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EMPLOYING META-HEURISTIC ALGORITHM FOR ESTIMATING THE SURVIVAL FUNCTION OF INVERSE KUMARASWAMY DISTRIBUTION

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ABSTRACT

For the creation of mathematical models, parameter estimation is crucial in many different domains. Therefore, this paper suggests a new estimation technique related to Meta-heuristic (M-H) algorithms and specifically Particular Swarm Optimization which will be mixed with Maximum Likelihood Estimation (PSOMLE) to estimate the parameters as well as the survival function of Inverse Kumaraswamy Distribution. For determine the effectiveness of the suggested estimator (PSOMLE), a simulation study was considered and make a comparison between the considered estimator with the maximum likelihood (MLE) Based on Mean Squared Error. The findings showed that the suggested estimator (PSOMLE) provides accurate and satisfactory estimates for the survival function. Since it has less Mean Squared Error than Maximum Likelihood Estimation.

KEY WORDS

Inverse Kumaraswamy Distribution, Particular Swarm Optimization, Maximum Likelihood Method, Survival Functions, Mean Squared Error and Simulation.

1. INTRODUCTION

Survival analysis has important applications in the fields of engineering, medicine, economics, epidemiology, biology, public health, and physics. Also, the Survival function is one of the most continually utilized approaches in statistics of medical [1-3]. Recently, numerous statisticians have been interested to Estimate survival functions. The research in survival analysis increased greatly over the Life testing problems to apply inverse distribution. The researchers have been interested in applying the inverse distributions in Life testing problems [4-6]. When it was first introduced in 1980 [8]. Abdul Fatah et al. [7] provided the Inverse Kumaraswamy Distribution (IKD), it was supported in various applications, including those involving test results, individual heights, air temperature, and a wide range of other data [9-13].

However, the nonlinearity of the IKD produces a hard estimation of its parameters and creates a challenging and complicated statistical analysis of parameter estimates. As well as, sometimes the traditional methods fail to estimate the parameters of a model [14-16]. Meta Heuristics Algorithm good choice to give a near optimal solution in real time [17-19]. Numerous benefits of using meta-heuristic algorithms include their reliably effectively, and robustly simple implementation. In 2023, Batah and other researchers constructed new systems of the version of X – Exponential distribution [21-25]. Therefore, it will be used to estimate the parameters of Inverse Kumaraswamy Distribution based for survival functions by adopting Particular Swarm Optimization (PSO) as well as maximum likelihood estimator. Since, PSO was the best option for many practitioners in of physical, medical, sciences, statistics, and engineering fields.

The construction of this paper will be as: the material of Inverse Kumaraswamy Distribution is clarified in Section 2. Sections 3 and 4 offers the Maximum Likelihood method (MLE), Particular Swarm Optimization (PSO) and explain the proposed mixed estimation method (PSOMLE), respectively. Section 5 presents the Simulation study and numerical results of the comparison the proposed method and MLE. In addition, a conclusion is provided in Section 6.

2. INVERS KUMARASWAMY DISTRIBUTION (IKD)

Abd Al-Fattah et al. [11] suggested to derive the two parameters invers Kumaraswamy distribution IKD (θ,γ) using the transformation

$$Z=\frac{1}{T}\ ;\ T\sim KD\ (\theta,\gamma)$$

when comparing the IKum Distribution with other common distributions, IKD has a long right tail. Hence, it will produce optimistic predictions of rare events occurring in the right tail of the distribution. Also, the IKD gives a good fit to many data in the literature [4], [6].

The probability density function (PDF) of r.v. Z which is distributed as IKD is,

$$F(z;\theta,\gamma)=\theta\gamma(z)^{-(\theta+1)}\left(1-(z)^{-\theta}\right)^{\gamma-1},z>1\ ;\ \theta,\gamma>0. \quad (1)$$

where, θ, and γ are shape parameters.

