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A NEW CLASS OF ROBUST REGRESSION-IN-RATIO ESTIMATOR FOR POPULATION MEAN IN RANKED SET SAMPLING SCHEMES

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ABSTRACT

The ordinary least square (OLS) produced inefficient estimates of population mean when data have outliers. The redescending M-estimators are used as alternate method to tackle the effect of outliers. In this article, we have proposed an efficient class of regression-in-ratio estimators to estimate the population mean in the presence of outliers using robust regression in ranked set sampling schemes. Performance of proposed class of regression-in-ratio estimators is compared with existing estimators in considered sampling schemes using mean square error. The proposed class of estimators is found to be more efficient in all considered ranked set sampling schemes. A real-life data example and extensive simulation study are included to validate the results.

KEYWORDS

Outliers, Ranked Set Sampling, Redescending M-Estimator, Robust Regression.

1. INTRODUCTION

Regression-in-ratio estimators are used to improve the efficiency of estimators for population mean when auxiliary information is available. Kadilar and Cingi (2004) suggested that use of information regarding the population parameters such as coefficient of kurtosis and coefficient of variation of auxiliary variable can be used to increase the efficiency of ratio estimators. Al-Hadhrami (2009) proposed a class of regression-in-ratio estimators in ranked set sampling (*RSS*) and concluded that this proposed class of estimators performed well as compare to considered estimators in *SRS* design to estimate the population mean.

It is well-known phenomena that ordinary least square (*OLS*) estimators of population mean do not produce reliable estimates when data contain outliers and only a single value of outlier can disturb the performance of *OLS*. In this situation, robust estimators called M- estimators are used to remove the effect of outliers. Since the invention of M-estimator, a number of researchers developed their M-estimators to minimize the effect of outliers. One can consult, Huber (1964), Beaton and Tukey (1974), Andrew et al. (1972), Hample et al. (1986), Qadir (1996), Khalil et al. (2016), Noor-ul-Amin et al. (2018) and Subzar et al.(2020) for further details of M-estimators. Robust Ratio type estimators for population mean are also formulated using M-estimators to deal with data having outliers, such as

Kadilar et al. (2007), Noor-ul-Amin et al. (2016), Raza et al. (2019), Menezes et al. (2021), Noor-ul-Amin and Raza (2021) and Ahmed et al. (2022).

Huber (1964) has developed following objective function $\rho_2(e)$, where '*e*' is the OLS residual obtained from $y = a_0 + bx + e$.

$$\rho_2(e_i) = \begin{cases} \dfrac{e^2}{2} & |e| \le c \\ c|e| - \dfrac{v^2}{2} & |e| > c \end{cases} \tag{1.1}$$

where *c* is tuning constant which is used to control the robustness of the objective function and its value in terms of standard deviation ($\hat{\sigma}$) of error terms is $1.5\hat{\sigma}$ to obtain 95% efficiency. The objective function proposed by Huber (1964) does not perform well for large residuals. To overcome this drawback, Raza et al. (2019) suggested following objective function $\rho_3(e)$

$$\rho_3(e) = \frac{c^2}{2w}\left[1 - \left\{1 + \left(\frac{e}{c}\right)^2\right\}^{-w}\right] \qquad |e| \ge 0 \tag{1.2}$$

where *c* and *w* are tuning constants which control the robustness of objective function. For current study tuning constants have values $c = 2.5$ and $w = 6$. The M-estimator uses the symmetrical loss function instead of sum of square of errors. The Robust estimate of *b* is found by minimizing the $\sum_{i=1}^{n} \rho_j(y_i - a_0 - bx_i), j = 2,3$ with respect to *b* and ρ_j are the considered objective functions. Raza et al. (2019) used objective functions defined in (1.2) to develop a new class robust regression-in-ratio estimator in *SRS* design to obtain more efficient estimates of population mean in the presence of outliers. Robust estimation in *RSS* design has discussed some authors i.e. Majd and Saba (2018) but much work is needed on robust estimation in *RSS* schemes.

