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INTRODUCING AN EFFICIENT VARIANT OF DIAGONAL SYSTEMATIC SAMPLING UNDER AN INCREASING OR DECREASING LINEAR TREND

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ABSTRACT

In survey sampling, systematic random sampling is commonly employed by researchers to draw a random sample from a population under study. Assuming the population size being finite, an efficient modified variant of the systematic random sampling is proposed for dealing with linear trend data sets. The findings of the study indicate that the suggested sampling procedure attains much more improvement in efficiency over some of the available popular sampling designs in those circumstances where the units of a finite population show an increasing or decreasing type of a linear trend. Besides numerical illustration, the mathematical expressions for efficiency have also been obtained.

KEYWORDS

Survey sampling; systematic sampling; efficiency comparison; linear trend; diagonal systematic sampling.

1. INTRODUCTION

In recent decades, systematic sampling has attracted survey statisticians due to its simplicity of application to real-life surveys. Survey researchers find systematic sampling even simpler than the usual simple random sampling as only the first unit (or the first few units in some cases) of the population is selected randomly. The remaining units, after selection of the first unit, are obtained by following a pre-defined pattern. First introduced by Madow and Madow (1944), the systematic sampling along with its various forms have been analyzed by different researchers under various real-life circumstances. Madow and Madow (1944) presented the novel idea of selecting the units from the population according to a pre-defined rule. The Madow and Madow (1944) method had a drawback in the sense that it was only useful in circumstances in which the size of the finite population is a multiplicative factor of the sample size. In order to overcome this problem, Lahiri (1951) presented a new method widely known as the circular systematic random sampling. Chang and Huang (2000) presented a new modification of the linear systematic sampling commonly called the remainder systematic random sampling which is superior to the linear systematic sampling because of its applicability in those cases in which the size of a finite population cannot be necessarily expressed as a multiplicative factor of the

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sample size. Subramani (2000) introduced the principle of a new approach of systematic sample selection called diagonal systematic sampling. As its name suggests, the units are selected diagonally in the Subramani's (2000) sampling scheme. Sampath and Varalakshmi (2008) and Subramani (2009) suggested new modified forms of the originally developed diagonal systematic sampling design. In situations in which the size of the sample is an odd number, Subramani (2012) proposed a new variant of the systematic random sampling and proved that it was more efficient as compared to other existing forms of the systematic sampling. Another improved form of systematic random sampling was developed by Subramani and Gupta (2014). A benefit of the Subramani and Gupta (2014) sampling scheme was that it didn't necessarily need the size of the population to be expressed as a multiplicative factor of the required sample size, which improved its applicability. Azeem (2021) analyzed estimation of proportion under the diagonal systematic random sampling. For further literature on different aspects of the systematic sampling designs under various scenarios, one may refer to the studies of Khan et al. (2017), Sharma (2017), Mostafa and Ahmad (2018), Naidoo et al. (2018), Gupta et al. (2018), Rao et al. (2019), Khan et al. (2020), and Magnussen et al. (2020).

In a recent study, Azeem et al. (2021) presented an efficient variant of the diagonal systematic random sampling which improved the efficiency of the existing sampling schemes. Motivated by the studies of Chang and Huang (2000) and Azeem et al. (2021), the current study presents a new modification of the Chang and Huang and Azeem et al. (2021) methods. Our findings indicate that our proposed sampling method is more efficient than both Chang and Huang and Azeem et al. (2021) methods in situations in which a certain degree of linear trend occurs in the units of a finite population. The sampling variance of the mean on the basis of the new suggested sampling scheme is derived. The improvement in efficiency is shown for a real data set as well as for situations with a perfect linear trend.

2. CHANG AND HUANG (2000) SAMPLING DESIGN

Chang and Huang (2000) suggested a sampling scheme where the entire population is divided into two non-overlapping subsets. A linear systematic random sample is obtained from each subset, as shown in Figure 1. This method is popularly known as remainder systematic sampling.

