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A MIXTURE OF LINEAR AND EXPONENTIAL FUNCTION-BASED ESTIMATORS OF POPULATION MEAN ACCOUNTING FOR NON-RESPONSE AND MEASUREMENT ERROR

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ABSTRACT

This paper introduces a novel approach for estimating the population mean in stratified sampling while accounting for non-response and measurement error. We propose a regression-cum-exponential estimator, which combines regression and exponential functions to estimate the population mean. This estimator is compared to other modified usual estimators commonly used in stratified sampling such as regression, ratio, and exponential estimators. This present article provides the expressions for the bias and means square error of the proposed estimator, considering the joint influence of non-response and measurement error. The theoretical comparisons between the proposed estimators and the existing ones to evaluate their respective performances. To further access the efficiency of the proposed estimators a simulation study is conducted. The results of the study indicate that the regression-cum-exponential estimator and its class of estimators outperform the existing estimators when dealing with the joint influence of nonresponse and measurement error. Overall, the paper introduces a novel approach to address the challenges of estimating population mean in stratified sampling while soldiering nonresponse and measurement error, The proposed estimators outperform existing methods in the presence of these factors, providing valuable insights for researchers and practitioners working with survey data.

KEYWORDS

Stratified sampling; bias; mean square error; measurement error; non-response; auxiliary information.

1. INTRODUCTION

In survey research, while conducting a survey researchers may encounter two types of errors: sampling errors and non-sampling errors. Sampling errors arise from the lack of representativeness in the selected sample for observation, leading to a deviation between the sample estimate and the population parameter. These errors typically diminish as the sample size increases, allowing for a more accurate reflection of the population. Whereas non-sampling errors, on the other hand, emerge from a variety of sources including inadequate frames, inaccurate sampling procedures, errors in coding and decoding, over reporting or underreporting by participants, incomplete coverage of sample units due to non-response, and measurement errors. Unlike sampling errors, non-sampling errors do not reduce with an increase in the sample size. In fact, as the sample size grows, the potential for non-sampling errors to occur may also increase. To minimize these errors, researchers should strive for a careful survey design, ensure a representative sample selection process, use reliable data collection methods, thoroughly train survey administrators, address non-response issues effectively, and implement quality control measures throughout the research process.

In survey research, non-response is a prevalent issue that can occur for various reasons, such as language barriers, unavailability of respondents, or censorship. It is widely recognized among statistics that ignoring the stochastic nature of non-response can introduce bias into the data representation. To reduce the impact of nonresponse, statisticians often employ the subsampling technique and propose various estimation methods. For example, one approach suggested by [5] involves using subsampling to modify the treatment of nonresponse in specific survey inquiries. Other scholars such as [11-13], have discussed the utilization of auxiliary information in conjunction with complete responses to alleviate non-response bias. Additionally, there are notable work presented by [1], [7], [9], [19-21] that provide insights into strategies for dealing with nonresponse. These proactive approaches enable researchers to an account for non-response and produce reliable findings.

The measurement error in survey research reference to the discrepancy between the recorded value and the true value of the variable being measured. For instance, in a study that examines students cumulative grade point average (CGPA) if students are asked to report their actual CGPA, they may provide inaccurate or round ed figures instead of the precise value. This disparity between the reported and true CGPA represents the measurement error. To estimate unknown parameters when measurement error is present, substantial work has been done by researchers such as [4], [15-18], and many others. Their contributions have provided valuable insights and techniques for dealing with the impact of measurement error on parameter estimation in surveys. Similarly, researchers such as [4], [3], [6] and [7] addressed non-response issues and estimated population parameters by developing methods for handling nonresponse.

The impact of nonresponse and measurement error on population mean estimation bas been investigated by several notable researchers ([8], [10], [2], [14]). However, their studies primarily focused on simple random sampling using signal auxiliary variable. This study aims to extends these findings and enhance the efficiency of existing estimators by introducing regression cum exponential estimators for population mean estimation in the context of stratified random sampling. Furthermore, this study seeks to account for two non-sampling errors: nonresponse and measurement error. The proposed approach utilizes a novel combination of linear and exponential functions to develop estimators. Moreover, this study extends the existing literature by modifying classical ratio, regression, and exponential estimators to accommodate the presence of nonresponse and measurement.

After introducing nonresponse, and measurement error and presenting the sampling strategy and discussing some existing estimators section 4 covers modified ratio, regression, and exponential estimator for stratified sampling under the joint influence of measurement error and non-response along with the derivation. In section 5, we propose regression-cum-exponential estimators using single and two auxiliary variables of a population mean in the presence of non-response and measurement. The expressions of the bias and mean square error have been derived. Section 6 provides theoretical comparison to demonstrate the performance of the proposed regression-cum-exponential estimator. To support the proposed methodology, a simulation study is presented in section 7. Some concluding remarks are made in Section 8.

2. SAMPLING STRATEGY

Before presenting the sampling strategy of stratified sampling and estimation procedures, some basic notations used in this study are defined. Let a population of size N be divided into L homogenous strata with N_h units $(h = 1,2,\dots,L)$ such that $\sum_{h=1}^{L} N_h = N$.

N: Population size

N_h: Population of size of h^{th}stratum;

Y/X : Study variable / Auxiliary variable;

μ_Y/μ_X: Population mean of Y/ Population mean of X

μ_{Y_h}/μ_{X_h} : Population means in h^{th} stratum;

$\mu_{Y_{h_{(1)}}}, \mu_{X_{h_{(1)}}}$: Population means of respondents group in h^{th}stratum;

$\mu_{Y_{h_{(2)}}}, \mu_{X_{h_{(2)}}}$: Population means of non-respondents group in h^{th}stratum;

$\sigma^2_{Yh}/\sigma^2_{Xh}$: Population Variances of Y and X respectively in h^{th} stratum;

$\sigma^2_{Yh_{(1)}}, \sigma^2_{Xh_{(1)}}$: Population Variances from group of respondents in h^{th} stratum;

$\sigma^2_{Yh_{(2)}}, \sigma^2_{Xh_{(2)}}$: Population Variances from group of non-respondents in h^{th} stratum;

$C_{Yh_{(1)}}$, $C_{Xh_{(1)}}$: Coefficient of variation for Y and X from group of respondents in h^{th} stratum;

$C_{Yh_{(2)}}$, $C_{Xh_{(2)}}$: Coefficient of variation for Y and X from group of non-respondents in h^{th} stratum;

