics for controlling production procedure.

$$UCL = (1.128379 + 0.602810 \times L)\sqrt{Var(P_j)} \tag{7}$$

where L is control constant and for in control process it is calculated to attain the required average run length, which is denoted by ARL_0, and variance is $Var(P_j) = Var(Q_j) = \sqrt{\frac{\lambda}{(2-\lambda)}}$.

3. PROPOSED MAX-EEWMA CONTROL CHART FOR JOINT MONITORING

In this part, the suggested Max-EEWMA control chart is established. We assumed that the quality characteristics U_i and V_i follows normal distribution. We proposed the Max- extended EWMA control chart by combining equation 4 and 5 with extended EWMA statistics proposed by (Naveed et al., 2018). Note that a critique paper is presented by Haq and Woodal (2023) on the modified EWMA versions and they specifically discussed the EEWMA version. The Khan et al. (2023) presented a rebuttal paper on the use of EEWMA statistic. The reader may consult these papers for further clarity. The design of the proposed control chart is given by:

Step 1: Choose a sample of "n" size. Calculate the two proposed EEWMA statistics.

The two proposed EEWMA statistics for mean and variance are

$$P_i = \psi_1 U_i - \psi_2 U_{i-1} + (1 - \psi_1 + \psi_2) P_{i-1}, \tag{8}$$

$$Q_i = \psi_1 V_i - \psi_2 V_{i-1} + (1 - \psi_1 + \psi_2) Q_{i-1}, \tag{9}$$

where $P_0 = Q_0 = 0$ for first sample and $0 < \psi_1 \leq 1$ and $0 \leq \psi_2 < \psi_1$.

In proposed statistics the total weight is one. The quantities U_{i-1} and V_{i-1} denotes the variable's preceding value and P_{i-1} and Q_{i-1} signifies the statistics preceding values.

For in-control, the mean and variance of EEWMA statistics are

$$E(P_j) = E(Q_j) = \mu \tag{10}$$

$$Var(P_j) = Var(Q_j) \tag{11}$$

$$Var(P_j) = \sigma^2 \left[(\psi_1^2 + \psi_2^2) \left\{ \frac{1 - a^{2i}}{2(\psi_1 - \psi_2) - (\psi_1 - \psi_2)^2} \right\} - 2a\psi_1\psi_2 \left\{ \frac{1 - a^{2i-2}}{2(\psi_1 - \psi_2) - (\psi_1 - \psi_2)^2} \right\} \right] \tag{12}$$

We will use only a single maximum absolute statistic for joint examining of mean and variance instead of both statistics separately.

Step 2: Choose the Max statistics from among two of proposed EEWMA statistics.

$$\text{Max-EEWMA} = \text{Max}\ (|P_i|, |Q_i|), \tag{13}$$

where $i = 1,2,3,\ldots$ Max-EEWMA statistics is maximum statistic out of two absolute statistics for mean and variance.

According to (Xie, 1999) by using absolute values only upper control limit (UCL) is adequate for plotting Max-EEWMA statistics for controlling production procedure.

Step 3: The process will be declared in-control when the proposed EEWMA statistic is beyond the limit otherwise it is out-of-control. The upper control limit is expressed by "*UCL*".

$$UCL = (1.128379 + 0.602810 \times L)\sqrt{Var(P_j)} \tag{14}$$

where L is control constant and for in control process it is calculated to attain the required average run length, which is denoted by ARL_0, and variance is $Var(P_j) = Var(Q_j)$ for large repetitive samples when both P_j and Q_j follow the standard normal distribution.

4. SIMULATION STUDY

In this segment the suggested chart is evaluated by an extensive study for this purpose the statistic is generating from normal distribution such that $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$. Algorithm for calculating ARL and SDRL for Max-EEWMA is given below:

Step 1: Control statistics for sample variance and mean.

i. For the in-control process, generate 30000 random samples of "n" size from normal distribution.
ii. Compute the proposed statistic for every sample.

Step 2: Establishing Control limits.

i. Choose the initial values of two parameters L and λ.
ii. Calculate the statistic for both mean and variance.
iii. Determine the transformed statistic for mean and variance to attain Max-EEWMA statistics (Haq, 2017).
iv. Evaluate the proposed statistic for out-of-control signal. When procedure is declared as in-control, then proceed to step v, Else record samples number as run length for in-control.
v. Replicate steps from II to IV 30000 times to evaluate ARL_0. If we attain the required ARL_0 then go to step 3 with existing λ value. Otherwise, change value of λ and repeat steps II to V of step-2.

Step 3: Calculating the *ARL* for out-of-control

i. Generate a random variable Y from normal distribution at each sample by considering a mean shift, i.e. $Y \sim N(\delta, 1)$.
ii. Evaluate the proposed statistic for the sample.
iii. Steps I and II from step 3 will repeat till the procedure is considered as out-of-control. The number of samples will be recorded as a run-length.
iv. The entire procedure will be repeated 10,000 times to evaluate values of *ARL* and *SDRL*.
v. Compute upper control limit and compare the *UCL* and Max-EEWMA statistics. The process will be in control if Max-EEWMA remain under the *UCL*. After that, generate another sample and repeat steps I to IV.
vi. The procedure has been shifted to be out of control if Max-EEWMA statistic is outside *UCL* limit, and the production procedure will stop, and the required run length will be achieved.

We have utilized 30,000 replicates to compute every *ARL* and *SDRL*. We arrived at $L = 2.278$ to attain $ARL_0 = 370$ for $\psi_1 = 0.15$ and $\psi_2 = 0.03$. Several combinations for mean shift *i.e.,* $a = 0.0$, 0.1, 0.25, 0.50, 0.75, 1, 1.5, 2.0 and for variance shift i.e.,

$b = 0.25, 0.50, 0.75, 0.90, 1.00, 1.10, 1.25, 1.50, 2.00, 2.50, 3.00$ are used in this study. We can observe from Table 1 that Max-EEWMA chart can efficiently detect procedure mean and variance shifts simultaneously. It can be easily observed that *ARLs* decay as mean shift increases and, similarly, *ARLs* decrease with the increase in variance shift as well, which indicates the capability of detecting both shifts at an early stage.