The Cumulative Distribution Function (CDF) of Z has the form as below:

$$F(z;\theta,\gamma)=\left(1-(z)^{-\theta}\right)^{\gamma},z>1\ ;\ \theta,\gamma>0. \quad (2)$$

The Survival and hazard functions of Z given as:

$$S(z;\theta,\gamma)=1-F(z-1;\theta,\gamma)=1-\left(1-(z)^{-\theta}\right)^{\gamma},z>1\ ;\theta,\gamma>0. \quad (3)$$

The hazard functions of Z given as:

$$h(z;\theta,\gamma)=\frac{f(z;\theta,\gamma)}{S(z;\theta,\gamma)}=\frac{\theta\gamma(z)^{-(\theta+1)}\left(1-(z)^{-\theta}\right)^{\gamma-1}}{1-(1-(z)^{-\theta})^{\gamma}},z>1\ ;\theta,\gamma>0. \quad (4)$$

The following Figures of specific probability functions of IKD for some arbitrary parameters are listed below.

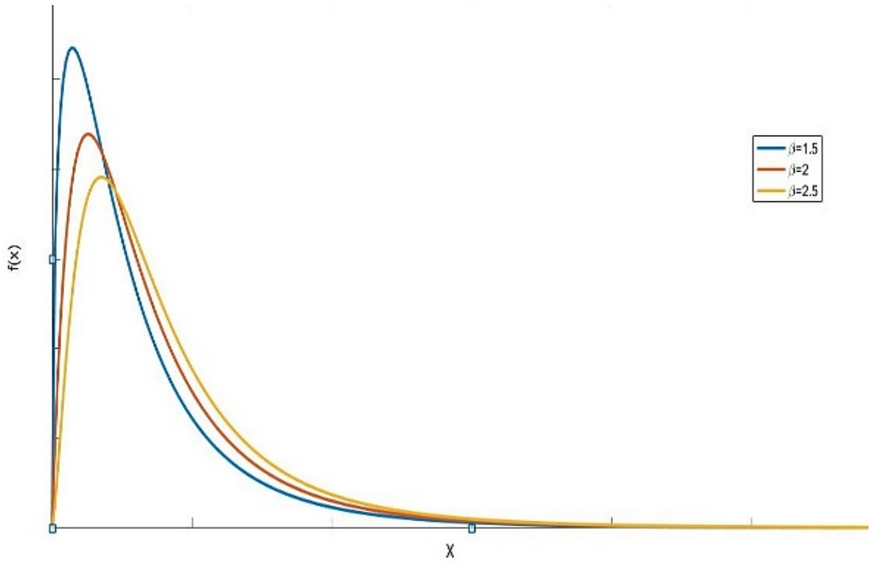

Figure 1: PDF curve of IKD when $\theta = 5$

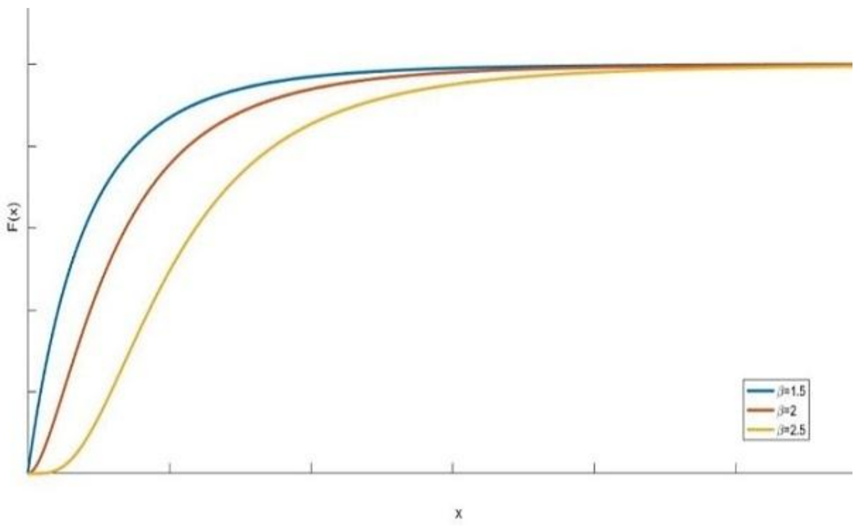

Figure 2: CDF curve of IKD when $\theta = 5$

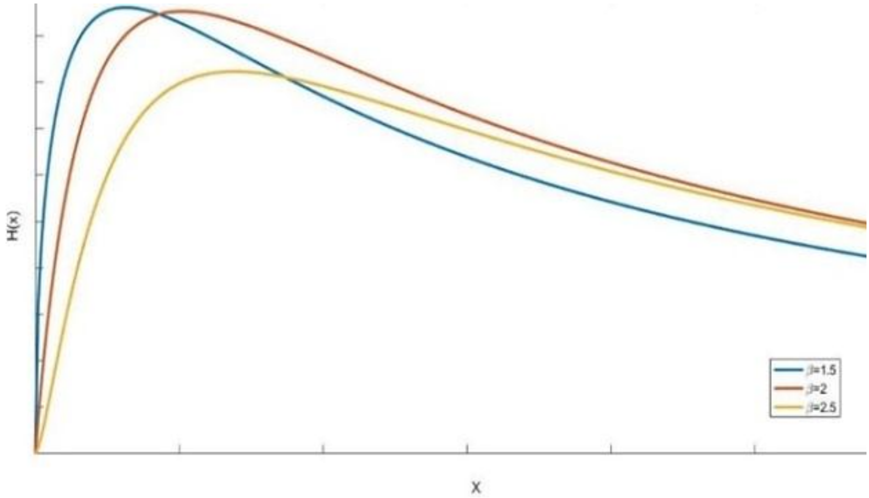

Figure 3: Survival curve of IKD when $\theta = 5$