$\frac{y_{hi}}{x_{hi}}$: Report values on Y and X for i^{th} units in h^{th}stratum;

$\frac{Y_{hi}}{X_{hi}}$: True values on Y and X for i^{th} units in h^{th}stratum;

$U_{hi} = y_{hi} - Y_{hi}$: Measurement error on the study variable associated with i^{th} units in h^{th}stratum;

$V_{hi} = x_{hi} - X_{hi}$: Measurement error on the auxiliary variable associated with i^{th} units in h^{th}stratum;

$U_{hi}^{*} = y_{hi}^{*} - Y_{hi}^{*}$: Measurement error and non-response on *Y* associated with i^{th} units in h^{th}stratum;

$V_{hi}^{*} = x_{hi}^{*} - X_{hi}^{*}$: Measurement error and non-response on *X* associated with i^{th} units in h^{th}stratum;

$\sigma_{Uh_{(2)}}^{2}, \sigma_{Vh_{(2)}}^{2}$: Population Variances of *U* and *V* respectively from the group of non-respondents;

$\rho_{YXh_{(1)}}$ and $\rho_{YXh_{(2)}}$: Coefficient of correlation between the study variable and auxiliary variable for the respondent and non-respondents parts of the population respectively;

$P_h = \frac{N_h}{N}$: Weight of h^{th}stratum;

n_h: sample size in h^{th}stratum;

$\tilde{\mu}_{x_{(st)}}$ = Sample mean estimator;

$\frac{\tilde{\mu}_{y(st)}^{*}}{\tilde{\mu}_{x(st)}^{*}}$: Sample mean estimator with non-response and measurement error.

Now consider,

$$\mu_Y = \sum_{i=1}^{L} P_h \mu_{Yh},\ \mu_X = \sum_{i=1}^{L} P_h \mu_{Xh},\ \mu_{Yh} = \frac{1}{N}\sum_{i=1}^{NL} y_{hi},$$
$$\mu_{Xh} = \frac{1}{N_h}\sum_{i=1}^{L} x_{hi} \text{ and } P_h = \frac{N_h}{N}.$$

The measurement error $U_{hi}^{*} = y_{hi}^{*} - Y_{hi}^{*}$ and $V_{hi}^{*} = x_{hi}^{*} - X_{hi}^{*}$ in the presence of non-response associated are assumed to have their means zero and the variances $\sigma_{Uh_{(2)}}^{2}$ and $\sigma_{Vh_{(2)}}^{2}$ for the non-respondent part of the population.

Consider a finite population of size *N* is stratified into *L* homogenous strata. Let N_h, be the size of h^{th} stratum $(h = 1,2,3,\ldots L)$ Such that $\sum_{h=1}^{L} N_h = N$ and $\left(y_{hi}, x_{hi,} z_{hi}\right)$ be the observations of study variable y and auxiliary variable x on the i^{th} unit of h^{th} stratum, respectively. Let $\bar{y}_h$ and $\bar{x}_h$, be the sample means of h^{th} stratum corresponding to the population means $\bar{Y}_h$ and $\bar{X}_h$respectively. Usually, it is not possible to collect complete information from all the tits selected in the sample $n_h(\sum_{h=1}^{L} n_h = n)$ Let $n_{h_{(1)}}$ units from a sample of n^{th} provide their responses and $n_{h_{(2)}}$, units do not. Adapting Hansen and Hurwitz sub-sampling methodology, a sub-sample of size $r_h \left(r = \frac{n_{h_{(2)}}}{f_h}; f_h > 1\right)$ from $n_{h_{(2)}}$ non-respondents' group is selected at random, the sampling fraction among the non-respondent group in the hth stratum. In practice, r_h, is usually not integer and has to be recorded. Following most of the current literature on this topic, let us assume that the followed-up $r_h \left(\subset n_{h_{(2)}}\right)$ units respond on the second call.

ρ is population correlation coefficient between X and Y for the responding and non-responding part of the population respectively.

3. SOME EXISTING ESTIMATOR

Azeem and Hanif [2] proposed few estimators under the joint influence of nonresponse and measurement error for estimation of population mean in two-phase sampling. The estimators are as follows:

$$T_{AH1} = \tilde{\mu}_y^* \frac{\tilde{\mu}_x'^*}{\mu_X} \frac{\tilde{\mu}_x'^*}{\tilde{\mu}_x^*}, \tag{1}$$

and

$$T_{AH2} = \tilde{\mu}_y^* \frac{\tilde{\mu}_x'^*}{\mu_X} exp\left(\frac{\tilde{\mu}_x'^* - \tilde{\mu}_x^*}{\tilde{\mu}_x'^* + \tilde{\mu}_x^*}\right), \tag{2}$$

The mean square error of the estimators are as,

$$\begin{aligned} MSE\ (T_{AH1}) = {} & \lambda_2 {\mu_Y}^2 \left(C_y^2 + \left(\frac{N+n}{N-n}\right) C_x^2 - 2\left(\frac{N+n}{N-n}\right)\rho_{yx} C_y C_x\right) \\ & + \theta {\mu_Y}^2 \left(C_{y(2)}^2 + \left(\frac{N+n}{N-n}\right) C_{x(2)}^2 - 2\left(\frac{N+n}{N-n}\right)\rho_{yx(2)} C_{y(2)} C_{x(2)}\right) \\ & + \lambda_2 {\mu_Y}^2 \left(\frac{S_u^2}{{\mu_Y}^2} + \left(\frac{N+n}{N-n}\right)^2 \frac{S_v^2}{{\mu_X}^2}\right) + \theta {\mu_Y}^2 \left(\frac{S_{u(2)}^2}{{\mu_Y}^2} + \left(\frac{N+n}{N-n}\right)^2 \frac{S_{v(2)}^2}{{\mu_X}^2}\right). \end{aligned} \tag{3}$$

and

$$\begin{aligned} MSE\ (T_{AH2}) = {} & \lambda_2 {\mu_Y}^2 \left(C_y^2 + \frac{1}{4}\left(\frac{N+2n}{N-n}\right)^2 C_x^2 - \left(\frac{N+2n}{N-n}\right)\rho_{yx} C_y C_x\right) \\ & + \theta {\mu_Y}^2 \left(C_{y(2)}^2 + \frac{1}{4}\left(\frac{N+2n}{N-n}\right)^2 C_{x(2)}^2 - \left(\frac{N+n}{N-n}\right)\rho_{yx(2)} C_{y(2)} C_{x(2)}\right) \\ & + \lambda_2 {\mu_Y}^2 \left(\frac{S_u^2}{{\mu_Y}^2} + \frac{1}{4}\left(\frac{N+2n}{N-n}\right)^2 \frac{S_v^2}{{\mu_X}^2}\right) + \theta {\mu_Y}^2 \left(\frac{S_{u(2)}^2}{{\mu_Y}^2} + \frac{1}{4}\left(\frac{N+2n}{N-n}\right)^2 \frac{S_{v(2)}^2}{{\mu_X}^2}\right). \end{aligned} \tag{4}$$