In present study, we have developed an efficient class of robust regression-in-ratio estimators in *RSS*, Median *RSS* (*MRSS*), Quartile *RSS* (*QRSS*) and Even Order *RSS* (*EORSS*) by using objective functions mentioned in (1.1) and (1.2) to deal with data having outliers. Superiority of the proposed class of regression-in-ratio estimators to the existing class of ratio estimators in *SRS* design and *RSS* schemes is supported with help of a real life data example and by using extensive simulation study. In upcoming section *2*, the *RSS*, *MRSS*, *QRSS* and *EORSS* schemes are discussed. Ratio-in-regression estimators in *SRS* design suggested by Kadilar and Cingi (2004) are presented in section *3*. Section *4* deals with the regression-in-ratio estimator proposed by Al-Hadhrami (2009) in *RSS* design. The proposed class of robust regression-in-ratio estimators for population mean in *RSS* schemes is cited in section *5*. Example based on real life data and simulation analysis is described in section *6*. Conclusions drawn on the basis of results obtain in section 6 are placed in section *7*.

2. RANKED SET SAMPLING DESIGNS

The *RSS* was invented by McIntyre (1952) to obtain efficient estimate of population mean. This scheme was remains abeyant for many years but the cost effective nature of *RSS* is highlighted significantly in last two decades. Many authors proposed modified versions of selection procedures in *RSS* design to increase the efficiency of estimates. Here, some renowned modifications selection schemes in *RSS* design are discussed which will be used in upcoming sections for construction of robust regression-in-ratio estimators of population mean. The procedures and mathematical expressions of mean, variance and covariance in these schemes are described as follow.

2.1 Ranked Set Sampling

The *RSS* procedure is used to obtain efficient estimator of population mean by ordering the observations according to some characteristic of interest. The expressions of mean and variance for the mean of variable of interest are described as follows for single cycle i.e. $r=1$.

$$\bar{y}_{RSS}=\frac{1}{t}\sum_{i=1}^{t}y_{i[i]} \tag{2.1}$$

where $y_{i(i)}$ is the ranked-set sample of size *t*.

$$Var\left(\bar{y}_{RSS}\right)=\frac{\sigma_y^2}{t}-\frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]}-\mu_y\right)^2 \tag{2.2}$$

with covariance between x and y is

$$COV\left(\bar{x}_{(RSS)},\bar{y}_{[RSS]}\right)=\frac{\sigma_{xy}}{t}-\frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]}-\mu_y\right)\left(\mu_{x(i)}-\mu_x\right) \tag{2.3}$$

For mean and variance of auxiliary variable in *RSS* simply replace variable X with Y in all above expressions. For more details, consult Patil (2002).

2.2 Median Ranked Set Sampling

The *MRSS* was introduced by Muttlak (1997) to estimate the population mean with greater efficiency. The estimator of population mean, variance and covariance in MRSS scheme for even number of observations with single cycle are described as follows

$$\bar{y}_{(MRSS)e}=\frac{1}{t}\left[\sum_{i=1}^{t/2}y_{i\left[\frac{t}{2}\right]}+\sum_{i=1}^{t/2}y_{\frac{t}{2}+i\left[\frac{t+2}{2}\right]}\right] \tag{2.4}$$

where $y_{i\left[\frac{i}{2}\right]}$ is the median ranked sample of size t.

$$Var\left(\bar{y}_{(MRSS)e}\right)=\frac{1}{2t}\left[\sigma^2_{y\left[\frac{t}{2}\right]}+\sigma^2_{y\left[\frac{t+2}{2}\right]}\right] \tag{2.5}$$

$$COV\left(\bar{x}_{(MRSS)e}, \bar{y}_{[MRSS]e}\right) = \beta_{Me} Var\left(\bar{x}_{(MRSS)e}\right) \tag{2.6}$$

where $\beta_{Me} = \rho\sqrt{\frac{Var\left(\bar{y}_{[MRSS]e}\right)}{Var\left(\bar{x}_{(MRSS)e}\right)}}$

For more details see Al-Omari (2012).

For odd number of observation, estimator of mean, variance and covariance for $r = 1$ are given as

$$\bar{y}_{(MRSS)o} = \frac{1}{t}\left[\sum_{i=1}^{t} y_{i\left[\frac{t+1}{2}\right]}\right] \tag{2.7}$$

$$Var\left(\bar{y}_{(MRSS)o}\right) = \frac{1}{t}\left[\sigma^2_{y\left[\frac{t+1}{2}\right]}\right] \tag{2.8}$$

$$COV\left(\bar{x}_{(MRSS)o}, \bar{y}_{[MRSS]o}\right) = \beta_{Mo} Var\left(\bar{x}_{(MRSS)o}\right) \tag{2.9}$$

where $\beta_{Mo} = \rho\sqrt{\frac{Var\left(\bar{y}_{[MRSS]o}\right)}{Var\left(\bar{x}_{(MRSS)o}\right)}}$

To obtain the mean and variance of concomitant variable in *MRSS*, replace variable X in place of Y in all above expressions in section *2.2*.