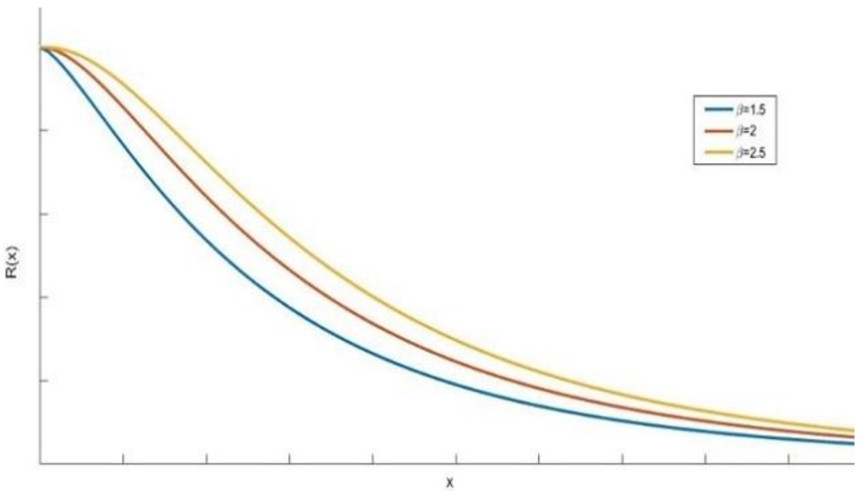

Figure 4: Hazard curve of IKD when $\theta = 5$

3. MAXIMUM LIKELIHOOD ESTIMATION

Let $z_1, z_2, \dots, z_n$ be a random sample distributed as IKD (θ, γ) when θ and γ are unknown, and the likelihood function was given as bellow

$$l = L(z_1, z_2, \dots, z_n; \theta, \gamma) = \prod_{i=1}^{n} f(z_i) = \prod_{i=1}^{n} \theta\gamma (z_i)^{-(\theta+1)} \left(1 - (z_i)^{-\theta}\right)^{\gamma-1}$$

$$= \theta^n \gamma^n \prod_{i=1}^{n} (z_i)^{-(\theta+1)} . \prod_{i=1}^{n} \left(1 - (z_i)^{-\theta}\right)^{\gamma-1} \quad (5)$$

Take normal logarithm (ln) to both sides and then the partial derivative will be made depend w.r.t. θ and γ, respectively as follows;

$$ln\, l = n\, ln\, \theta + n\, ln\, \gamma - (\theta + 1) \sum_{i=1}^{n} ln(z_i) + (\gamma - 1) \sum_{i=1}^{n} ln\left(1 - (z_i)^{-\theta}\right)$$

$$\frac{\partial\, ln\, l}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n} ln\left(1 - (z_i)^{-\theta}\right) = 0$$

