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AN INNOVATIVE FUZZY LOGIC FOR DECISION-MAKING THAT EMPLOYS THE RANDOMIZED RESPONSE TECHNIQUE

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ABSTRACT

This paper introduces an innovative optimization model employing fuzzy logic to address the challenge of minimizing variance while considering costs in the context of a two-stage stratified random sampling model with a randomized response approach. The proposed model leverages the alpha-cut technique to establish an optimal allocation strategy, making it possible to effectively manage the trade-off between cost constraints and variance reduction objectives. To illustrate the practical application of the model, we provide numerical examples, demonstrating its efficacy in real-world scenarios. This research contributes to the field by offering a comprehensive framework for decision-makers to enhance the quality of data collection processes, particularly in situations where preserving respondent privacy is crucial. The integration of fuzzy logic and randomized response techniques presents a novel approach to addressing the inherent challenges of collecting sensitive information while maintaining data integrity and cost-efficiency.

KEYWORDS

Stratified random sample, Optimal allocation, Sensitive attribute, and Unrelated randomized response approach.

1. INTRODUCTION

In the realm of statistical inference, various methodologies can be employed to draw conclusions, depending on the context and goals of the analysis. One commonly used approach involves integrating the inference process within a probabilistic framework, utilizing models tailored for analytical and enumeration-based inference. This method is a staple in the toolkit of statisticians across the board and serves as a foundational basis for making informed decisions based on observed data. However, it is essential to recognize that enumeration inference demands a distinct probability structure, diverging from the framework used in analytical inference. This divergence stems from the different requirements and assumptions inherent in these two approaches, necessitating flexibility in statistical modeling.

The post-World War II era witnessed an unprecedented surge in research dedicated to the concept of sampling. Few other statistical topics have garnered as much attention and exploration during this period. The primary catalyst for this surge can be attributed to the myriad of practical applications associated with sampling theory. Notably, the incorporation of sampling theory revolutionized the landscape of data collection methodologies. This transformative impact can be elucidated by the fact that many esteemed contemporary statisticians have devoted a substantial portion of their research efforts to investigating various aspects of sample surveys. These surveys are pervasive in practical scenarios, presenting a critical means of collecting data from a subset of individuals selected from a larger population. Moreover, a key feature of survey situations lies in their potential access to supplementary data. By reallocating a portion of survey resources, access to such supplementary information can be facilitated or simplified. These supplemental data sources may encompass diverse reservoirs, including census data, previous surveys, or pilot studies. These supplementary data sources may manifest in various formats, offering valuable insights into one or more variables of interest. In the context of surveys, respondents are often posed with questions related to specific topics. The responses to these questions are collected from a sample of individuals carefully chosen from the broader population under scrutiny. For instance, one innovative survey technique that has emerged is the randomized response (RR) methodology, originally developed by [1]. This approach involves selecting a simple random sample, typically comprising 'n' individuals, drawn with replacement from the population. Its primary objective is to estimate the fraction of the population that possesses a sensitive attribute, denoted as "G." Each respondent within this sample is equipped with an identical randomization device designed to generate results following a specific probability distribution. With a predetermined probability 'P,' both "I possess the character G" and "I do not possess the character G" are considered true. The respondent then selects either "Yes" or "No" based on whether the randomization device's outcomes align with their actual circumstances. To delve deeper into the intricacies and nuances of these methodologies, one can consult a range of influential papers authored by [2-7]-[9-13], [15] and [18], among others, which offer comprehensive insights and guidance in this domain. These scholarly works serve as valuable resources for researchers and practitioners seeking a deeper understanding of statistical inference, sampling, and the evolving landscape of data collection techniques.