Set – 1	**Set -2**
Linear Systematic Sampling	**Linear Systematic Sampling**

Figure 1: Chang and Huang (2000) Sampling Scheme

3. AZEEM et al. (2021) SAMPLING SCHEME

Azeem et al. (2021) suggested the use of the diagonal variant of systematic sampling in Set-1 rather than linear systematic sampling, while retaining linear systematic sampling in Set-2, as shown in Figure 2 below. Moreover, the group sizes used in this method were different from those of Chang and Huang (2000) method.

Set – 1	Set -2
Diagonal Systematic Sampling	**Linear Systematic Sampling**

Figure 2: Azeem et al. (2021) Sampling Scheme

4. PROPOSED SAMPLING SCHEME

Motivated by Chang and Huang (2000) and Azeem et al. (2021), we suggest an efficient variant of the diagonal systematic random sampling which uses diagonal systematic sampling in both sets, as shown in Figure 3.

Set – 1	Set -2
Diagonal Systematic Sampling	**Diagonal Systematic Sampling**

Figure 3: Proposed Sampling Scheme

It is worth noting that the suggested sampling design retains the group sizes of the Azeem et al. (2021) sampling scheme but uses diagonal systematic sampling in place of linear systematic sampling in Set-2.

To develop an estimator of finite population mean, let the population of interest consists of a total of N units and suppose that a random sample of n units is needed to be selected such that $N = nk = k \cdot k + (n-k)k$. The proposed method selects the sample in the following steps:

1) Arrange the finite population of interest into two non-overlapping sets of units: Set-1 and Set-2, so that Set-1 gets the first $k \times k = k^2$ units y_i (i=1, 2, …, k^2) which can be placed in a matrix having order $k \times k$, Set-2 gets the next $(n-k)k$ units y_i ($i = kk+1,\ kk+2,\ kk+3, \ldots, nk$).
2) Organize the units of Set-1 in a $k \times k$ square matrix. Arrange the $(n-k)k$ units of Set-2 so as to form a matrix having order $(n-k) \times k$ as presented in Table 1.
3) Generate two numbers at random, say, r_1 and r_2, for $1 \le r_1 \le k$ and $1 \le r_2 \le k$. The units from Set-1 are selected so that the chosen k units are the elements of the diagonal of the resulting matrix. Likewise, In Set-2, the chosen $n-k$ units belong to the r_2th diagonal or broken diagonal. At the final stage, the units obtained from both sets can be combined together to achieve the required systematic sample.

Table 1
Units of Population Arranged in Two Sets

Set-1					**Set-2**				
S#	**1**	**2**	$\cdots$	k	**S#**	**1**	**2**	$\cdots$	k
1	y_1	y_2	$\cdots$	y_k	k+1	y_{kk+1}	y_{kk+2}	$\cdots$	$y_{kk+k=(k+1)k}$
2	y_{k+1}	y_{k+2}	$\cdots$	y_{2k}	k+2	$y_{(k+1)k+1}$	$y_{(k+1)k+2}$	$\cdots$	$y_{(k+2)k}$
3	y_{2k+1}	y_{2k+2}	$\cdots$	y_{3k}	k+3	$y_{(k+2)k+1}$	$y_{(k+2)k+2}$	$\cdots$	$y_{(k+3)k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$y_{(k-1)k+1}$	$y_{(k-1)k+2}$	$\cdots$	y_{kk}	n-1	$y_{(n-1)k+1}$	$y_{(n-1)k+2}$	$\cdots$	y_{nk}

The number of all possible samples that can be selected under the new sampling scheme is $k \times k = k^2$, each having size n. The probabilities of inclusion based on the proposed systematic random sampling are as follows:

$$\pi_i = \frac{1}{k} \tag{1}$$

and

$$\pi_{ij} = \begin{cases} \frac{1}{k}, & \text{if } i\text{th and } j\text{th units belong to the same diagonal of Set-1} \\ \frac{1}{k}, & \text{if } i\text{th and } j\text{th units belong to the same diagonal of Set-2} \\ \frac{1}{k^2}, & \text{if } i\text{th and } j\text{th units belong to two different sets,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Generally, for $r_1 = 1, 2, ..., k$, $r_2 = 1, 2, ..., k$, the units selected under the proposed sampling scheme depend on four possible cases.