Table 1

***ARLs* and *SDRLs* of Max-EEWMA Chart and Max-EWMA Chart for $\psi_1 = \lambda = 0.154$**

Shifts		ψ_2=0.03 L=2.278		ψ_2=0.05 L=1.628		ψ_2=0.07 L=0.984		λ=0.15 L=3.168	
b	***a***	***ARL***	***SDRL***	***ARL***	***SDRL***	***ARL***	***SDRL***	***ARL***	***SDRL***
0.25	0.00	1.536	0.581	1.259	0.475	1.082	0.285	3.034	0.444
	0.10	1.528	0.581	1.258	0.474	1.083	0.288	3.034	0.448
	0.25	1.529	0.584	1.261	0.477	1.084	0.289	3.034	0.442
	0.50	1.533	0.582	1.260	0.476	1.081	0.286	3.032	0.448
	0.75	1.535	0.583	1.251	0.467	1.067	0.256	3.032	0.443
	1.00	1.441	0.508	1.114	0.320	1.007	0.083	2.978	0.37
	1.50	1.000	0.016	1.000	0.000	1.000	0.000	2.084	0.278
	2.00	1.000	0.000	1.000	0.000	1.000	0.000	2.000	0.000
0.50	0.00	4.381	2.108	3.913	2.203	3.264	2.224	5.882	1.688
	0.10	4.356	2.080	3.894	2.205	3.218	2.215	5.892	1.693
	0.25	4.371	2.084	3.905	2.193	3.244	2.238	5.878	1.684
	0.50	4.165	1.864	3.670	1.991	2.897	1.966	5.725	1.484
	0.75	3.016	1.201	2.502	1.270	1.864	1.128	4.667	0.874
	1.00	1.978	0.774	1.522	0.705	1.218	0.503	3.52	0.585
	1.50	1.051	0.220	1.009	0.097	1.001	0.035	2.249	0.432
	2.00	1.000	0.005	1.000	0.000	1.000	0.000	1.995	0.067
0.75	0.00	20.686	14.521	19.213	13.888	17.274	13.404	23.942	16.262
	0.10	20.587	14.542	18.950	13.585	17.334	13.772	23.742	16.217
	0.25	17.128	11.146	15.757	10.771	13.956	10.831	20	12.194
	0.50	7.210	3.866	6.470	4.039	5.387	4.154	8.944	3.463
	0.75	3.463	1.793	2.956	1.801	2.374	1.687	5.116	1.441
	1.00	2.070	1.023	1.739	0.960	1.449	0.803	3.614	0.833
	1.50	1.144	0.366	1.066	0.258	1.022	0.153	2.333	0.485
	2.00	1.005	0.072	1.001	0.035	1.000	0.015	1.963	0.211
0.90	0.00	151.393	147.460	138.618	137.888	122.703	129.558	171.722	162.445
	0.10	110.456	106.430	99.204	97.557	89.191	93.411	127.085	117.567
	0.25	29.201	23.424	26.620	21.914	23.227	20.978	34.504	26.3
	0.50	7.368	4.639	6.681	4.771	5.616	4.750	9.287	4.283
	0.75	3.488	2.023	2.998	2.008	2.496	1.890	5.172	1.725
	1.00	2.100	1.140	1.826	1.079	1.530	0.924	3.632	0.994
	1.50	1.197	0.437	1.110	0.338	1.049	0.233	2.375	0.526
	2.00	1.016	0.126	1.005	0.074	1.002	0.044	1.942	0.294
1.00	0.00	371.998	386.464	370.102	402.895	370.918	447.495	373.574	369.346
	0.10	144.198	144.138	132.407	139.297	118.257	137.058	158.957	150.107
	0.25	27.580	23.214	24.830	22.382	21.379	21.309	32.336	25.219
	0.50	7.139	4.832	6.408	4.914	5.341	4.832	9.213	4.565
	0.75	3.464	2.144	3.007	2.128	2.505	1.988	5.196	1.887
	1.00	2.138	1.217	1.850	1.141	1.582	0.997	3.668	1.096
	1.50	1.234	0.486	1.141	0.396	1.076	0.296	2.387	0.564
	2.00	1.025	0.158	1.011	0.107	1.005	0.073	1.928	0.348