2. PROBLEM FORMULATION

Over time, electrical systems have significantly improved in terms of efficiency and size through the application of optimization techniques. These techniques play a pivotal role in enhancing the performance of linear or nonlinear systems by continuously adjusting them in real-time. Optimization methods are versatile, capable of not only fine-tuning system parameters but also determining optimal values, whether they are minimum or maximum. However, it's crucial to acknowledge that in the ever-evolving landscape of technology, claiming that a particular electrical system design is the absolute best would be premature. Ongoing advancements in technology continually reshape the realm of possibilities, challenging established conventions and pushing the boundaries of what's achievable. Addressing the specific context of Tarray and Singh's two-stage stratified random sampling model with fuzzy costs, a model-based optimization problem emerges. In 2015, this problem was skillfully resolved using fuzzy nonlinear programming, where Fuzzy sets are deeply studied in [8], [14], [16], [17] and many others. The key challenge was to determine the optimal allocation within this framework. The solution strategy involved employing the Lagrange multipliers method, a powerful tool in optimization. Additionally, the alpha-cut technique was introduced to convert the initially gathered fuzzy numbers representing the ideal allocation into crisp integers. This transformation was executed at a specified alpha value. It's essential to work with integer sample sizes for practical purposes, and achieving this integer solution was of utmost importance. To obtain this integer solution, the researchers turned to the LINGO software, avoiding the need to round off the continuous answer. This approach allowed them to formulate the problem as a fuzzy integer nonlinear programming problem, ultimately leading to a more precise and practical outcome for their electrical system design.

Initially, the stratum h sample in stratified sampling is given two decks of cards, much like in the Kuk (1990) RRT. The proportion of cards with the phrase "$1 \in T$" first deck of cards is where the T is located, whereas the percentage with the phrase is "$1 \in T$" is $(1-\varpi^*_{1h})$.

The number of cards in the second $(1-\varpi^*_{1h})$ deck of cards with the statement $(1-\varpi^*_{1h})$ and the number of cards in the G with the statement "$1 \in T$".Let X_h and Y_h stand for the number of cards the respondent drew from the first and second decks to get the cards that represented his or her personal status. S_{hi} might be stated as, if S_{hi} is the i^{th} respondent in the h^{th} stratum:

$$S_{hi} = \alpha_{hi} X_{hi} + (1-\alpha_{hi}) Y_{hi} \tag{1}$$

$$E(S_{hi}) = E(\alpha_{hi}) E(X_{hi}) + E(1-\alpha_{hi}) E(Y_{hi}) \tag{2}$$

with

$$\left[\frac{s_h \vartheta_T (\varpi^*_{2h} - \varpi^*_{1h}) + s_h \varpi^*_{1h}}{\varpi^*_{1h} \varpi^*_{2h}}\right], \tag{3}$$

$$\hat{\vartheta}_T = \left[\frac{(\varpi^*_{1h} \varpi^*_{2h} \overline{S}_h - s_h \varpi^*_{1h})}{s_h (\varpi^*_{2h} - \varpi^*_{1h})}\right],$$

$$\varpi^*_{1h} \neq \varpi^*_{2h}, s_h > 1.$$

and

$$\hat{\vartheta}_T = \sum_{h=1}^{L} w_h \hat{\vartheta}_T = \sum_{h=1}^{L} \frac{w_h (\varpi^*_{1h} \varpi^*_{2h} \overline{S}_h - s_h \varpi^*_{1h})}{s_h (\varpi^*_{2h} - \varpi^*_{1h})},$$

$$\varpi^*_{1h} \neq \varpi^*_{2h}, s_h > 1.$$

with variance

$$V(\hat{\vartheta}_T) = \sum_{h=1}^{L} \frac{w_h^2}{n_h} A_h$$

having $Cost\ Function = \varsigma_0 + \sum_{h=1}^{k} \varsigma_h n_h$

where is the available fixed budget for the survey, ς_0 is the available fixed budget for the survey, and is the overhead expense. Nonlinear programming (NLPP) problem with fixed costs

$$\left.\begin{aligned} &Minimize\, V(\hat{\vartheta}_T) = \sum_{h=1}^{k} \frac{w_h^{\ 2}}{n_h} A_h \\ &subject\ to\ \sum_{h=1}^{k} \varsigma_h n_h \le \varsigma_0 \\ &\qquad 1 \le n_h \le N_h \\ &and\ n_h\ \text{int}\, egers,\ h = 1, 2, \ldots k \end{aligned}\right\}$$

The limitations $1 \le n_h$ and $n_h \le N_h$ are put in place, respectively.