Case 1: If $r_1 = 1,\ r_2 = 1$, then the selected units are:

$$S_{r_1 r_2} = \Big\{ y_{r_1}, y_{(k+1)+r_1}, ..., y_{(k-1)(k+1)+r_1}, y_{kk+r_2}, y_{(k+1)+kk+r_2}, \\ y_{2(k+1)+kk+r_2}, ..., y_{(n-1)(k+1)+kk+r_2} \Big\}.$$

Case 2: If $r_1 = 1,\ r_2 > 1$, then the selected units are:

$$S_{r_1 r_2} = \Big\{ y_{r_1}, y_{(k+1)+r_1}, ..., y_{(k-1)(k+1)+r_1}, y_{kk+r_2}, y_{(k+1)+kk+r_2}, \\ ..., y_{t_2(k+1)+kk+r_2}, y_{(t_2+1)k+1}, y_{(t_2+2)+2}, ..., y_{(n-1)k+k-t_2-1} \Big\}.$$

Case 3: If $r_1 > 1$, $r_2 = 1$, then the selected units are:

$$S_{r_1 r_2} = \left\{ y_{r_1}, y_{(k+1)+r_1}, \ldots, y_{t_1(k+1)+r_1}, y_{(t_1+1)k+1}, y_{(t_1+2)k+2}, \ldots, y_{(k-1)k+k-t_1-1}, \right.$$
$$\left. y_{kk+r_2}, y_{(k+1)+kk+r_2}, y_{2(k+1)+kk+r_2}, \ldots, y_{(n-1)(k+1)+kk+r_2} \right\}.$$

Case 4: If $r_1 > 1$, $r_2 > 1$, then the selected units are:

$$S_{r_1 r_2} = \left\{ y_{r_1}, y_{(k+1)+r_1}, \ldots, y_{t_1(k+1)+r_1}, y_{(t_1+1)k+1}, y_{(t_1+2)k+2}, \ldots, y_{(k-1)k+k-t_1-1}, \right.$$
$$\left. y_{kk+r_2}, y_{(k+1)+kk+r_2}, \ldots, y_{t_2(k+1)+kk+r_2}, y_{(t_2+1)k+1}, y_{(t_2+2)+2}, \ldots, y_{(n-1)k+k-t_2-1} \right\},$$

where $t_1 = k - r_1$ and $t_2 = k - r_2$. The sample mean is given by:

$$\bar{y}_{msy} = w_1 \bar{y}_1 + w_2 \bar{y}_2, \tag{3}$$

where,

$$\bar{y}_1 = \begin{cases} \dfrac{1}{k} \sum\limits_{l=0}^{k-1} y_{l(k+1)+r_1}, & \text{if } r_1 = 1, \\ \dfrac{1}{k} \left(\sum\limits_{i=0}^{t} y_{i(k+1)+r_1} + \sum\limits_{i=1}^{k-t-1} y_{(t+i)k+i} \right), & \text{if } r_1 > 1. \end{cases} \tag{4}$$

and,

$$\bar{y}_2 = \begin{cases} \dfrac{1}{n-k} \sum\limits_{l=k}^{n-1} y_{l(k+1)+kk+r_2}, & if\ r_2 = 1 \\ \dfrac{1}{n-k} \left(\sum\limits_{i=0}^{t_2} y_{i(k+1)+kk+r_2} + \sum\limits_{i=1}^{k-t_2-1} y_{(t_2+i)k+kk+i} \right), & if\ r_2 > 1 \end{cases}, \tag{5}$$

where $w_1 = \dfrac{k}{n}$, $w_2 = \dfrac{n-k}{n}$, $w_1 + w_2 = 1$.