The MLE method for the unknown shape parameters γ is given by;

$$\hat{\gamma} = \frac{-n}{\sum_{i=1}^{n} ln(1 - (z_i)^{-\gamma_0})}. \quad (6)$$

$$\frac{\partial\, ln\, l}{\partial \theta} = \frac{n}{\gamma} - \sum_{i=1}^{n} ln(z_i) + (\gamma - 1) \sum_{i=1}^{n} \frac{ln(z_i)}{((z_i)^{\theta} - 1)} = 0$$

$$\frac{n}{\theta} = \sum_{i=1}^{n} ln(z_i) - (\gamma - 1) \sum_{i=1}^{n} \frac{ln(z_i)}{((z_i)^{\theta} - 1)}$$

As a result, we obtained

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} ln(z_i) - (\gamma - 1) \sum_{i=1}^{n} \frac{ln(z_i)}{((z_i)^{\theta_0} - 1)}}. \quad (7)$$

where θ_0 is an initial value for θ. The initial value can be obtained by use median method as bellow:

$$F(z; \theta, \gamma) = 0.5 \;;\; \theta, \gamma > 0$$

$$(1 - (z)^{-\alpha})^{\beta} = 0.5$$

$$1 - (z)^{-\alpha} = 0.5^{\frac{1}{\gamma}}$$

$$z^{-\theta} = 1 - 0.5^{\frac{1}{\gamma}}$$

$$Z = \left(1 - 0.5^{\frac{1}{\gamma}}\right)^{\frac{-1}{\theta}}$$

$$Z_{median} = \left(1 - 0.5^{\frac{1}{\gamma}}\right)^{\frac{-1}{\theta}}$$

By equating the population median (Z_{median}) with sample median (z_{median}) we get:

$$Z_{median} = z_{median}$$

$$\left(1 - 0.5^{\frac{1}{\gamma}}\right)^{\frac{-1}{\theta}} = z_{median}$$

$$\left(1 - 0.5^{\frac{1}{\gamma}}\right)^{\frac{-1}{\theta}} = z_{median}$$

$$1 - 0.5^{\frac{1}{\gamma}} = (z_{median})^{-\theta}$$

$$ln(1 - 0.5^{\frac{1}{\gamma}}) = ln(x_{median} + 1)^{-\alpha}$$

$$ln\left(1 - 0.5^{\frac{1}{\gamma}}\right) = \theta\, ln(z_{median})^{-1}$$

$$\theta_0 = \frac{ln\left(1 - 0.5^{\frac{1}{\gamma}}\right)}{ln(z_{median})^{-1}}$$

Substitute equations (6) and (7) in equation (5), then the MLE estimator of survival analysis (S) as below:

$$\hat{S}_{MLE}(z) = 1 - \left(1 - (z)^{-\hat{\theta}}\right)^{\hat{\gamma}}, z > 1\,; \hat{\theta}, \hat{\gamma} > 0 \quad (8)$$

4. PRACTICAL SWARM OPTIMIZATION MIXED WITH MAXIMUM LIKELIHOOD ESTIMATION (PSOMLE) METHOD

In 1995, Eberhart et al. introduced particle swarm optimization (PSO) algorithm at first time traveling birds to find food was the idea of the PSO algorithm [20]. A particle containing position and speed represents the member of the population of an algorithm. According to its own experience and the collective experience of the population, each particle changes its position and speed as it moves through multidimensional space. Based on the following equations, position and speed are updated:

$$V_{ij}(t+1) = V_{ij}(t) + C_1 R_1 \left(P_{ij}(t) Z_{ij}(t)\right) + C_2 R_2 \left(P_{gj}(t) Z_{ij}(t)\right) \quad (9)$$

$$Z_{ij}(t+1) = Z_{ij}(t) + V_{ij}(t+1) \quad (10)$$

where $j = 1,2, \dots, n\,, i = 1, 2, \dots, N, n, N$ are dimension of search space and number of particles respectively, and A$\subset R^n$; R_1 and R_2 are random variables distributed uniform distribution [0,1], t denotes the iteration counter, C_1, C_2 are weighting factors. A set

$S = \{Z_1, Z_2, \ldots., Z_N\}$ to be optimized, it was defined as probable solutions of an objective function. The position with the best fitness was represented by $P_{ij}(t)$ for the ith particle of search space in the jth dimension. While, the best position $(P_{gj}(t))$ was discovered by the all particles. $Z_i = (Z_{i1}, Z_{i2}, \ldots \ldots, Z_{in})^T \in A$ represent the position of ith particle. In addition, assumed that each particle moves within the search space iteratively. V_i called velocity when $V_i = (V_{i1}, V_{i2}, \ldots, V_{in})^T$.