Shifts		ψ_2=0.03 L=2.278		ψ_2=0.05 L=1.628		ψ_2=0.07 L=0.984		λ=0.15 L=3.168	
b	*a*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*	*ARL*	*SDRL*
1.10	0.00	77.946	78.310	71.205	76.808	61.025	74.903	88.954	83.21
	0.10	55.939	55.130	50.578	53.464	42.559	50.720	64.791	58.352
	0.25	21.069	18.277	18.777	17.659	15.513	16.881	25.574	19.328
	0.50	6.736	4.954	5.927	4.868	4.857	4.670	8.994	4.673
	0.75	3.422	2.269	2.941	2.187	2.422	1.981	5.181	2.024
	1.00	2.145	1.285	1.871	1.205	1.601	1.035	3.688	1.189
	1.50	1.268	0.529	1.165	0.428	1.098	0.345	2.41	0.598
	2.00	1.039	0.197	1.020	0.145	1.009	0.096	1.92	0.392
1.25	0.00	15.417	14.031	13.481	13.717	10.822	12.653	19.829	14.413
	0.10	14.515	13.035	12.507	12.431	9.993	11.787	18.587	13.22
	0.25	10.573	9.135	9.236	8.961	7.382	8.523	14.207	9.238
	0.50	5.435	4.291	4.704	4.189	3.749	3.803	8.043	4.134
	0.75	3.151	2.260	2.746	2.161	2.236	1.897	5.081	2.12
	1.00	2.112	1.342	1.858	1.249	1.581	1.050	3.68	1.304
	1.50	1.296	0.577	1.196	0.489	1.120	0.383	2.427	0.665
	2.00	1.061	0.249	1.033	0.185	1.016	0.131	1.912	0.455
1.50	0.00	4.711	4.005	3.935	3.751	3.103	3.252	7.309	3.949
	0.10	4.570	3.895	3.876	3.671	3.035	3.241	7.188	3.836
	0.25	4.236	3.558	3.557	3.339	2.798	2.868	6.815	3.512
	0.50	3.337	2.673	2.799	2.446	2.271	2.107	5.637	2.638
	0.75	2.500	1.843	2.131	1.649	1.773	1.369	4.441	1.856
	1.00	1.915	1.258	1.680	1.125	1.473	0.950	3.505	1.318
	1.50	1.316	0.520	1.217	0.533	1.139	0.432	2.446	0.763
	2.00	1.088	0.306	1.055	0.245	1.030	0.183	1.91	0.546
2.00	0.00	1.814	1.234	1.601	1.077	1.400	0.863	3.406	1.44
	0.10	1.817	1.239	1.598	1.082	1.393	0.842	3.395	1.429
	0.25	1.779	1.194	1.579	1.044	1.391	0.845	3.338	1.397
	0.50	1.704	1.111	1.504	0.936	1.345	0.771	3.215	1.309
	0.75	1.570	0.972	1.411	0.821	1.275	0.661	2.992	1.193
	1.00	1.435	0.802	1.317	0.692	1.216	0.560	2.75	1.046
	1.50	1.247	2.930	1.168	0.465	1.105	0.366	2.254	0.804
	2.00	1.105	0.328	1.071	0.279	1.043	0.428	1.857	0.64
2.50	0.00	1.303	0.635	1.217	0.544	1.143	0.429	2.352	0.93
	0.10	1.307	0.644	1.219	0.543	1.144	0.434	2.357	0.932
	0.25	1.293	0.626	1.206	0.524	1.138	0.421	2.349	0.92
	0.50	1.276	0.600	1.196	0.506	1.129	0.407	2.295	0.901
	0.75	1.251	0.570	1.170	0.465	1.117	0.389	2.227	0.871
	1.00	1.209	0.511	1.148	0.429	1.101	0.352	2.134	0.817
	1.50	1.132	0.396	1.093	0.330	1.062	0.269	1.927	0.729
	2.00	1.080	0.300	1.060	1.222	1.054	2.422	1.704	0.64
3.00	0.00	1.134	0.394	1.092	0.323	1.064	0.270	1.859	0.741
	0.10	1.137	0.399	1.094	0.333	1.061	0.259	1.854	0.731
	0.25	1.136	0.396	1.093	0.330	1.062	0.266	1.857	0.74
	0.50	1.128	0.384	1.091	0.324	1.056	0.251	1.828	0.728
	0.75	1.121	0.372	1.081	0.303	1.054	0.250	1.799	0.719
	1.00	1.107	0.350	1.075	0.294	1.048	0.231	1.759	0.694
	1.50	1.079	0.295	1.056	0.253	1.036	0.198	1.646	0.649
	2.00	1.052	0.234	1.036	0.196	1.024	0.218	1.521	0.594