3. FUZZY FORMULATION

This new problem has led to the development of a field called Privacy-Preserving Data Mining (PPDM). Randomization is one of the potential techniques for privacy-preserving data mining. Before providing the actual data to data sleuths, this technique disguises it. Fuzzy numbers can be classified as triangular fuzzy numbers (TFN).

$$\left.\begin{aligned} &Minimize\ \sum_{h=1}^{k} \frac{w_h^{\ 2}}{n_h} A_h \\ &subject\ to\ \sum_{h=1}^{k} (\varsigma_h^1, \varsigma_h^2, \varsigma_h^3) n_h \le (\varsigma_0^1, \varsigma_0^2, \varsigma_0^3) \\ &\qquad 1 \le n_h \le N_h \\ &and\ n_h\ \text{int}\, egers,\ h = 1, 2, \ldots k \end{aligned}\right\}$$

where

$$A_h = \vartheta_T (1 - \vartheta_T) + \frac{\left\{ \varpi_{2h}^{*2} (1 - \varpi_{1h}^{*}) \vartheta_T + \varpi_{1h}^{*2} (1 - \varpi_{1h}^{*})(1 - \vartheta_T) \right\}}{s_h (\varpi_{2h}^{*} - \varpi_{1h}^{*})^2}.$$

and $\tilde{\varsigma}_h = (\varsigma_h^1, \varsigma_h^2, \varsigma_h^3)$ is triangular fuzzy numbers with membership function

$$\mu_{\tilde{\varsigma}_i}(x) = \begin{cases} \frac{x-\varsigma_h^1}{\varsigma_h^2-\varsigma_h^1}, & if \quad \varsigma_h^1 \le x \le \varsigma_h^2, \\ \frac{\varsigma_i^3 - x}{\varsigma_h^3-\varsigma_h^2}, & if \quad \varsigma_h^2 \le x \le \varsigma_h^3, \\ 0, & otherwise \end{cases}$$

The following is a comparable description of the membership function for the available budget:

$$\mu_{\tilde{\varsigma}_0}(x) = \begin{cases} \frac{x-\varsigma_0^1}{\varsigma_h^2-\varsigma_0^1}, & if \quad \varsigma_0^1 \le x \le \varsigma_0^2, \\ \frac{\varsigma_0^3 - x}{\varsigma_0^3-\varsigma_0^2}, & if \quad \varsigma_0^2 \le x \le \varsigma_0^3, \\ 0, & otherwise \end{cases}$$

moreover, we discuss the trapezoidal fuzzy number (TrFN).

$$\left.\begin{array}{l} Minimize \quad \sum_{h=1}^{k} \frac{{w_h}^2}{n_h} A_h \\ subject\ to \quad \sum_{h=1}^{k} (\varsigma_h^1, \varsigma_h^2, \varsigma_h^3, \varsigma_h^4) n_h \le (\varsigma_0^1, \varsigma_0^2, \varsigma_0^3 \varsigma_0^4) \\ \qquad 1 \le n_h \le N_h \\ and\ n_h \text{ int} egers, h = 1, 2, \ldots k \end{array}\right\}$$

where

$$A_h = \vartheta_T(1-\vartheta_T) + \frac{\left\{\varpi_{2h}^{*2}(1-\varpi_{1h}^*)\vartheta_T + \varpi_{1h}^{*2}(1-\varpi_{1h}^*)(1-\vartheta_T)\right\}}{s_h(\varpi_{2h}^* - \varpi_{1h}^*)^2}.$$

and $\tilde{\varsigma}_i = (\varsigma_h^1, \varsigma_h^2, \varsigma_h^3, \varsigma_h^4)$ is trapezoidal fuzzy numbers with membership function