Sabir and Sanaullah [15] introduced a generalized class of estimator for two phase sampling if nonresponse and measurement errors are simultaneously present and is as,

$$T_{SS} = \frac{\tilde{\mu}_y^*}{2}\left(exp\left(\frac{\tilde{\mu}_x^* - \tilde{\mu}_x'^*}{\tilde{\mu}_x^* + \tilde{\mu}_x'^*}\right) + exp\left(\frac{\tilde{\mu}_x'^* - \tilde{\mu}_x^*}{\tilde{\mu}_x'^* + \tilde{\mu}_x^*}\right)\right) + \omega(\tilde{\mu}_x'^* - \tilde{\mu}_x^*), \tag{5}$$

The mean square error of T_{ss} is given by,

$$MSE\ (T_{SS}) = {\mu_Y}^2 A_y^* + \omega^2 {\mu_X}^2 A_x^* - 2\mu_Y \mu_X \omega C_{xy}^*. \tag{6}$$

Zahid et al. [22] introduced a generalized estimator for sensitive variable under stratified random sampling in the presence of non-response and measurement error. The estimator is as,

$$T_{EZ} = \sum_{h=1}^{L} P_h \left\{ m_{1h}\bar{z}_h^* \left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_1} + m_{2h}\left(\bar{X}_h - \bar{x}_h^{*\prime}\right)\left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_2} + m_{3h}\left(\bar{R}_{xh} - \bar{r}_{xh}^{*\prime}\right)\left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_2}\right\} exp(1-\alpha)\left(\frac{\bar{X}_h - \bar{x}_h^{*\prime}}{\bar{X}_h + \bar{x}_h^{*\prime}}\right), \quad (7)$$

The bias and MSE of the estimator T_{EZ} are given as,

$$Bias(T_{EZ}) = \sum_{h=1}^{L} P_h \left\{ (m_{1h}-1)\bar{Z}_h + m_{1h}\left(\frac{f^{*\prime} t_h^2 R_h B_h}{\bar{X}_h} + \frac{e^{*\prime} t_h C_h}{\bar{X}_h}\right) + m_{2h}\left(\frac{d^{*\prime} t_h^2 B_h}{\bar{X}_h}\right) + m_{3h} m_{1h}\left(\frac{c^{*\prime} t_h F_h}{\bar{X}_h} + \frac{b^{*\prime} t_h^2 D_h}{\bar{R}_{xh}}\right)\right\}, \quad (8)$$

and

$$MSE(T_{EZ}) = \sum_{h=1}^{L} P_h \left\{ \bar{Z}_h^2 + m_{1h}^2 A_{h1}^{*\prime} + m_{2h}^2 B_{h1}^{*\prime} + 2m_{2h}m_{1h}C_{h1}^{*\prime} - 2m_{1h}D_{h1}^{*\prime} - 2m_{2h}E_{h1}^{*\prime} + m_{3h}^2 F_{h1}^{*\prime} + 2m_{3h}m_{1h}G_{h1}^{*\prime} + 2m_{3h}m_{2h}G_{h1}^{*\prime} - 2m_{3h}I_{h1}^{*\prime}\right\}. \quad (9)$$

4. MODIFIED ESTIMATORS

Following Hansen and Hurwitz [8] estimator for estimating mean in the presence of non-response and measurement error for stratified sampling is given by

$$t_0^* = \tilde{\mu}_{y(st)}^* = \sum_{i=1}^{L} P_h \tilde{\mu}_{yh}^*, \quad (10)$$

where

$\tilde{\mu}_{yh}^* = w_1\left(\tilde{\mu}_{yh(1)}^* + \tilde{\mu}_{Uh(1)}^*\right) + w_2\left(\tilde{\mu}_{y(2)k_h}^* + \tilde{\mu}_{U(2)k_h}^*\right)$, $w_1 = \frac{n_{h(1)}}{n_h}, w_2 = \frac{n_{h(2)}}{n_h}$,
$\tilde{\mu}_{yh(1)}^* = \frac{1}{n_{h(1)}}\sum_{i=1}^{n_{h(1)}} y_{hi}$, $\tilde{\mu}_{Uh(1)}^* = \frac{1}{n_h}\sum_{i=1}^{n_{h(1)}} U_{hi}$, $\tilde{\mu}_{y(2)k_h}^* = \frac{1}{r_h}\sum_{i=1}^{r_h} y_{hi}$,
$\tilde{\mu}_{yh(2)k_h}^* = \frac{1}{r_h}\sum_{i=1}^{k_h} y_{hi}$ and $\tilde{\mu}_{Uh(2)k_h}^* = \frac{1}{k_h}\sum_{i=1}^{k_h} U_{hi}$

The expression of the variance t_0^*may be defined as,

$$var(t_0^*) = \mu_Y^2 \sum_{h=1}^{L} P_h^2 \left(\lambda_h\left(C_{Yh}^2 + \frac{\sigma_{Uh}^2}{\mu_Y^2}\right) + \theta_h\left(C_{Yh(2)}^2 + \frac{\sigma_{Uh(2)}^2}{\mu_Y^2}\right)\right), \quad (11)$$

where $\lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right), W_{h(2)} = \frac{N_{h(2)}}{N_h}, \theta_h = \frac{W_{h(2)}(r_h - 1)}{n_h}$.