Table 2

***ARLs* and *SDRLs* of Max-EEWMA Chart and Max-EWMA Chart for $\psi_1 = \lambda = 0.2$**

Shifts		ψ_2=0.03 L=2.630		ψ_2=0.05 L=2.167		ψ_2=0.07 L=1.688		ψ_2=0.1 L=0.977		λ=0.2 L=3.250	
b	***a***	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
0.25	0.00	1.718	0.613	1.505	0.594	1.297	0.510	1.091	0.301	2.817	0.495
	0.10	1.721	0.609	1.504	0.589	1.297	0.506	1.093	0.301	2.823	0.488
	0.25	1.716	0.611	1.511	0.595	1.297	0.507	1.088	0.297	2.820	0.495
	0.50	1.727	0.613	1.505	0.598	1.298	0.509	1.090	0.301	2.819	0.493
	0.75	1.724	0.613	1.506	0.598	1.297	0.511	1.073	0.275	2.817	0.497
	1.00	1.648	0.526	1.400	0.509	1.154	0.366	1.008	0.091	2.779	0.437
	1.50	1.005	0.072	1.000	0.011	1.000	0.000	1.000	0.000	2.002	0.047
	2.00	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.997	0.059
0.50	0.00	4.828	2.221	4.568	2.304	4.202	2.397	3.460	2.480	5.781	1.920
	0.10	4.798	2.200	4.564	2.301	4.213	2.389	3.436	2.456	5.779	1.945
	0.25	4.800	2.227	4.554	2.298	4.220	2.401	3.469	2.472	5.781	1.944
	0.50	4.619	1.991	4.363	2.094	3.994	2.190	3.133	2.206	5.614	1.696
	0.75	3.364	1.235	3.116	1.329	2.725	1.389	1.986	1.259	4.447	0.954
	1.00	2.174	0.784	1.914	0.827	1.622	0.785	1.245	0.550	3.279	0.574
	1.50	1.105	0.307	1.043	0.204	1.013	0.114	1.001	0.038	2.078	0.268
	2.00	1.000	0.019	1.000	0.005	1.000	0.000	1.000	0.000	1.910	0.286
0.75	0.00	26.146	20.651	24.449	18.947	22.784	17.554	20.278	16.644	29.282	22.840
	0.10	26.071	20.457	24.283	18.693	22.679	17.684	20.071	16.595	29.230	22.876
	0.25	21.821	16.239	20.087	14.598	18.858	13.865	16.295	13.414	24.383	17.620
	0.50	8.163	4.458	7.764	4.489	7.190	4.521	5.987	4.755	9.326	4.354
	0.75	3.827	1.859	3.529	1.903	3.179	1.959	2.512	1.831	4.924	1.583
	1.00	2.284	1.071	2.020	1.087	1.828	1.025	1.483	0.856	3.376	0.868
	1.50	1.217	0.438	1.133	0.359	1.077	0.283	1.025	0.165	2.170	0.403
	2.00	1.010	0.101	1.004	0.073	1.001	0.039	1.000	0.012	1.816	0.390
0.90	0.00	198.424	194.081	188.241	186.146	173.917	173.672	154.140	167.376	214.507	206.942
	0.10	147.386	143.397	135.694	132.815	126.643	127.677	111.522	120.248	159.722	153.218
	0.25	38.146	32.776	35.117	30.579	32.039	28.357	27.554	26.687	42.633	35.954
	0.50	8.270	5.161	7.819	5.172	7.244	5.181	5.979	5.189	9.583	5.090
	0.75	3.830	2.136	3.556	2.158	3.203	2.157	2.596	2.018	4.964	1.879
	1.00	2.312	1.194	2.116	1.192	1.902	1.150	1.586	0.998	3.401	1.018
	1.50	1.277	0.502	1.195	0.446	1.124	0.366	1.058	0.260	2.206	0.479
	2.00	1.028	0.166	1.013	0.118	1.006	0.083	1.001	0.043	1.780	0.429
1.00	0.00	369.800	374.773	371.766	385.269	370.499	394.751	370.973	440.843	369.211	365.107
	0.10	169.640	169.406	162.023	164.178	153.926	160.530	137.670	161.214	180.288	174.944
	0.25	33.342	29.477	31.289	27.931	28.826	26.728	24.347	25.339	37.803	32.299
	0.50	7.943	5.376	7.483	5.302	6.902	5.322	5.678	5.248	9.369	5.271
	0.75	3.788	2.262	3.526	2.294	3.174	2.264	2.586	2.092	4.968	2.008
	1.00	2.333	1.277	2.136	1.264	1.936	1.228	1.626	1.055	3.421	1.121
	1.50	1.305	0.543	1.229	0.489	1.160	0.422	1.081	0.313	2.220	0.528
	2.00	1.043	0.206	1.023	0.153	1.014	0.121	1.004	0.071	1.759	0.453

Shifts		ψ_2=0.03 L=2.630		ψ_2=0.05 L=2.167		ψ_2=0.07 L=1.688		ψ_2=0.1 L=0.977		λ=0.2 L=3.250	
b	***a***	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
1.10	0.00	87.720	88.458	84.122	86.471	77.400	83.589	67.287	82.881	95.971	90.800
	0.10	65.056	63.328	60.437	61.438	55.847	59.500	47.252	56.974	71.193	65.244
	0.25	24.378	21.856	22.405	20.413	20.722	20.044	17.025	18.988	27.866	22.882
	0.50	7.422	5.362	6.951	5.296	6.263	5.166	5.151	5.004	9.026	5.277
	0.75	3.702	2.366	3.426	2.388	3.094	2.319	2.494	2.075	4.971	2.164
	1.00	2.331	1.351	2.149	1.335	1.944	1.259	1.652	1.117	3.457	1.216
	1.50	1.337	0.585	1.257	0.530	1.185	0.469	1.103	0.358	2.233	0.585
	2.00	1.059	0.241	1.037	0.193	1.021	0.149	1.010	0.105	1.746	0.478
1.25	0.00	17.420	15.809	16.125	15.219	14.566	14.635	11.379	13.762	20.488	16.080
	0.10	16.290	14.576	15.004	14.235	13.468	13.639	10.564	12.631	19.423	15.129
	0.25	11.879	10.145	11.039	9.876	9.841	9.608	7.664	8.810	14.358	10.235
	0.50	5.998	4.562	5.542	4.503	4.932	4.349	3.877	3.999	7.778	4.431
	0.75	3.472	2.376	3.140	2.291	2.815	2.217	2.296	1.939	4.807	2.199
	1.00	2.296	1.409	2.113	1.379	1.918	1.306	1.620	1.111	3.438	1.317
	1.50	1.369	0.637	1.289	0.589	1.219	0.528	1.128	0.417	2.260	0.663
	2.00	1.084	0.287	1.056	0.239	1.036	0.197	1.017	0.136	1.742	0.514
1.50	0.00	5.190	4.259	4.677	4.098	4.120	3.909	3.147	3.352	7.006	4.097
	0.10	5.062	4.092	4.593	3.975	4.028	3.806	3.104	3.344	6.953	4.064
	0.25	4.655	3.785	4.229	3.600	3.714	3.440	2.856	2.940	6.455	3.626
	0.50	3.628	2.781	3.290	2.692	2.905	2.523	2.291	2.151	5.266	2.700
	0.75	2.712	1.928	2.472	1.852	2.189	1.713	1.812	1.432	4.162	1.889
	1.00	2.071	1.331	1.899	1.270	1.725	1.173	1.483	0.960	3.252	1.336
	1.50	1.392	0.687	1.306	0.630	1.238	0.565	1.145	0.452	2.258	0.767
	2.00	1.119	0.350	1.084	0.300	1.059	0.257	1.033	0.198	1.731	0.572
2.00	0.00	1.963	1.325	1.788	1.216	1.637	1.122	1.405	0.858	3.143	1.425
	0.10	1.962	1.319	1.792	1.225	1.627	1.098	1.405	0.851	3.132	1.435
	0.25	1.918	1.280	1.759	1.185	1.606	1.062	1.393	0.845	3.091	1.388
	0.50	1.821	1.185	1.666	1.078	1.533	0.973	1.350	0.776	2.944	1.305
	0.75	1.665	1.014	1.558	0.959	1.434	0.929	1.281	0.666	2.742	1.162
	1.00	1.533	0.880	1.435	0.803	1.347	0.723	1.222	0.573	2.518	1.025
	1.50	1.279	0.576	1.125	0.554	1.281	6.769	1.211	8.990	2.044	0.785
	2.00	1.118	0.361	1.094	0.322	1.258	9.559	1.040	0.218	1.684	0.633
2.50	0.00	1.358	0.693	1.288	0.620	1.226	0.554	1.144	0.427	2.157	0.924
	0.10	1.356	0.686	1.291	0.625	1.223	0.546	1.149	0.438	2.154	0.913
	0.25	1.356	0.696	1.287	0.629	1.226	0.549	1.143	0.436	2.131	0.914
	0.50	1.325	0.650	1.267	0.595	1.206	0.521	1.132	0.407	2.103	0.891
	0.75	1.293	0.611	1.240	0.588	1.178	0.476	1.119	0.389	2.029	0.846
	1.00	1.253	0.561	1.202	0.504	1.153	0.438	1.099	0.346	1.951	0.809
	1.50	1.173	0.450	1.137	0.412	1.101	0.348	1.070	0.445	1.753	0.709
	2.00	1.110	1.108	1.092	1.211	1.112	4.028	1.065	1.867	1.549	0.611
3.00	0.00	1.168	0.444	1.131	0.388	1.100	0.340	1.064	0.267	1.696	0.710
	0.10	1.163	0.434	1.129	0.386	1.098	0.336	1.064	0.264	1.695	0.712
	0.25	1.162	0.437	1.130	0.386	1.099	0.341	1.061	0.262	1.690	0.712
	0.50	1.154	0.421	1.119	0.366	1.093	0.323	1.058	0.252	1.670	0.699
	0.75	1.145	0.406	1.112	0.363	1.086	0.312	1.056	0.246	1.646	0.690
	1.00	1.129	0.384	1.104	0.345	1.080	0.303	1.050	0.235	1.604	0.667
	1.50	1.098	0.334	1.078	0.297	1.056	0.251	1.036	0.254	1.502	0.614
	2.00	1.070	0.447	1.051	0.236	1.039	0.278	1.024	0.171	1.397	0.552