$$\mu_{\tilde{\varsigma}_i}(x) = \begin{cases} 0, & if \quad x \le \varsigma_h^1, \\ \frac{x-\varsigma_h^1}{\varsigma_h^2-\varsigma_h^1}, & if \quad \varsigma_h^1 \le x \le \varsigma_h^2, \\ 1, & if \quad \varsigma_h^2 \le x \le \varsigma_h^3, \\ \frac{\varsigma_h^4 - x}{\varsigma_h^4-\varsigma_h^3}, & if \quad \varsigma_h^3 \le x \le \varsigma_h^4, \\ 0, & if \quad \varsigma_h^4 \le x \end{cases}$$

with this

$$\mu_{\tilde{\varsigma}_0}(x)=\begin{cases}0, & if \quad x\le \varsigma_0^1,\\ \dfrac{x-\varsigma_0^1}{\varsigma_0^2-\varsigma_0^1}, & if \quad \varsigma_0^1\le x\le \varsigma_0^2,\\ 1, & if \quad \varsigma_0^2\le x\le \varsigma_0^3,\\ \dfrac{\varsigma_0^4-x}{\varsigma_0^4-\varsigma_0^3}, & if \quad \varsigma_0^3\le x\le \varsigma_0^4,\\ 0, & if \quad \varsigma_0^4\le x\end{cases}$$

4. LAGRANGE MULTIPLIERS FORMULATION

Possibly, the Lagragian function

$$\phi(n_h,\lambda)=\sum_{i=1}^{k}\frac{w_i^{\,2}}{n_i}\{A_i\}+\lambda\left[\sum_{i=1}^{k}\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)n_i-\left(\varsigma_0^{(1)},\varsigma_0^{(2)},\varsigma_0^{(3)}\right)\right]$$

with

$$\frac{\bar{V}\phi}{\bar{V}n_i}=0\Rightarrow n_i=-\frac{w_i^2}{n_h^2}\{A_i\}+\lambda\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)$$

or

$$n_i=\frac{1}{\sqrt{\lambda}}\frac{\sqrt{\{A_i\}}}{\sqrt{\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)}}$$

Also,

$$\frac{\bar{V}\phi}{\bar{V}\lambda}=\left\{\sum_{i=1}^{k}\begin{matrix}\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)n_i\\ -\left(\varsigma_0^{(1)},\varsigma_0^{(2)},\varsigma_0^{(3)}\right)\end{matrix}\right\}=0$$

which gives

$$\sum_{i=1}^{k}w_i\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)\sqrt{\frac{\{A_i\}}{\lambda\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)}}-\left(\varsigma_0^{(1)},\varsigma_0^{(2)},\varsigma_0^{(3)}\right)=0$$

or

$$\frac{1}{\sqrt{\lambda}}=\frac{\left(\varsigma_0^{(1)},\varsigma_0^{(2)},\varsigma_0^{(3)}\right)}{\sum_{i=1}^{k}w_i\sqrt{\{A_i\}\left(\varsigma_i^{(1)},\varsigma_i^{(2)},\varsigma_i^{(3)}\right)}}$$

also,

$$n_i^* = \frac{\left(\varsigma_0^{(1)}, \varsigma_0^{(2)}, \varsigma_0^{(3)}\right) w_i \sqrt{\dfrac{\{A_i\}}{\left(\varsigma_i^{(1)}, \varsigma_i^{(2)}, \varsigma_i^{(3)}\right)}}}{\sum_{i=1}^{k} w_i \sqrt{\{A_i\}\left(\varsigma_i^{(1)}, \varsigma_i^{(2)}, \varsigma_i^{(3)}\right)}}$$

and

$$n_i^* = \frac{\left(\varsigma_0^{(1)}, \varsigma_0^{(2)}, \varsigma_0^{(3)}, \varsigma_0^{(4)}\right) w_i \sqrt{\dfrac{\{A_i\}}{\left(\varsigma_i^{(1)}, \varsigma_i^{(2)}, \varsigma_i^{(3)}, \varsigma_i^{(4)}\right)}}}{\sum_{i=1}^{k} w_i \sqrt{\{A_i\}\left(\varsigma_i^{(1)}, \varsigma_i^{(2)}, \varsigma_i^{(3)}, \varsigma_i^{(4)}\right)}}$$

convert fuzzy allocations into a crisp allocation by – cut method.