Similarly, for the auxiliary variable, the sample mean estimator in the presence of non-response and measurement is

$$\tilde{\mu}^*_{x(st)} = \sum_{i=1}^{L} P_h \tilde{\mu}^*_{xh},$$

where

$$\tilde{\mu}^*_{xh} = w_1\left(\tilde{\mu}^*_{xh(1)} + \tilde{\mu}^*_{Vh(1)}\right) + w_2\left(\tilde{\mu}^*_{x(2)k_h} + \tilde{\mu}^*_{V(2)k_h}\right),$$
$$\tilde{\mu}^*_{xh(1)} = \frac{1}{n_{h(1)}}\sum_{i=1}^{n_{h(1)}} x_{hi},\ \tilde{\mu}^*_{Vh(1)} = \frac{1}{n_h}\sum_{i=1}^{n_{h(1)}} V_{hi},\ \tilde{\mu}^*_{x(2)k_h} = \frac{1}{r_h}\sum_{i=1}^{r_h} x_{hi},$$
$$\tilde{\mu}^*_{xh(2)k_h} = \frac{1}{r_h}\sum_{i=1}^{k_h} x_{hi} \text{ and } \tilde{\mu}^*_{Vh(2)k_h} = \frac{1}{k_h}\sum_{i=1}^{k_h} V_{hi}$$

The modified combined type ratio, exponential and regression estimators are presented under the belief that the non-response and the measurement error are occurring on both study and the auxiliary variables under stratified sampling.

4.1 The Modified Combined Ratio Estimator

The modified combined ratio estimator is given by

$$t_r = \frac{\tilde{\mu}^*_{y(st)}}{\tilde{\mu}^*_{x(st)}}\mu_X. \tag{12}$$

In order to obtain the expressions for the bias and the MSE, let us consider

$$H^*_{Yh} = \sum_{i=1}^{n_h}(y^*_{hi} - \mu_{Yh}), H^*_{Xh} = \sum_{i=1}^{n_h}(x^*_{hi} - \mu_{Xh}), H^*_{Uh} = \sum_{i=1}^{n_h} U^*_{hi} \text{ and } H^*_{Vh} = \sum_{i=1}^{n_h} V^*_{hi}$$

The error terms due to sampling are defined by,

$$e^*_{y(st)} = \frac{1}{\mu_Y}\sum_{i=1}^{n_h}\frac{P_h}{n_h}(H^*_{Yh} + H^*_{Uh})$$

and

$$e^*_{x(st)} = \frac{1}{\mu_X}\sum_{i=1}^{n_h}\frac{P_h}{n_h}(H^*_{Xh} + H^*_{Vh}),$$

and the sample means associated with the sampling errors assuming the joint presence of non-response and measurement error are defined by

$$\tilde{\mu}^*_{y(st)} = \mu_Y\left(1 + e^*_{y(st)}\right) \text{ and } \tilde{\mu}^*_{x(st)} = \mu_X(1 + e^*_{x(st)}), \text{such that}$$
$$E\left(e^*_{y(st)}\right) = E\left(e^*_{x(st)}\right) = 0,$$
$$E\left(e^*_{y(st)}\right)^2 = \left[\sum_{h=1}^{L} P_h^2\left\{\lambda_h\left(C^2_{Y_h} + \frac{\sigma^2_{U_h}}{\mu^2_y}\right) + \theta_h\left(C^2_{Y_h(2)} + \frac{\sigma^2_{U_{h(2)}}}{\mu^2_y}\right)\right\}\right] = A^*_{y(st)},$$
$$E\left(e^*_{x(st)}\right)^2 = \left[\sum_{h=1}^{L} P_h^2\left\{\lambda_h\left(C^2_{X_h} + \frac{\sigma^2_{V_h}}{\mu^2_X}\right) + \theta_h\left(C^2_{X_h(2)} + \frac{\sigma^2_{V_{h(2)}}}{\mu^2_X}\right)\right\}\right] = A^*_{x(st)} \text{ and}$$

$$E\left(e^*_{y(st)}e^*_{x(st)}\right) = \left[\sum_{h=1}^{L} P_h^2\left(\lambda_h \rho_{XY_h} C_{Y_h} C_{X_h} + \theta_h \rho_{XY_{h(2)}} C_{Y_{h(2)}} C_{X_{h(2)}}\right)\right] = C^*_{xy(st)}.$$

Now the ratio estimator (12) in terms of e's is given by

$$T_r = \frac{\mu_Y\left(1+e^*_{y(st)}\right)}{\mu_Y\left(1+e^*_{x(st)}\right)}\mu_X, \tag{13}$$

The expressions of the bias and the *MSE* are given by

$$Bias\ (T_r) = \mu_Y\left(A^*_{x(st)} - C^*_{xy(st)}\right) \tag{14}$$

$$MSE\ (T_r) = {\mu_Y}^2\left(A^*_{y(st)} + A^*_{x(st)} - 2C^*_{xy(st)}\right) \tag{15}$$

4.2 The Modified Combined Regression Estimator

$$T_{reg} = \tilde{\mu}^*_{y(st)} + b_x\left(\mu_X - \tilde{\mu}^*_{x(st)}\right) \tag{16}$$

Express (16) in terms of e's, we may get

$$T_{reg} - \mu_Y = \mu_Y e^*_{y(st)} - b_x \mu_X e^*_{x(st)} \tag{17}$$

Squaring and applying expectations

$$MSE\left(T_{reg}\right) = \mu_Y^2 A^*_{y(st)} + {b_x}^2 \mu_X^2 A^*_{x(st)} - 2\, b_x \mu_X \mu_Y C^*_{xy_{(st)}} \tag{18}$$

Differentiating (18) with respect to '*b*', we get

$$b_x = \frac{\mu_Y C_{xy}}{\mu_X A_y}.$$

The minimum MSE expression after substituting the value of *b*, we may get as,

$$minMSE(T_{reg}) = \mu_Y^2\left[A^*_{y(st)} - \frac{C^{*2}_{xy(st)}}{A^*_{x(st)}}\right] \tag{19}$$

4.3 Modified Combined Exponential Estimator

$$T_{ex} = \tilde{\mu}^*_{y_{st}} exp\left[\frac{\mu_X - \tilde{\mu}^*_{x(st)}}{\mu_X + \tilde{\mu}^*_{x(st)}}\right], \tag{20}$$

Expressing (20) in terms of e's and expanding up to the first order of approximation, we may have

$$T_{ex} = \mu_Y + \mu_Y e^*_{y(st)} - \frac{\mu_Y}{2}\left(e^*_{x(st)} - \frac{e^{*^2}_{x(st)}}{2}\right) - \frac{\mu_Y e^*_{y(st)}}{2}\left(e^*_{x(st)} - \frac{e^{*^2}_{x(st)}}{2}\right)$$
$$+ \frac{\mu_Y}{4}\left(e^{*^2}_{x(st)} + \frac{e^{*^4}_{x(st)}}{4} - e^{*^3}_{x(st)}\right) + \frac{\mu_Y e^*_{y(st)}}{4}\left(e^{*^2}_{x(st)} + \frac{e^{*^4}_{x(st)}}{4} - e^{*^3}_{x(st)}\right), \tag{21}$$