Table 3
ARLs and SDRLs of Max-EEWMA Chart and Max-EWMA Chart for ψ_1=λ =0.3

Shifts		ψ_2=0.03 L=3.002		ψ_2=0.05 L=2.737		ψ_2=0.07 L=2.459		ψ_2=0.1 L=2.019		λ=0.3 L=3.344	
b	***a***	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
0.25	0.00	1.927	0.635	1.819	0.653	1.693	0.661	1.486	0.629	2.535	0.549
	0.10	1.932	0.639	1.818	0.658	1.688	0.658	1.491	0.623	2.541	0.550
	0.25	1.931	0.627	1.817	0.654	1.689	0.662	1.492	0.626	2.537	0.549
	0.50	1.934	0.634	1.822	0.663	1.703	0.675	1.498	0.633	2.534	0.547
	0.75	1.928	0.638	1.830	0.667	1.707	0.679	1.503	0.644	2.537	0.547
	1.00	1.856	0.547	1.748	0.577	1.626	0.589	1.381	0.540	2.501	0.506
	1.50	1.043	0.202	1.010	0.103	1.002	0.045	1.000	0.011	1.999	0.036
	2.00	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.397	0.489
0.50	0.00	5.629	2.809	5.517	2.806	5.435	2.845	5.126	2.893	6.161	2.767
	0.10	5.667	2.894	5.536	2.818	5.399	2.833	5.104	2.876	6.176	2.758
	0.25	5.634	2.800	5.579	2.845	5.421	2.818	5.155	2.907	6.174	2.763
	0.50	5.421	2.534	5.352	2.563	5.210	2.584	4.957	2.681	5.948	2.467
	0.75	3.877	1.413	3.798	1.450	3.677	1.511	3.449	1.642	4.382	1.225
	1.00	2.459	0.827	2.370	0.879	2.246	0.929	2.008	0.980	3.041	0.661
	1.50	1.206	0.406	1.134	0.343	1.087	0.285	1.035	0.188	1.999	0.189
	2.00	1.001	0.036	1.000	0.021	1.000	0.016	1.000	0.008	1.449	0.497
0.75	0.00	43.155	39.134	40.819	36.343	38.848	34.644	35.907	31.594	47.359	42.514
	0.10	42.507	38.004	40.708	36.365	38.741	34.266	35.370	30.735	47.197	42.589
	0.25	36.014	31.491	34.247	29.639	32.208	27.661	29.949	25.623	39.550	34.971
	0.50	10.695	6.970	10.281	6.612	9.985	6.423	9.451	6.218	11.665	7.309
	0.75	4.379	2.179	4.265	2.171	4.146	2.229	3.848	2.281	4.959	2.029
	1.00	2.558	1.144	2.463	1.179	2.354	1.212	2.141	1.227	3.157	0.980
	1.50	1.313	0.504	1.253	0.477	1.198	0.437	1.128	0.368	2.001	0.408
	2.00	1.020	0.143	1.013	0.116	1.008	0.089	1.003	0.062	1.462	0.499
0.90	0.00	287.249	286.347	276.117	275.772	269.983	272.165	254.857	257.569	299.460	293.569
	0.10	217.433	215.160	210.320	208.816	203.714	202.804	191.203	191.939	230.117	224.655
	0.25	61.090	58.194	57.704	53.868	54.440	51.245	51.413	48.718	66.002	61.581
	0.50	10.289	7.245	9.958	7.070	9.652	6.814	9.040	6.594	11.269	7.645
	0.75	4.325	2.409	4.210	2.441	4.079	2.462	3.778	2.488	4.946	2.283
	1.00	2.558	1.293	2.475	1.315	2.389	1.345	2.200	1.342	3.190	1.135
	1.50	1.367	0.563	1.311	0.542	1.261	0.520	1.188	0.467	2.002	0.508
	2.00	1.046	0.212	1.031	0.176	1.023	0.151	1.013	0.116	1.471	0.503
1.00	0.00	371.865	376.106	369.095	373.644	370.813	375.749	371.947	386.804	373.733	369.780
	0.10	209.774	208.190	205.970	206.217	200.174	200.798	192.295	197.184	218.830	215.287
	0.25	47.401	44.696	45.349	42.734	43.728	41.413	40.396	38.588	50.703	46.826
	0.50	9.609	7.161	9.240	6.842	8.968	6.691	8.399	6.554	10.520	7.262
	0.75	4.231	2.534	4.147	2.550	3.998	2.556	3.723	2.581	4.909	2.416
	1.00	2.574	1.377	2.488	1.395	2.378	1.417	2.210	1.427	3.206	1.235
	1.50	1.400	0.604	1.346	0.586	1.289	0.560	1.223	0.516	2.002	0.564
	2.00	1.063	0.248	1.050	0.221	1.037	0.193	1.023	0.155	1.480	0.509