Theorem:

The sample mean under the suggested sampling design may be expressed in the Horvitz and Thompson (1952) form. Moreover, the sample mean is unbiased with sampling variance given by:

$$Var(\bar{y}_{msy}) = \frac{1}{N^2}\left[k^4\left\{\frac{1}{k}\sum_{i=1}^{k}(\bar{y}_{1i} - \bar{Y}_1)^2\right\} + (n-k)^2 k^2 \left\{\frac{1}{k}\sum_{i=1}^{k}(\bar{y}_{2i} - \bar{Y}_2)^2\right\}\right],$$

where $\bar{y}_1$ and $\bar{y}_2$ denote the mean of the sample obtained from the units in Set-1 and Set-2, respectively. Moreover, $\bar{Y}_1$ and $\bar{Y}_2$ denote the means based on Set-1 and Set-2, respectively, whereas k denotes the size of the group.

Proof: By definition

$$\bar{y}_{msy} = \frac{k^2}{N}\bar{y}_1 + \frac{(n-k)k}{N}\bar{y}_2 = \frac{1}{N}\left(k\sum_{i\in s_1} y_{1i} + k\sum_{i\in s_2} y_{2i}\right), \tag{6}$$

where s_1 and s_2 are the samples obtained from the units of Set-1 and Set-2, respectively.

$$\bar{y}_{msy} = \frac{1}{N}\left(\sum_{i\in s_1}\frac{y_{1i}}{1/k} + \sum_{i\in s_2}\frac{y_{2i}}{1/k}\right) = \frac{1}{N}\sum_{i\in s}\frac{y_i}{\pi_i} = \bar{y}_{HT}, \tag{7}$$

where '*s*' is the sample selected from the total units of the entire population. Appling expectation on equation (6) gives:

$$E\left(\bar{y}_{msy}\right) = \frac{k^2}{N}E\left(\bar{y}_1\right) + \frac{(n-k)k}{N}E\left(\bar{y}_2\right). \tag{8}$$

Now,

$$E\left(\bar{y}_1\right) = E\left(\frac{1}{k}\sum_{i=1}^{k} y_{1i}\right) = \frac{1}{k}\sum_{i=1}^{k} E\left(y_{1i}\right).$$

$$E\left(\bar{y}_1\right) = E\left(\frac{1}{k}\sum_{i=1}^{k} y_{1i}\right) = E\left(y_{1i}\right) = \bar{Y}_1 \tag{9}$$

Similarly,

$$E\left(\bar{y}_2\right) = E\left(\frac{1}{n-k}\sum_{i=1}^{n-k} y_{2i}\right) = E\left(y_{2i}\right) = \bar{Y}_2. \tag{10}$$

Now using equation (9) and (10) in equation (8) gives:

$$E\left(\bar{y}_{msy}\right) = \bar{Y}.$$

Now applying variance on equation (3) gives:

$$Var\left(\bar{y}_{msy}\right) = \frac{k^4}{N^2}Var\left(\bar{y}_1\right) + \frac{(n-k)^2 k^2}{N^2}Var\left(\bar{y}_2\right), \tag{11}$$

where,

$$Var\left(\bar{y}_1\right) = \frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{1i} - \bar{Y}_1\right)^2, \tag{12}$$

since each possible sample in Set-1 has probability equal to *1/k*. Similarly,

$$Var\left(\bar{y}_2\right) = \frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{2i} - \bar{Y}_2\right)^2. \tag{13}$$

Using (12) and (13) in (11), the sampling variance of $\bar{y}_{msy}$ under the suggested sampling design is obtained as:

$$Var\left(\bar{y}_{msy}\right) = \frac{1}{N^2}\left[k^4\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{1i} - \bar{Y}_1\right)^2\right\} + (n-k)^2 k^2\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{2i} - \bar{Y}_2\right)^2\right\}\right] \tag{14}$$