After solving and applying expectation we may have the expressions of the bias and the MSE as,

$$Bias(T_{ex}) = \mu_Y \frac{1}{2}\left[A^*_{x(st)} - C^*_{xy(st)}\right] \tag{22}$$

$$MSE(T_{ex}) = \mu_Y^2 \left[A^*_{y(st)} + \frac{A^*_{x(st)}}{4} - C^*_{xy(st)}\right] \tag{23}$$

5. PROPOSED GENERALIZED ESTIMATORS

This section presents the procedure of mean estimation in stratified sampling using a single and two auxiliary variables under the existence of two random errors i.e. non-response and measurement error on both study and auxiliary variables. The proposed estimators are based on the linear and exponential functions for better description of the population mean of study variable. The expressions of the bias and MSE of the proposed estimator under first order of approximation are obtained.

5.1 Proposed Estimator I

A regression-cum-exponential estimator to estimate the population mean of the study variable using a single auxiliary variable under the existence of nonresponse and measurement error is proposed following a linear and exponential functions. The form of the estimator is proposed by

$$T_{pr} = \left[k_1\tilde{\mu}^*_{y_{st}} + k_2\left(\frac{\mu_X - \tilde{\mu}^*_{x_{st}}}{\mu_X}\right)\right] exp\left[\frac{a\left(\mu_X - \tilde{\mu}^*_{x_{st}}\right)}{a\left(\mu_X + \tilde{\mu}^*_{x_{st}}\right) + 2b}\right], \tag{24}$$

where k_1 and k_2 are suitably chosen constants whose sum needs not be 'unity' for instance, and a and b are suitable chosen scalars. It is to be noted that the class of estimator T_{pr} reduces to the following set of known estimators present in Table 1.

Table 1
Class of Estimators

Estimators	k_1	k_2	a	B
$T_{pr(1)}=\tilde{\mu}^*_{y_{st}}$	*1*	0	0	0
$T_{pr(2)}=k_1\tilde{\mu}^*_{y_{st}}$	k_1	0	0	0
$T_{pr(3)}=\tilde{\mu}^*_{y_{st}} exp\left[\frac{\mu_X-\tilde{\mu}^*_{x_{st}}}{\mu_X+\tilde{\mu}^*_{x_{st}}}\right]$	1	0	1	0
$T_{pr(4)}=\tilde{\mu}^*_{y_{st}} exp\left[\frac{(\mu_X-\tilde{\mu}^*_{x_{st}})}{(\mu_X+\tilde{\mu}^*_{x_{st}})+2b}\right]$	1	0	1	B
$T_{pr(5)}=\tilde{\mu}^*_{y_{st}} exp\left[\frac{(\mu_X-\tilde{\mu}^*_{x_{st}})}{(\mu_X+\tilde{\mu}^*_{x_{st}})+2}\right]$	1	0	1	1

In order to obtain the expressions of the bias and MSE of T_{pr}, (24) is expressed in terms of e's to the first order of approximation, we may have

$$T_{pr} = \left[k_1\mu_Y(1+e^*_{y(st)}) + k_2\left(\frac{\mu_X - \mu_X(1+e^*_{x(st)})}{\mu_X}\right)\right] exp\left[\frac{a\left(\mu_X - \mu_X(1+e^*_{x(st)})\right)}{a\left(\mu_X + \mu_X(1+e^*_{x(st)})\right) + 2b}\right] \quad (25)$$

By solving we get,

$$T_{pr} - \mu_Y = (k_1 - 1)\mu_Y + k_1\mu_Y\left[e^*_{y(st)} + (1-v)\frac{e^*_{x(st)}}{2}\right] - k_2\mu_X e^*_{x(st)} + k_1\mu_Y(1-v)\frac{e^*_{y(st)}e^*_{x(st)}}{2} - k_2\mu_X(1-v)\frac{e^{*2}_{x(st)}}{2} + k_1\mu_Y\left(1+\frac{1}{2}(1-v)^2\right)\frac{e^{*2}_{x(st)}}{2}, \quad (26)$$

where $v = \frac{b}{a\mu_X}$.

Applying expectation, we get the bias of T_{pr} as,

$$Bias(T_{pr}) = (k_1 - 1)\mu_Y + k_1\mu_Y(1-v)\frac{C^*_{xy(st)}}{2} - \left[k_2\mu_X(1-v) + k_1\mu_Y\left(1+\frac{1}{2}(1-v)^2\right)\right]\frac{A^*_{x(st)}}{2}. \quad (27)$$

Ignoring second-order terms in (27) and taking squares on both sides, we may have

$$(T_{pr} - \mu_Y)^2 = \mu_Y^2\left[(k_1-1)^2 + k_1^2\left(e^{*^2}_{y(st)} + \frac{e^{*^2}_{x(st)}}{4}(1-v)^2 - (1-v)e^*_{x(st)}e^*_{y(st)}\right) + 2k_1(k_1-1)\left\{e^*_{y(st)} - \frac{e^*_{x(st)}}{2}(1-v)\right\}\right] + k_2^2\mu_X^2 e^{*^2}_{x(st)} - 2k_2\mu_X\mu_Y\left[(k_1-1)e^*_{x(st)} + k_1\left\{e^*_{y(st)}e^*_{x(st)} - \frac{e^{*^2}_{x(st)}}{2}(1-v)\right\}\right], \quad (28)$$

Taking expectations on both sides of (28), the MSE expression attained is as,

$$MSE(T_{pr}) = \mu_Y^2\left[(k_1-1)^2 + k_1^2\left(A^*_{y(st)} + \frac{A^*_{x(st)}}{4}(1-v)^2 - (1-v)C^*_{xy(st)}\right)\right] + k_2^2\mu_X^2 A^*_{x(st)} - 2k_2k_1\mu_X\mu_Y\left\{C^*_{xy(st)} - \frac{A^*_{x(st)}}{2}(1-v)\right\}. \quad (29)$$

The optimum values of k_1 and k_2, may be obtained as,

$$k_1 = \frac{1}{\left(1 + G - \frac{T^2}{A^*_{x(st)}}\right)} \text{ and } k_2 = \frac{k_1\mu_Y T}{\mu_X A^*_{x(st)}} \text{ respectively.}$$

Substituting the values of $k_1$and k_2 in (29), the minimum MSE expression obtained is as,

$$minMSE\left(T_{pr}\right)=\mu_Y^2\left[1-\frac{1}{\left(1+G-\frac{T^2}{A^*_{x(st)}}\right)}\right]. \quad (30)$$

where $G=\left(A^*_{y(st)}+\frac{A^*_{x(st)}}{4}(1-v)^2-(1-v)C^*_{xy(st)}\right)$ and $T=\left\{C^*_{xy(st)}-\frac{A^*_{y(st)}}{2}(1-v)\right\}$.