Shifts		ψ_2=0.03 L =3.002		ψ_2=0.05 L =2.737		ψ_2=0.07 L =2.459		ψ_2=0.1 L =2.019		λ =0.3 L =3.344	
b	***a***	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
1.10	0.00	102.208	101.355	99.405	101.402	95.766	98.212	93.106	97.426	106.428	102.871
	0.10	78.103	76.511	75.667	75.570	73.649	74.048	69.460	72.252	82.146	78.781
	0.25	30.996	28.960	29.470	27.316	28.851	27.360	26.392	25.645	33.191	29.442
	0.50	8.667	6.604	8.296	6.353	8.070	6.307	7.486	6.106	9.656	6.598
	0.75	4.107	2.608	3.982	2.619	3.845	2.608	3.588	2.609	4.858	2.496
	1.00	2.563	1.465	2.466	1.466	2.366	1.464	2.199	1.455	3.221	1.336
	1.50	1.427	0.642	1.374	0.633	1.327	0.611	1.260	0.572	2.006	0.617
	2.00	1.086	0.290	1.068	0.260	1.052	0.231	1.035	0.197	1.487	0.517
1.25	0.00	20.616	19.125	19.830	18.906	18.847	17.922	17.533	17.529	22.791	19.560
	0.10	19.220	17.657	18.221	16.982	17.634	16.606	16.189	16.136	21.296	18.406
	0.25	13.885	12.097	13.274	11.780	12.687	11.541	11.735	11.188	15.390	12.346
	0.50	6.765	5.189	6.567	5.204	6.304	5.137	5.751	4.962	7.896	5.247
	0.75	3.804	2.542	3.650	2.546	3.539	2.543	3.243	2.488	4.613	2.460
	1.00	2.519	1.516	2.409	1.509	2.309	1.511	2.180	1.519	3.199	1.414
	1.50	1.460	0.694	1.421	0.695	1.367	0.671	1.287	0.617	2.024	0.697
	2.00	1.112	0.333	1.094	0.310	1.078	0.289	1.053	0.242	1.495	0.532
1.50	0.00	5.691	4.628	5.442	4.535	5.131	4.353	4.659	4.253	6.849	4.556
	0.10	5.568	4.472	5.346	4.407	5.118	4.354	4.588	4.158	6.715	4.471
	0.25	5.149	4.052	4.908	4.006	4.696	3.943	4.199	3.768	6.293	4.050
	0.50	4.019	2.969	3.793	2.893	3.653	2.901	3.305	2.779	4.970	2.913
	0.75	2.979	2.065	2.848	2.031	2.706	1.993	2.483	1.922	3.816	1.998
	1.00	2.234	1.408	2.170	1.415	2.080	1.408	1.908	1.334	2.964	1.388
	1.50	1.474	0.748	1.428	0.732	1.378	0.708	1.305	0.660	2.005	0.793
	2.00	1.152	0.393	1.126	0.363	1.110	0.351	1.079	0.299	1.516	0.569
2.00	0.00	2.109	1.414	2.008	1.367	1.925	1.322	1.756	1.203	2.827	1.437
	0.10	2.095	1.410	2.008	1.362	1.899	1.302	1.756	1.196	2.830	1.460
	0.25	2.055	1.342	1.969	1.311	1.874	1.270	1.744	1.198	2.786	1.415
	0.50	1.937	1.236	1.858	1.215	1.779	1.161	1.669	1.099	2.649	1.309
	0.75	1.791	1.105	1.715	1.060	1.645	1.023	1.539	0.936	2.436	1.166
	1.00	1.626	0.945	1.571	0.908	1.505	0.866	1.426	0.802	2.229	1.028
	1.50	1.354	1.042	1.274	0.587	1.272	0.592	1.246	0.553	1.805	0.773
	2.00	1.156	0.397	1.114	0.394	1.094	0.311	1.096	0.353	1.480	0.597
2.50	0.00	1.425	0.743	1.390	0.723	1.337	0.678	1.280	0.611	1.896	0.889
	0.10	1.428	0.750	1.385	0.712	1.341	0.669	1.274	0.615	1.891	0.897
	0.25	1.412	0.729	1.375	0.706	1.321	0.647	1.271	0.597	1.892	0.878
	0.50	1.393	0.716	1.353	0.677	1.310	0.638	1.256	0.583	1.848	0.853
	0.75	1.350	0.661	1.310	0.629	1.285	0.612	1.231	0.546	1.787	0.820
	1.00	1.303	0.611	1.272	0.580	1.246	0.556	1.196	0.495	1.715	0.768
	1.50	1.207	0.813	1.191	0.899	1.164	0.448	1.162	2.482	1.541	0.666
	2.00	1.128	0.822	1.138	2.023	1.177	4.254	1.158	4.093	1.369	0.550
3.00	0.00	1.204	0.482	1.177	0.450	1.150	0.420	1.127	0.384	1.503	0.665
	0.10	1.201	0.483	1.173	0.445	1.155	0.423	1.122	0.378	1.501	0.662
	0.25	1.196	0.470	1.174	0.448	1.156	0.430	1.122	0.375	1.504	0.663
	0.50	1.190	0.466	1.171	0.445	1.145	0.406	1.119	0.372	1.484	0.653
	0.75	1.180	0.453	1.157	0.427	1.136	0.395	1.110	0.356	1.461	0.631
	1.00	1.159	0.423	1.136	0.394	1.122	0.376	1.100	0.337	1.424	0.609
	1.50	1.115	0.354	1.105	0.341	1.094	0.329	1.072	0.362	1.344	0.548
	2.00	1.082	0.307	1.075	0.381	1.066	0.453	1.051	0.376	1.261	0.480