Remark 1:

Following the approach given by Sen-Yates-Grundy [20,21], the sampling variance of $\bar{y}_{msy}$ may be expressed as:

$$Var\left(\bar{y}_{msy}\right)=\frac{1}{N^2}\left\{\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right\}=Var_{SYG}\left(\bar{y}_{HT}\right) \tag{15}$$

Remark 2:

A Sen-Yates-Grundy type estimator of the sampling variance given in (15), is given as:

$$var\left(\bar{y}_{msy}\right)=\frac{1}{N^2}\left\{\frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right\}=var_{SYG}\left(\bar{y}_{HT}\right) \tag{16}$$

One can use the mathematical expressions of π_i and π_{ij} given in equation (1) and equation (2) in equation (15) and equation (16) to get the variance of the sample mean as well as its estimator based on the proposed method.

5. LINEAR TREND AND ITS PRACTICAL EXISTENCE

Linear trend refers to the arithmetic progression which the population units may follow, which may either be in increasing or decreasing order. In some practical circumstances, a moderate to a high degree of linear trend can occur. As an illustration, educational institutes almost everywhere in the world offer admissions on merit basis in various academic departments. Many universities tend to allocate roll numbers to their students on the basis on their quantified academic scores during admission process. In such cases, intelligent students tend to occupy the top enrollment numbers. Thus, upon admission, if the examination marks obtained by students are observed in order of students' enrollment roll numbers, one can expect a moderate level of increasing trend in this data set since the top enrolled students, being the merit toppers, tend to perform better in subsequent examinations.

Likewise, another common occurrence of linear trend can be observed in milk-yield data sets. Consider the daily record of the milk yield data, starting from the day of calving. One can naturally expect that the daily milk yield quantity decreases over time, resulting in a high degree of decreasing trend.

Suppose the population under consideration exhibits a complete linear trend. Thus,

$$y_i = a + ib, \qquad i = 1,\ 2,\ 3,\ \ldots, N\ . \tag{17}$$

The sampling variance of the sample mean of simple random sampling under perfect linear trend is as follows:

$$Var\left(\bar{y}_r\right)=\left(k-1\right)\left(N+1\right)\frac{b^2}{12} \tag{18}$$

The variance of the sample mean under systematic random sampling is as follows:

$$Var\left(\bar{y}_{sy}\right)=\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{19}$$

The sampling variance using the diagonal systematic random sampling is as follows:

$$Var\left(\bar{y}_{dsy}\right)=\left(k-n\right)\left[n\left(k-n\right)+2\right]\frac{b^2}{12n} \tag{20}$$

where $N=nk+r$. The variance under the modified systematic sampling suggested by Subramani (2012) can be derived as:

$$Var\left(\bar{y}_{ssy}\right)=\left(\frac{\left(n-1\right)^2+1}{n^2}\right)\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{21}$$

The variance of the Azeem et al. (2021) sampling scheme is given as:

$$Var\left(\bar{y}_{mdsy}\right)=\left(\frac{n-k}{n}\right)^2\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{22}$$

Finally, under complete linear trend in the population units, the variance of the suggested sampling design can be derived as:

$$Var\left(\bar{y}_{msy}\right)=w_1^2Var\left(\bar{y}_1\right)+w_2^2Var\left(\bar{y}_2\right) \tag{23}$$

In order to obtain the variance of the proposed sampling scheme under perfect linear trend, we first need to obtain the variance expressions for $\bar{y}_1$ and $\bar{y}_2$. Since the total number of units in Set-1 are $k\times k=k^2$, so using $k=n$ in (20) leads to

$$Var\left(\bar{y}_1\right)=0 \tag{24}$$

Moreover, Set-2 consists of a total of $(n-k)k$ units where the diagonal systematic sampling method is utilized. Substituting $n=n-k$ in the right-hand side of (20) yields:

$$Var\left(\bar{y}_2\right)=\left(2k-n\right)\left[\left(n-k\right)\left(2k-n\right)+2\right]\frac{b^2}{12\left(n-k\right)} \tag{25}$$