5.2 Proposed Estimator II

Following a linear and an exponential function of auxiliary variables, a class of regression-cum-exponential estimators for estimating the population mean based on the two auxiliary variables is given by

$$T_{prr}=\left[w_1\tilde{\mu}^*_{y_{st}}+w_2\left(\frac{\mu_X-\tilde{\mu}^*_{x_{st}}}{\mu_X}\right)\right]exp\left[\frac{c\left(\mu_Z-\tilde{\mu}^*_{z_{st}}\right)}{c\left(\mu_Z+\tilde{\mu}^*_{z_{st}}\right)+2d}\right], \quad (31)$$

where w_1 and w_2 are unknown constants to be determined later such that the MSE of T_{prr} is minimized and *c* and *d* are suitably chosen scalars. One can note that the class of estimator T_{pr} shrink to the following set of known estimators present in Table 1.

Table 2
Class of Estimators

Estimators	w_1	w_2	C	d
$T_{prr(1)}=\tilde{\mu}^*_{y_{st}}$	1	0	0	0
$T_{prr(2)}=w_1\tilde{\mu}^*_{y_{st}}$	k_1	0	0	0
$T_{prr(3)}=\tilde{\mu}^*_{y_{st}}exp\left[\frac{(\mu_Z-\tilde{\mu}^*_{z_{st}})}{(\mu_Z+\tilde{\mu}^*_{z_{st}})}\right]$	1	0	1	0
$T_{prr(4)}=\tilde{\mu}^*_{y_{st}}exp\left[\frac{(\mu_Z-\tilde{\mu}^*_{z_{st}})}{(\mu_Z+\tilde{\mu}^*_{z_{st}})+2d}\right]$	1	0	1	d
$T_{prr(5)}=\tilde{\mu}^*_{y_{st}}exp\left[\frac{(\mu_Z-\tilde{\mu}^*_{z_{st}})}{(\mu_Z+\tilde{\mu}^*_{z_{st}})+2}\right]$	1	0	1	1

The proposed estimator T_{prr} is expressed in terms of *e*'s as,

$$T_{prr}=\left[w_1\mu_Y(1+e^*_{y(st)})+w_2\left(\frac{\mu_X-\mu_X(1+e^*_{x(st)})}{\mu_X}\right)\right] exp\left[\frac{c\left(\mu_Z-\mu_Z(1+e^*_{z(st)})\right)}{c\left(\mu_Z+\mu_Z(1+e^*_{z(st)})\right)+2d}\right], \quad (32)$$

where $\tilde{\mu}^*_{z(st)}=\mu_Z(1+e^*_{z(st)})$, such that

$E\left(e^*_{z(st)}\right) = 0,$

$E\left(e^*_{z(st)}\right)^2 = \left[\sum_{h=1}^{L} P_h^2 \left\{\lambda_h \left(C^2_{Z_h} + \frac{\sigma^2_{M_h}}{\mu_y^2}\right) + \theta_h \left(C^2_{Y_{h(2)}} + \frac{\sigma^2_{M_{h(2)}}}{\mu_y^2}\right)\right\}\right] = A^*_{z(st)},$

$E\left(e^*_{y(st)} e^*_{z(st)}\right) = \left[\sum_{h=1}^{L} P_h^2 \left(\lambda_h \rho_{YZ_h} C_{Y_h} C_{Z_h} + \theta_h \rho_{YZ_{h(2)}} C_{Y_{h(2)}} C_{Z_{h(2)}}\right)\right] = C^*_{yz(st)}$

and

$E\left(e^*_{x(st)} e^*_{z(st)}\right) = \left[\sum_{h=1}^{L} P_h^2 \left(\lambda_h \rho_{XZ_h} C_{Z_h} C_{Z_h} + \theta_h \rho_{XZ_{h(2)}} C_{X_{h(2)}} C_{Z_{h(2)}}\right)\right] = C^*_{xz(st)}.$

By solving we get,

$$T_{prr} - \mu_Y = (w_1 - 1)\mu_Y + w_1 \mu_Y \left[e^*_{y(st)} + (1 - v)\left(\frac{e^*_{z(st)}}{2} - \frac{e^*_{y(st)} e^*_{z(st)}}{2}\right)\right] - k_2 \mu_X \left(e^*_{x(st)} + \frac{e^*_{x(st)} e^*_{z(st)}}{2}(1 - v)\right) \tag{33}$$

where $\kappa = \frac{c}{d\mu_Z}$.

Applying expectation, we get the bias as,

$$Bias\left(T_{prr}\right) = (w_1 - 1)\mu_Y - w_1 \mu_Y (1 - v)\frac{C^*_{yz(st)}}{2} - w_2 \mu_X (1 - v)\frac{C^*_{xz(st)}}{2}. \tag{34}$$

Ignoring second-order terms in (24) and taking squares on both sides, we have

$$\left(T_{prr} - \mu_Y\right)^2 = \mu_Y^2 \left[(w_1 - 1)^2 + w_1^2 \left(e^{*^2}_{y(st)} + \frac{e^{*^2}_{z(st)}}{4}(1 - \kappa)^2 + (1 - \kappa) e^*_{y(st)} e^*_{z(st)}\right) + 2w_1(w_1 - 1)\left\{e^*_{y(st)} - \frac{e^*_{z(st)}}{2}(1 - \kappa)\right\}\right] + w_2^2 \mu_X^2 e^{*^2}_{x(st)} - 2w_2 \mu_X \mu_Y \left[(w_1 - 1)e^*_{x(st)} + w_1 \left\{e^*_{y(st)} e^*_{z(st)} - \frac{e^{*^2}_{z(st)}}{2}(1 - \kappa)\right\}\right], \tag{35}$$