5. ILLUSTRATION EXAMPLE

We used data set given by (Costa & Castagliola, 2011) and (Hu et al., 2015), to evaluate the implementation of developed chart. The data consists of 200 values during the procedure of yogurt packing in a cup with the quality characteristic "Y" set at 125 grams. The in-control parameters $\mu_o = 124.9$ and $\sigma_o = 0.76$ mean and standard deviation, respectively, have been calculated after the long-term phase-I study. To retain the quality of the cup filling, after every hour the quality control specialists pick a sample and record its weight twice. In the above-mentioned papers, 20 samples of size 5 have been formed, and we utilize the same data set in this paper, but we consider the first 100 data values as in-control and the rest of the 100 values as shifted.

Table 4
Max-EEWMA Statistics and Max-EWMA Statistics for Yogurt Filling Data Set

Sample	MAX-EEWMA	*UCL*	MAX-EWMA	UCL_1
1	0.2718588	0.9001873	0.2718588	1.064736
2	0.1502446	0.9001873	0.1931716	1.064736
3	0.1888789	0.9001873	0.2618548	1.064736
4	0.3364976	0.9001873	0.3248089	1.064736
5	0.3199126	0.9001873	0.4363788	1.064736
6	0.2831080	0.9001873	0.3959245	1.064736
7	0.3084560	0.9001873	0.4240572	1.064736
8	0.4238823	0.9001873	0.4607316	1.064736
9	0.3518010	0.9001873	0.4645682	1.064736
10	0.8265119	0.9001873	0.9259101	1.064736
11	0.3869105	0.9001873	0.4904952	1.064736
12	0.5703074	0.9001873	0.5805294	1.064736
13	0.8086730	0.9001873	1.1434820	1.064736
14	0.8602045	0.9001873	1.3867137	1.064736
15	1.5515119	0.9001873	2.1226628	1.064736
16	1.6040796	0.9001873	2.4124951	1.064736
17	1.8161535	0.9001873	2.6596604	1.064736
18	1.9464108	0.9001873	2.8044331	1.064736
19	2.1572042	0.9001873	2.9873333	1.064736
20	2.1780362	0.9001873	2.9983125	1.064736

We attain $\text{ARL}_0 = 370$ by using $L = 3.427$, $A = 0$, $B = 1$, and $\lambda = 0.2$ by using the first 100 values of the data set. After that, we select 20 samples of each size 5 of the yogurt cup in such a way that the first 10 samples are selected from in-control values and the rest of the 10 values are selected from the last 100 shifted values. Afterward, we calculated the proposed statistic Max-EEWMA as well as *UCL* from all 20 values. Values of statistics and *UCLs* are given in Table 4.