Substituting (24) and (25) in (23), and after simplification, the variance of $\bar{y}_{msy}$ is obtained as:

$$Var\left(\bar{y}_{msy}\right)=\left[\frac{n-k}{n}\right]^2\left(2k-n\right)\left[\left(n-k\right)\left(2k-n\right)+2\right]\frac{b^2}{12\left(n-k\right)} \tag{26}$$

6. EFFICIENCY COMPARISON UNDER LINEAR TREND

In this section, the mathematical expressions for the conditions of the efficiency of the new suggested sampling design with other popular sampling designs are obtained. These conditions are derived based on the assumption that a linear trend exists among the units of finite population.

6.1 Comparison with Simple Random Sampling

In cases where the units form a linear trend, the suggested sampling scheme will be more precise than simple random sampling, if

$$Var\left(\bar{y}_{msy}\right) < Var\left(\bar{y}_{r}\right) \tag{27}$$

Substituting (18) and (26) in (27) and further simplification yields:

$$\left[\frac{n-k}{n}\right]^2 (2k-n)\left[(n-k)(2k-n)+2\right] < (n-k)(k-1)(N+1) \tag{28}$$

6.2 Comparison with Systematic Random Sampling

The suggested sampling design will be more efficient than the linear systematic random sampling if

$$Var\left(\bar{y}_{msy}\right) < Var\left(\bar{y}_{sy}\right) \tag{29}$$

Substituting (19) and (26) in (29) yields:

$$(n-k)(2k-n)\left[(n-k)(2k-n)+2\right] < n^2 (k-1)(k+1)\,. \tag{30}$$

6.3 Comparison with Subramani's (2012) Sampling Scheme

Under perfect linear trend, the suggested sampling scheme will be more efficient than Subramani's (2012) sampling procedure if,

$$Var\left(\bar{y}_{msy}\right) < Var\left(\bar{y}_{ssy}\right) \tag{31}$$

Using equation (21) and (26) in equation (31) yields:

$$(n-k)(2k-n)\left[(n-k)(2k-n)+2\right] < \left[(n-1)^2+1\right](k-1)(k+1)\,. \tag{32}$$

6.4 Comparison with Diagonal Systematic Sampling

Our proposed sampling scheme will be more efficient than Subramani's (2000) diagonal systematic random sampling if,

$$Var\left(\bar{y}_{msy}\right) < Var\left(\bar{y}_{dsy}\right) \tag{33}$$

Using equation (20) and (26) in equation (33) gives:

$$\left[\frac{n-k}{n}\right]^2 (2k-n)\left[(n-k)(2k-n)+2\right]\frac{b^2}{12(n-k)} < (k-n)\left[n(k-n)+2\right]\frac{b^2}{12n}\,,$$

or,

$$(n-2k)\left[(n-k)(2k-n)+2\right] < n\left[n(k-n)+2\right]. \tag{34}$$

6.5 Comparison with Azeem et al. (2021) Sampling Procedure

Our proposed sampling design will be more efficient than the Azeem et al. (2021) sampling scheme if,

$$Var\left(\bar{y}_{msy}\right) < Var\left(\bar{y}_{mdsy}\right) \tag{35}$$

Using (22) and (26) in (35) gives:

$$\left[\frac{n-k}{n}\right]^2 (2k-n)\left[(n-k)(2k-n)+2\right]\frac{b^2}{12(n-k)} < \left(\frac{n-k}{n}\right)^2 (k-1)(k+1)\frac{b^2}{12}$$

or

$$(2k-n)\left[(n-k)(2k-n)+2\right] < (n-k)(k-1)(k+1) \tag{36}$$

The above mathematical conditions seem to be complex at a glance. Since the proposed sampling design uses $n > k$, so usually these conditions are true for $n > k$. It is also to be noted that unlike the Subramani's (2000) diagonal systematic sampling which is applicable for $k > n$, the suggested sampling design is applicable in circumstances where $n > k$.