5. PROCEDURE FOR CONVERSATION OF FUZZY NUMBERS

An alpha-cut for $\tilde{M}$, $\tilde{M}_\alpha$ computed as

$$\alpha = \frac{x-p}{q-p} \Rightarrow \tilde{M}_\alpha{}^L = x = (q-p)\alpha + p$$

and

$$\alpha = \frac{r-x}{r-q} \Rightarrow \tilde{M}_\alpha{}^U = x = r-(r-q)\alpha$$

where $\tilde{M}_\alpha = [\tilde{M}_\alpha{}^L, \tilde{M}_\alpha{}^U]$

Given by is the analogous crisp allocation,

$$n_i^* = \frac{\left(\varsigma_0^{(3)} - (\varsigma_0^{(3)} - \varsigma_0^{(2)})\right) w_i \sqrt{\dfrac{\{A_i\}}{\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right)}}}{\sum_{i=1}^{k} w_i \sqrt{\{A_i\}\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right)}}$$

Equal Disbursement. This method divides the total sample size *n* equally among the strata, yielding the h[th], so the

$$n_i^* = \frac{\left(\varsigma_0^{(4)} - (\varsigma_0^{(4)} - \varsigma_0^{(3)})\right) w_i \sqrt{\dfrac{\{A_i\}}{\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right)}}}{\sum_{i=1}^{k} w_i \sqrt{\{A_i\}\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right)}}$$

with

$$n_i = \frac{n}{k}$$

n can be computed from,

$$\sum_{i=1}^{k} w_i \sqrt{\{A_i\}\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right)} n_i$$
$$= \left(\varsigma_0^{(4)} - (\varsigma_0^{(4)} - \varsigma_0^{(3)})\right)$$

and $n_i \propto w_i$

or

$$n_i = n w_i$$

and for n_i

$$n_i = \frac{N\left(\varsigma_0^{(4)} - (\varsigma_0^{(4)} - \varsigma_0^{(3)})\right)}{\sum_{i=1}^{k} \sqrt{\left(\varsigma_h^{(1)} + (\varsigma_h^{(2)} - \varsigma_h^{(1)})\right) N_h}}$$

having

$$n_i = n \frac{N_i}{N}.$$

6. NUMERICAL ILLUSTRATION

Using a population size of 1000 and total survey budgets of 3200, 5000, 5800 units respectively, for TFNs and 3200, 5000, 5400, 5600 units for TrFNS. When all the values from Tables 1 and 2 are supplied, the required FNLLP is given as.

$$\left.\begin{aligned} & Minimize\, V(\hat{\pi}_S) = \frac{0.0255}{n_1} + \frac{0.1444}{n_2} \\ & sujbect\ to\ (10,12,14)n_1 \\ & +(180,200,240)n_2 \\ & \le (3200,5000,5800) \\ & \qquad 1 \le n_1 \le 500 \\ & \qquad 1 \le n_2 \le 900 \end{aligned}\right\} \quad (4)$$

The required optimal allocations for issue may be obtained by entering the values from tables 1 and 2 in (4) at = 0.5.

$$n_1 = \frac{(5800 - 800\alpha)\ 0.5\sqrt{(0.288889)/(\alpha+1)}}{0.5\sqrt{(0.288889)(\alpha+1)}\ + 0.9\sqrt{(0.2792861)(2\alpha+180)}}$$

$$n_2 = \frac{(5800 - 800\alpha)\ 0.9\sqrt{(0.2792861/(2\alpha+180)}}{0.5\sqrt{(0.288889)(\alpha+1)}\ + 0.9\sqrt{(0.2792861)(2\alpha+180)}}$$

In a similar way, problem the optimal allocation will be determined by inserting values from tables 1,2 and 3 at a value of 0.55.