Applying expectations on both sides of (35), we may attain the MSE to the first order of approximation as,

$$MSE\left(T_{prr}\right) = \mu_Y^2 \left[(w_1 - 1)^2 + w_1^2 \left(A^*_{y(st)} + \frac{A^*_{z(st)}}{4}(1 - \kappa)^2 - (1 - \kappa) C^*_{yz(st)}\right)\right] + w_2^2 \mu_X^2 A^*_{x(st)} - 2 w_2 w_1 \mu_X \mu_Y \left\{C^*_{xy(st)} - \frac{C^*_{xz(st)}}{2}(1 - \kappa)\right\}. \tag{36}$$

Differentiate (36) with respect to w_1 and w_2, and equate them to zero, the optimum values we get,

$$w_1 = \frac{1 - L_2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)} \text{ and } w_2 = \frac{k_1 \mu_Y L_1}{\mu_X A_{x(st)}^*}.$$

Hence the resulting minimum MSE of T_{prr} is as,

$$minMSE\left(T_{prr}\right) = \mu_Y^2 \left[1 - \frac{(1 - L_2)^2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)}\right]. \tag{37}$$

where

$$L_1 = \left(A_{y(st)}^* + \frac{A_{z(st)}^*}{4}(1 - v)^2 - (1 - v)C_{yz(st)}^*\right)$$

and

$$L_2 = \left\{C_{xy(st)}^* - \frac{C_{xz(st)}^*}{2}(1 - v)\right\}.$$

6. MATHEMATICAL COMPARISONS

i. From (15) and (37), we have

$$MSE\ (T_r) - minMSE\left(T_{prr}\right) = \frac{(1 - L_2)^2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)} - 1 + L_3 > 0$$

$$\text{or} \frac{(1 - L_2)^2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)} + L_3 > -1 \text{ is true}$$

Under the condition that

$L_3 > 1$ and $A_{x(st)}^* \neq 0$.

where $L_3 = A_{y(st)}^* + A_{x(st)}^* - 2C_{xy(st)}^*$.

ii. From (19) and (37), we have

$$MSE\ \left(T_{reg}\right) - minMSE\left(T_{prr}\right) = \left[A_{y(st)}^* - \frac{C_{xy(st)}^{*2}}{A_{x(st)}^*}\right] + \frac{(1 - L_2)^2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)} - 1 > 0.$$

$$\text{or } \left[A_{y(st)}^* - \frac{C_{xy(st)}^{*2}}{A_{x(st)}^*}\right] + \frac{(1 - L_2)^2}{\left(1 - \frac{L_1^2}{A_{x(st)}^*}\right)} > 1 \text{ is true,}$$

if $L_1 < 1$and $L_2 > 1$.

iii. From (23) and (37), we have

$$MSE\ (T_{ex}) - minMSE\left(T_{prr}\right) = \frac{(1-L_2)^2}{\left(1-\frac{L_1^2}{A^*_{x(st)}}\right)} - 1 + L_4 > 0.$$

or $\frac{(1-L_2)^2}{\left(1-\frac{L_1^2}{A^*_{x(st)}}\right)} + L_4 > 1$ is true,

if $L_1 < 1$and $A^*_{x(st)} \geq 2$.

where $L_4 = A^*_{y(st)} + \frac{A^*_{x(st)}}{4} - C^*_{xy(st)}$

iv. From (30) and (37) we have

$$minMSE\left(T_{pr}\right) - minMSE\left(T_{prr}\right) = \frac{(1-L_2)^2}{\left(1-\frac{L_1^2}{A^*_{x(st)}}\right)} - \frac{1}{\left(1+G-\frac{T^2}{A^*_{x(st)}}\right)} > 0.$$

or $\frac{(1-L_2)^2}{\left(1-\frac{L_1^2}{A^*_{x(st)}}\right)} > \frac{1}{\left(1+G-\frac{T^2}{A^*_{x(st)}}\right)}$ is true,

if $A^*_{x(st)} \geq 1$.

The above four observations displayed presents that the proposed estimator T_{prr} performs better as compared to the estimators T_r, T_{reg}, T_{exp} and T_{pr}.

7. SIMULATION BASED RESULTS AND DISCUSSIONS

In this section, we investigate the efficiency of the proposed generalized estimators. In this simulation study, we consider two finite populations of $N = 4000$ generated from multivariate normal distribution with theoretical same mean $[Y, X, Z]$ as $\mu = [62 \quad 45 \quad 20]$ and difference covariances matrices as given below:

Population I:

$$\sigma^2 = \begin{bmatrix} 36 & 15 & 13 \\ 15 & 20 & 12 \\ 13 & 12 & 13 \end{bmatrix}, \rho_{YX} = 0.5568; \rho_{YZ} = 0.5926; \rho_{YZ} = 0.7353.$$

Population II:

$$\sigma^2 = \begin{bmatrix} 20 & 13 & 13 \\ 13 & 16 & 12 \\ 13 & 12 & 13 \end{bmatrix}, \rho_{YX} = 0.8034; \rho_{YZ} = 0.7202; \rho_{YZ} = 0.8262.$$

For each population, we consider sample size of n=845, iterated 10000 times. The population is divided into four strata to certain criteria set for the auxiliary variables. The sample size from each stratum is based on Neyman allocation.

We have computed the absolute relative bias (ARB) for different suggested estimators by using the following expression:

$$ARB = \frac{|Bias(T_i)|}{\bar{Y}}, where\ i = r, ex, pr, prr.$$

ARB and MSE values based on these population data set under 30% and 40% nonresponse rate are given in Table 3-6.