7. EMPIRICAL RESULTS AND DISCUSSION

Consider the cases where the units exhibit a perfect linear trend. We have performed efficiency comparison for different choices of the values of *N*, *n*, and *k*. The variances of the sample mean under the suggested and other popular sampling methods were displayed in Table 2. The choice of values of *N*, *n*, and *k* for the purpose of efficiency comparison has been determined in a manner that $N = nk$ and $n > k$. It is also worth noting here that since the constant b^2 plays only the role of a multiplication factor in the variance expressions of all of the sampling procedures discussed in Section-5, so in order to make the analysis easier, $b = 1$ has been used in the calculation of variances. The findings from Table 2 clearly indicate that our suggested systematic sampling design is superior in terms of efficiency over the existing sampling schemes, including the sampling scheme suggested by Azeem et al. (2021).

As is the case with the Azeem et al. (2021) sampling scheme, the suggested sampling scheme also divides the finite population into two subsets. The difference is that the proposed method uses diagonal systematic sampling in both subsets as opposed to the Azeem et al. (2021) method which uses linear systematic sampling in the second subset. Following Chang and Huang (2000) and Azeem et al. (2021), the suggested method uses a weighting approach to estimate the population mean. Efficiency comparison has been carried out to analyze the performance of the new sampling scheme with regard to other available sampling designs, under the existence of a complete linear trend, and the improvement in efficiency has been shown. On the basis of the findings of the current study, the proposed method is recommended for use in practical situations where a high degree of an increasing or decreasing linear trend exists.

Table 2
Variances of Different Sampling Designs in the case of Perfect Linear Trend

n	k	$Var(\bar{y}_r)$	$Var(\bar{y}_{sy})$	$Var(\bar{y}_{dsy})$	$Var(\bar{y}_{ssy})$	$Var(\bar{y}_{mdsy})$	$Var(\bar{y}_{msy})$
10	4	10.25	1.25	2.90	1.03	0.45	0.10
	6	25.42	2.92	1.27	2.39	0.47	0.07
	8	47.25	5.25	0.30	4.31	0.21	0.14
30	10	225.75	8.25	33.22	7.72	3.67	3.67
	15	526.17	18.67	18.67	17.46	4.67	0.00
	20	951.58	33.25	8.28	31.11	3.69	0.94
	25	1502.00	52.00	2.06	48.65	1.44	0.94
50	20	1584.92	33.25	74.90	31.95	11.97	2.98
	30	3627.42	74.92	33.27	71.98	11.99	1.35
	35	4961.17	102.00	18.70	98.00	9.18	3.02
	40	6503.25	133.25	8.30	128.03	5.33	3.02
100	40	13003.25	133.25	299.90	130.61	47.97	11.98
	60	29504.92	299.92	133.27	293.98	47.99	5.35
	70	40255.75	408.25	74.95	400.17	36.74	12.02
	80	52673.25	533.25	33.30	522.69	21.33	12.02
300	140	486511.58	1633.25	2133.24	1622.40	464.57	9.48
	180	805514.92	2699.92	1199.93	2681.98	431.99	48.01
	220	1204518.25	4033.25	533.29	4006.45	286.81	116.17
	260	1683521.58	5633.25	133.31	5595.82	100.15	71.72
500	180	1342514.92	2699.92	8533.23	2689.14	1105.89	668.98
	260	2805854.92	5633.25	4799.92	5610.76	1297.90	7.68
	320	4253359.92	8533.25	2699.94	8499.19	1105.91	211.70
	400	6650033.25	13333.25	833.30	13280.02	533.33	300.02
	450	8418787.42	16874.92	208.32	16807.55	168.75	133.35

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