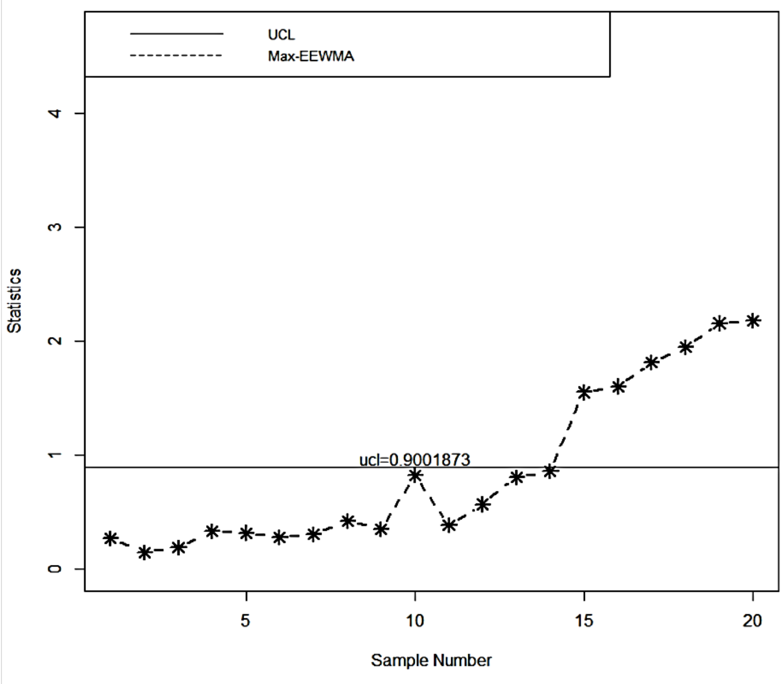

Figure 1: Max-EEWMA Control Chart

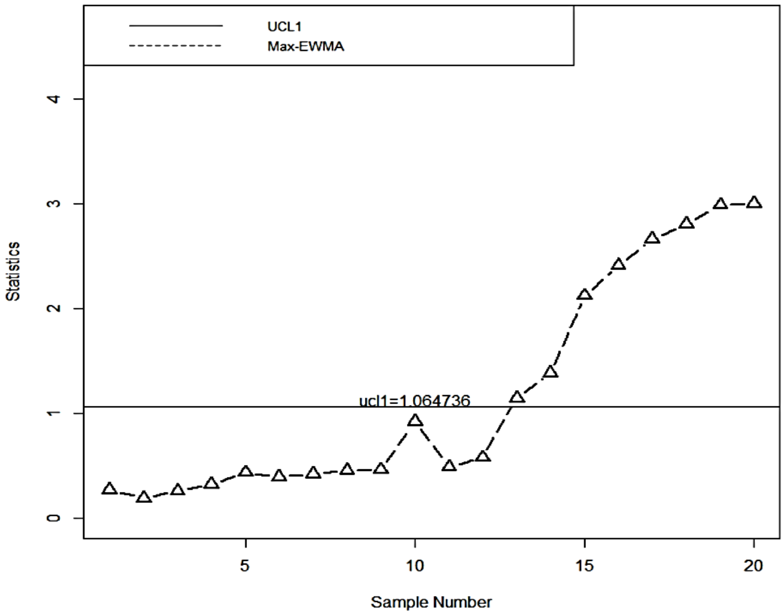

Figure 2: Max-EWMA Control Chart

Figure 1 shows a clear picture of Table 4 data, which shows the proposed Max-EEWMA control chart. In this chart, *UCL* for proposed chart and in Figure 2 UCL_1 for the existing chart is represented as a horizontal line. It can easily be observed that proposed chart indicates the out-of-control signal at 10th value while the existing chart indicates it at 13th sample. Therefore, it is clear from Figures 1-2 that proposed chart performed better than existing chart.

6. MAIN FINDINGS

Results regarding Max-EEWMA control chart are presented in Tables 1-3, Table 1 shows the *ARLs* and *SDRLs* for $\psi_1 = 0.15$, Table 2 for $\psi_1 = 0.2$ and Table 3 for $\psi_1 = 0.3$. The results of Max-EEWMA statistics are presented in Table 4 with Max-EWMA statistics. According to this study, the major findings of the Max-EEWMA chart are mentioned below.

1. When $a = 0$ and $b = 1$, then the process is without shift, and it indicates that process is in control with ARL_0. In Table 1, for $\psi_2 = 0.05$ the *ARL* is 370.10. In Table 2, for $\psi_2 = 0.07$ the *ARL* is 370.49. In Table 3, for $\psi_2 = 0.1$ the *ARL* is 371.94.
2. All table values reveal that when mean shifts "a" increases from 0.00 to 0.10, 0.25, 0.50, 0.75, 1.00, 1.50, and 2.00, values of respective *ARLs* and *SDRLs* minimize, for all various values in variance "b". It reveals that suggested chart is more efficient in identifying mean shifts early.
3. Same pattern is observed for "b", the variance shift. As the variance shift decreases from 1 to 0.25, 0.50, 0.75, 0.90, and increases to 1.10, 1.25, 1.50, 2.00, 2.50, and 3.00, then both values of *ARLs* & *SDRLs* are decreased respectively with variation in mean shift "a". It can also be observed that variance shift also detected early, which also indicates the efficiency of the developed chart.
4. In all Tables 1-3, the last column shows the comparison with existing Max-EWMA control chart in terms of *ARLs* and reveals efficiency of our proposed Max-EEWMA control chart for combined detection of mean and variance shifts.
5. As values of ψ_2 increase from 0.03 to 0.05, 0.07, and 0.1 for different values of $\psi_1 = 0.15$, 0.2, and 0.3, the *ARLs* and *SDRLs* decrease. In Table 2, when $\psi_1 = 0.2$ and $\psi_2 = 0.05$, the value of *ARL* is 24.283 for $b = 0.75$ and $a = 0.1$, for $\psi_2 = 0.07$ *ARL* is 22.679.
6. Real-Life application detects out of control earlier than existing one which also support the simulations result which as well can be observe from Figure 1 and Figure 2 that proposed chart detect out of control at 10th value and existing chart at 13th value.

From the above-mentioned results and facts from calculations in ARL values tables, we can conclude that our developed Max-EEWMA control chart outperforms existing control charts. The proposed chart efficiently detects even smaller mean and variance shifts combinedly. The suggested control chart can examine both mean and variance process shifts concurrently with single statistics rather than two separate statistics for mean and variance.

7. CONCLUSION

In quality control field different types of control chart have been developed by different researchers but many researchers ignored the joint examining of mean and variance simultaneously. In this article, we addressed the issue of joint monitoring and developed a new control chart and named it Max-EEWMA control chart. The values of ARLs and SDRLs have been estimated and tables were created for numerous smoothing constant values with different mean and variance shifts. The comparisons to the existing Max-EWMA control chart were made in terms of ARLs and SDRLs, which reveal that the proposed chart identifies both mean and variance shifts quicker than the Max-EWMA chart. The real-life application of the Max-EEWMA control chart was also illustrated, and it also shows the efficiency of the developed chart. The proposed chart can be used in the industry for joint monitoring of the mean and variance shifts in the production procedure.

DATA AVAILABILITY

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request. Further, no experiments on humans and/or the use of human tissue samples were involved in this study.

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