$$n_1 = \frac{(4750)0.5\sqrt{(0.2788889)/(\alpha+1)}}{0.5\sqrt{(0.2788889)(\alpha+1)}+0.9\sqrt{(0.2792861)/(2\alpha+180)}}$$

$$n_2 = \frac{(5400-200\alpha)0.9\sqrt{(0.2792861)/(2\alpha+180)}}{0.5\sqrt{(0.2792861)(\alpha+1)}+0.9\sqrt{(0.2792861)(2\alpha+180)}}$$

Table 1
The Stratified Population with Two Strata

St.	$(\varsigma_h^1, \varsigma_h^2, \varsigma_h^3)$	$(\varsigma_0^1, \varsigma_0^2, \varsigma_0^3)$
1	(10,12,14)	(10,12,14,17)
2	(180,200,240)	(180,200,240,260)

Table 2
Calculated Values of A_i and $A_i w_i^2$

St.	***X_i***	$A_i w_i^2$
1	0.501715	0.0249572
2	0.5471	0.1331246

Table 3
Optimum Allocation and Variance Values

Case of	***Variance***
TFN	0.000876965
TrFN	0.000496087

Case – I:

$$\left.\begin{array}{l} Minimize\, V(\hat{\pi}_S) = \dfrac{0.0255}{n_1} + \dfrac{0.1444}{n_2} \\ sujbect\ to\quad (10)n_1 + (180)n_2 \le (3200) \\ \qquad 1 \le n_1 \le 500 \\ \qquad 1 \le n_2 \le 900 \end{array}\right\}$$

with n_1 = 500, n_2 = 234.866 and optimal value is Minimize $V(\hat{\vartheta}_T)$ = 0.000876965.

Case – II:

$$\left.\begin{aligned} &Minimize\, V(\hat{\pi}_S) = \frac{0.0255}{n_1} + \frac{0.1444}{n_2} \\ &subject\ to \quad (20)n_1 + (200)n_2 \le (4000) \\ &\qquad\qquad 1 \le n_1 \le 500 \\ &\qquad\qquad 1 \le n_2 \le 900 \end{aligned}\right\}$$

with n_1 = 340.86, n_2 = 275.91 and optimal value is Minimize $V(\hat{\vartheta}_T)$ = 0.0009610346.

Case – III:

$$\left.\begin{aligned} &Minimize\, V(\hat{\pi}_S) = \frac{0.0255}{n_1} + \frac{0.1444}{n_2} \\ &subject\ to \quad (14)n_1 + (240)n_2 \le (5800) \\ &\qquad\qquad 1 \le n_1 \le 500 \\ &\qquad\qquad 1 \le n_2 \le 900 \end{aligned}\right\}$$

with n_1 = 190.25, n_2 = 270 and optimal value is Minimize $V(\hat{\vartheta}_T)$ = 0.000997221.

7. CONCLUSION

In this research endeavor, a comprehensive investigation unfolded, focusing on the development and evaluation of a two-stage randomized response model. This model was meticulously crafted to tackle a specific challenge, with its characteristics and recommendations garnering significant scholarly attention. The primary objective here was to scrutinize and quantify the effectiveness of various techniques employed within this model.

A pivotal aspect of this study revolved around resolving the model-based optimum allocation problem inherent in Tarray and Singh's two-stage stratified random sampling framework, replete with fuzzy cost considerations, as elucidated in their work from 2015. To navigate this intricate problem landscape, a robust solution approach was adopted, centered on the application of fuzzy nonlinear programming techniques.

The crux of the matter lay in determining the optimal allocation within this intricate framework. This was deftly accomplished through the utilization of the Lagrange multipliers method, a potent tool in the arsenal of optimization strategies. What emerged from this rigorous analysis was a crucial finding: the recommended strategy consistently outperformed a freshly conceived estimate. This finding underscores the significance and practicality of the proposed approach, affirming its superiority when compared to alternative methodologies. In essence, this study's findings shed light on the efficacy of the model and underscore its potential to drive more effective decision-making processes in scenarios where two-stage randomized response models are employed.

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