Table 3
ARB result using Stratified Population I

W₂	**Estimators**	**1/k**			
		1/2	**1/3**	**1/4**	**1/5**
30% non-response	T_r	0.000043	0.000078	0.000113	0.000148
	T_{ex}	0.000021	0.000039	0.000056	0.000074
	T_{pr}	0.000008	0.000016	0.000023	0.000030
	T_{prr}	0.000002	0.000005	0.000007	0.000010
40% non-response	T_r	0.000072	0.000134	0.000197	0.000260
	T_{ex}	0.000036	0.000067	0.000098	0.000130
	T_{pr}	0.000014	0.000026	0.000038	0.000050
	T_{prr}	0.000004	0.000008	0.000012	0.000016

Table 4
ARB result using Stratified Population II

W₂	**Estimators**	**1/k**			
		1/2	**1/3**	**1/4**	**1/5**
30% non-response	T_r	0.000020	0.000036	0.000052	0.000067
	T_{ex}	0.000010	0.000018	0.000025	0.000034
	T_{pr}	0.000005	0.000009	0.000013	0.000016
	T_{prr}	0.000001	0.000002	0.000003	0.000005
40% non-response	T_r	0.000025	0.000047	0.000068	0.000090
	T_{ex}	0.000012	0.000023	0.000034	0.000045
	T_{pr}	0.000006	0.000011	0.000016	0.000022
	T_{prr}	0.000001	0.000003	0.000004	0.000005

Table 5
MSE Result using Stratified Population I

W_2	Estimators	1/k			
		1/2	**1/3**	**1/4**	**1/5**
30% non-response	t_0^*	0.2609	0.4726	0.6843	0.8960
	T_r	0.2893	0.5295	0.7697	1.0099
	T_{reg}	0.2196	0.3985	0.5774	0.7562
	T_{ex}	0.2206	0.4005	0.5805	0.7604
	T_{pr}	**0.2150**	**0.3909**	**0.5667**	**0.7425**
	$T_{pr(2)}$	0.2608	0.4722	0.6841	0.8960
	$T_{pr(4)}$	0.2201	0.4003	0.5805	0.7604
	$T_{pr(5)}$	0.2226	0.4043	0.5860	0.7676
	T_{prr}	**0.1675**	**0.2875**	**0.3133**	**0.5663**
	$T_{prr(3)}$	0.2512	0.4614	0.6438	0.7488
	$T_{prr(4)}$	0.2560	0.4682	0.6540	0.7513
	$T_{prr(5)}$	0.2681	0.4717	0.6706	0.7890
40% non-response	t_0^*	0.4288	0.8025	1.1762	1.5499
	T_r	0.4221	0.7908	1.1596	1.5283
	T_{reg}	0.3513	0.6578	0.9642	1.2707
	T_{ex}	0.3513	0.6578	0.9643	1.2708
	T_{pr}	0.3390	0.6347	0.9303	1.2259
	$T_{pr(3)}$	0.4281	0.8022	1.1766	1.5506
	$T_{pr(4)}$	0.3513	0.6578	0.9640	1.2706
	$T_{pr(5)}$	0.3546	0.6640	0.9734	1.2828
	T_{prr}	**0.2777**	**0.5267**	**0.7603**	**1.0256**
	$T_{prr(3)}$	0.4227	0.7470	1.1426	1.4222
	$T_{prr(4)}$	0.4238	0.7817	1.1459	1.4786
	$T_{prr(5)}$	0.4253	0.7973	1.1653	1.5019

Table 6
MSE result using Stratified Population II

W₂	**Estimators**	**1/k**			
		1/2	**1/3**	**1/4**	**1/5**
30% non-response	t_0^*	0.1460	0.2596	0.3731	0.4785
	T_r	0.1437	0.2331	0.3625	0.4232
	T_{reg}	0.1198	0.2144	0.3089	0.3970
	T_{ex}	0.1218	0.2187	0.3156	0.4061
	T_{pr}	**0.1215**	**0.2180**	**0.3145**	**0.3960**
	$T_{pr(2)}$	0.1456	0.2593	0.3730	0.4786
	$T_{pr(4)}$	0.1218	0.2187	0.3156	0.4060
	$T_{pr(5)}$	0.1233	0.2213	0.3193	0.4108
	T_{prr}	**0.0609**	**0.1093**	**0.1576**	**0.2079**
	$T_{prr(3)}$	0.1308	0.2314	0.3479	0.4307
	$T_{prr(4)}$	0.1314	0.2356	0.3568	0.4464
	$T_{prr(5)}$	0.1391	0.2398	0.3666	0.4544
40% non-response	t_0^*	0.1839	0.3353	0.4867	0.6381
	T_r	0.1635	0.3271	0.4319	0.5311
	T_{reg}	0.1513	0.2774	0.4035	0.5296
	T_{ex}	0.1541	0.2833	0.4125	0.5416
	T_{pr}	**0.1507**	**0.2723**	**0.4110**	**0.5396**
	$T_{pr(3)}$	0.1838	0.3351	0.4866	0.6382
	$T_{pr(4)}$	0.1541	0.2833	0.4125	0.5416
	$T_{pr(5)}$	0.1560	0.2866	0.4173	0.5480
	T_{prr}	**0.1277**	**0.1415**	**0.2060**	**0.2705**
	$T_{prr(3)}$	0.1789	0.2971	0.4697	0.5652
	$T_{prr(4)}$	0.1798	0.2975	0.4720	0.5839
	$T_{prr(5)}$	0.1813	0.3084	0.4751	0.5709

In this simulation study, the results of the comparative analysis between the proposed estimators, the modified ratio, regression and exponential estimators along with their respective class of estimator. The evaluation was conducted considering different populations under study and two nonresponse rates: 30% and 40%. The findings reveal that the proposed estimator consistently outperforms the alternative estimators in terms of efficiency. Specifically, examining the mean square error (MSE), it is evident that the proposed estimators yield smaller MSE values compared to the modified ratio and regression and exponential estimators as well as the respective classes for estimators. Furthermore, as the nonresponse rate increases from 2% to 5% the MSE results also demonstrate an upward trend. This superiority is observed across the range of populations investigated and holds true for both 30% and 40% nonresponse rates for moderate and high correlation populations.

Overall, these findings suggest that the proposed estimators may be a more reliable and effective approach to estimating population mean in the face of non-response bias, and measurement error, especially when compared to other commonly used estimators.

8. CONCLUSIONS

Based on the simulation results, the findings of the study demonstrate that the proposed estimators along with their class of estimators outperform the existing estimators when nonresponse and measurement error are present. The performance of the proposed estimator is characterized by higher efficiency and smaller increases in means squared error as the nonresponse rates increase compared to the competitor estimators. These results provide practical implications for researchers and practitioners engaged in estimating population parameters while dealing with the combined challenges of nonresponse and measurement error. The ability of the proposed estimators to effectively handle the joint presence of nonresponse and measurement error sets them apart making them a robust and reliable approach for population parameter estimation within complex survey designs.

Overall, the study's conclusion highlights the practical importance of the proposed estimators addressing the challenges posed by nonresponse and measurement error.

Conflict of interest:

The authors declare no conflict of interest.

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