Possibly, they adjusting their position by using a proper position shift. In this scenario, the survival function of the invers Kumaraswamy distribution was estimated using PSOMLE and was dependent on its parameters. The likelihood function was maximized using particle swarm optimization as the objective function (fitness function).

$$f_{PSOMLE} \; = \; \theta^n \gamma^n \prod_{i=1}^{n} (z_i)^{-(\theta+1)} . \prod_{i=1}^{n} \left(1 - (z_i)^{-\theta}\right)^{\gamma-1}$$

The PSOMLE Procedure can be summarized as follows:

Step 1: Generate n solutions for χ when χ was represented the vector for all parameters required such as $\chi = [\theta \,, \gamma]$ and, and the maximum number of iteration K.

Step 2: Randomly generate each particle's location and velocity.

Step 3: As fitness function for PSO used

$$f_{PSOMLE} \; = \; \theta^n \gamma^n \prod_{i=1}^{n} (z_i)^{-(\theta+1)} . \prod_{i=1}^{n} \left(1 - (z_i)^{-\theta}\right)^{\gamma-1}$$

Step 4: Update p best, if the value of a new objective function is better than the previous one. After that, g best will be updated.

Step 5: For each particle update the velocity and position depending on Equations (9) and (10) respectively.

Step 6: Stop if any predefined criterion or the maximum number of iterations is met; otherwise, go to step 3 and repeat it. Then collect survival function

$$\hat{S}_{PSOMLE}(z) = 1 - \left(1 - (z)^{-\hat{\theta}}\right)^{\hat{\gamma}}, z > 0 \,; \hat{\theta}, \hat{\gamma} > 0. \qquad (11)$$

5. SIMULATION STUDY

To investigate the efficacy of the proposed technique to estimate the parameters of IKD (θ, γ), a simulation study was used in this section. Replicas 1000 times for each simulation. Various sample sizes (20, 40, 60, 90) are used to investigate the effect of the proposed approach. The simulation's steps, which are listed below, explain the statistical results using the Mean Squared Errors (MSE) criteria.

Step 1: The $f(z; \theta, \gamma)$ for inverse Kumaraswamy distribution in equation (2) was used for transforming it to a random sample as $u_1, u_2, \ldots, u_n$, follows uniform distribution (0,1) as follows:

$$F(z_i) = \left(1 - (z_i)^{-\theta}\right)^{\gamma}$$

$$u_i = \left(1 - (z_i)^{-\theta}\right)^{\gamma} \Rightarrow z_i = \left[1 - (u_i)^{\frac{1}{\gamma}}\right]^{-\frac{1}{\theta}}$$

Step 2: Initialize all the parameters of PSO, then used the equation (2) as fitness function for PSO algorithm.

Step 3: Reminisce the S in the equation (3).

Step 4: Utilizing equation (8) to compute $\hat{S}$ based on MLE.

Step 5: Compute $\hat{S}$ from the calculate the best solution for (f) based on PSOMLE.

Step 6: MSE will be calculated as Based on $L = 1000$ trials as follows;

$$MSE = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{S}_i - S\right)^2.$$

6. RESULTS AND DISCUSSION

The best outcome of the proposed estimation and MLE approaches to estimate the Survival function of Invers Kumaraswamy distribution is determined in this section using the simulation results. Four sample size (20, 40, 60, 90) were used. Then, the simulation results for PSOMLE and MLE are shown in Tables 1-4 depend MSE of Survival function. The set parameters of $(\theta, \gamma) =$ (1,5), (2,0.5), (1.5,2), (0.5,0.3) were considered. These tables showed that the particle swarm optimization mixed with MLE (PSOMLE) provided less Mean Square Error. This implies that the PSOMLE method better than MLE.

Table 1
MSE Values Depend of $\hat{S}, \hat{\gamma}, \hat{\theta}$ when θ= 1 , γ = 5

	Samples Size	Method	$\hat{\gamma}$	$\hat{\theta}$	$\hat{S}$	$Best$
$\theta = 1$, $\gamma = 5$	$n = 20$	PSOMLE	4.602051	0.97993	2.74E-07	PSOMLE
		MLE	4.375643	0.946432	1.40E-06	PSOMLE
	$n = 40$	PSOMLE	4.143561	1.097026	4.65E-05	PSOMLE
		MLE	3.441129	1.093784	0.00113	PSOMLE
	$n = 60$	PSOMLE	4.985154	0.97586	4.77E-06	PSOMLE
		MLE	3.355934	0.98436	0.328813	PSOMLE
	$n = 90$	PSOMLE	4.504176	0.946194	0.000156	PSOMLE
		MLE	3.328085	0.95217	0.002444	PSOMLE

Table 2
MSE Values Depend of $\hat{S}$, $\hat{\gamma}$, $\hat{\theta}$ when θ= 0.5 , γ = 2

	Samples size	Method	$\hat{\gamma}$	$\hat{\theta}$	$\hat{S}$	Best
θ =0.5, $\gamma = 2$	$n = 20$	PSOMLE	1.605329	0.503023	0.000422	PSOMLE
		MLE	1.605094	0.503047	0.000423	PSOMLE
	$n = 40$	PSOMLE	1.657425	0.630249	4.65E-05	PSOMLE
		MLE	1.432191	0.639723	0.000654	PSOMLE
	$n = 60$	PSOMLE	1.994062	0.470369	4.77E-06	PSOMLE
		MLE	1.272473	0.296214	0.011301	PSOMLE
	$n = 90$	PSOMLE	1.80167	0.435432	0.000156	PSOMLE
		MLE	1.765826	0.434967	0.000208	PSOMLE

Table 3
MSE Values Depend of $\hat{S}$, $\hat{\gamma}$, $\hat{\theta}$ when θ = 2 , γ = 1.5

	Samples size	Method	$\hat{\gamma}$	$\hat{\theta}$	$\hat{S}$	Best
$\theta = 2$, $\gamma = 1.5$	$n = 20$	PSOMLE	1.634843	1.607287	2.22E-08	PSOMLE
		MLE	1.55993	1.617361	8.98E-06	PSOMLE
	$n = 40$	PSOMLE	1.353494	3.053625	6.11E-05	PSOMLE
		MLE	1.366412	3.206306	0.000109	PSOMLE
	$n = 60$	PSOMLE	1.739796	2.025129	6.21E-05	PSOMLE
		MLE	0.62301	0.514148	0.078925	PSOMLE
	$n = 90$	PSOMLE	1.677681	2.034502	4.94E-06	PSOMLE
		MLE	0.646246	0.597056	0.029704	PSOMLE

Table 4
MSE Values Depend of $\widehat{S}, \widehat{\gamma}, \widehat{\theta}$ when θ= 0.3, γ = 0.5

	Samples size	Method	$\hat{\gamma}$	$\hat{\theta}$	$\hat{S}$	Best
$\theta = 0.3$ $\gamma = 0.5$	$n = 20$	PSOMLE	4.647871	0.310957	0.002458	PSOMLE
		MLE	5.513636	0.318303	0.045241	PSOMLE
	$n = 40$	PSOMLE	3.040542	0.300991	8.80E-05	PSOMLE
		MLE	2.780146	0.301625	0.000304	PSOMLE
	$n = 60$	PSOMLE	2.891885	0.29718	2.05E-05	PSOMLE
		MLE	2.843847	0.042401	0.42777	PSOMLE
	$n = 90$	PSOMLE	3.764535	0.311841	1.86E-05	PSOMLE
		MLE	4.731225	0.320741	9.23E-03	PSOMLE

7. CONCLUSIONS

This study proposed a new method by combining the PSO Algorithm and the MLE method for estimating the survival function of the Invers Kumaraswamy distribution. The effectiveness of the new method in comparison to a classical method (MLE) was investigated using a simulation study. As a result, it was clear that PSOMLE produced results that were superior to those of MLE in terms of Mean Square Error (MSE). Additionally, many optimization algorithms have different strengths and weaknesses can be used for this purpose. Therefore, we recommend the researcher to compare the methodology described above with any other classical estimating approach.

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