2.3 Quartile Ranked Set Sampling

Muttlak (2003) proposed a new scheme in *RSS*, named *QRSS* which provided efficient estimates of population mean. The estimator of population mean and variance in *QRSS* with single cycle for even sample size is described as

$$\bar{y}_{(QRSS)e} = \frac{1}{t}\left[\sum_{i=1}^{t/2} y_{i\left[\frac{t+1}{4}\right]} + \sum_{i=1}^{t/2} y_{\frac{t}{2}+i\left[\frac{3(t+1)}{4}\right]}\right] \tag{2.10}$$

where $y_{i\left[\frac{v(t+1)}{4}\right]}, v = 1,3$ is the Quartile ranked sample of size *t*.

$$Var\left(\bar{y}_{(QRSS)e}\right) = \frac{1}{2t}\left[\sigma^2_{y\left[\frac{t+1}{4}\right]} + \sigma^2_{y\left[\frac{3(t+1)}{4}\right]}\right] \tag{2.11}$$

and covariance

$$COV\left(\bar{x}_{(QRSS)e}, \bar{y}_{[QRSS]e}\right) = \beta_{Qe} Var\left(\bar{x}_{(QRSS)e}\right) \tag{2.12}$$

where $\beta_{Qe} = \rho\sqrt{\dfrac{Var\left(\bar{y}_{[QRSS]e}\right)}{Var\left(\bar{x}_{(QRSS)e}\right)}}$.

For odd number of observation, estimator of mean and its variance are given by

$$\bar{y}_{(QRSS)o} = \frac{1}{t}\left[\sum_{i=1}^{(t-1)/2} y_{i\left[\frac{t+1}{4}\right]} + \sum_{i=1}^{(t-1)/2} y_{i\left[\frac{t-1}{4}\right]+i\left[\frac{3(t+1)}{4}\right]} + y_{i\left[\frac{t+1}{2}\right]}\right] \tag{2.13}$$

$$Var\left(\bar{y}_{(QRSS)o}\right) = \frac{t-1}{2t^2}\left[\sigma^2_{y\left[\frac{t+1}{4}\right]} + \sigma^2_{y\left[\frac{3(t+1)}{4}\right]}\right] + \frac{1}{t^2}\left[+\sigma^2_{y\left[\frac{t+1}{2}\right]}\right] \tag{2.14}$$

with covariance

$$COV\left(\bar{x}_{(QRSS)o}, \bar{y}_{[QRSS]o}\right) = \beta_{Qo} Var\left(\bar{x}_{(QRSS)o}\right) \tag{2.15}$$

where $\beta_{Qo} = \rho\sqrt{\dfrac{Var\left(\bar{y}_{[QRSS]o}\right)}{Var\left(\bar{x}_{(QRSS)o}\right)}}$.

Mean and variance for auxiliary variable in *QRSS* can be expressed in similar fashion just replacing variable *Y* with *X* in all above stated equations in this section.

2.4 Even Order Ranked Set Sampling

Noor-ul-Amin et al. (2019) developed a new selection procedure in *RSS* named even order ranked set sampling (*EORSS*) to enhance the efficiency of estimator of population mean. For even number of sample observations, mean estimator with its variance for study variable in *EORSS* are expressed as

$$\bar{y}_{(EORSS)e} = \frac{1}{t}\left[\sum_{i=1}^{k} y_{i[2i]} + \sum_{i=1}^{k} y_{k+i[2i]}\right] \tag{2.16}$$

$$Var\left(\bar{y}_{(EORSS)e}\right) = \frac{2}{t}\left[\sum_{i=1}^{k} \sigma^2_{y_{[2i]}}\right] \tag{2.17}$$

and

$$COV\left(\bar{x}_{(EORSS)e}, \bar{y}_{[EORSS]e}\right) = \beta_{Ee} Var\left(\bar{x}_{(EORSS)e}\right) \tag{2.18}$$

where $\beta_{Ee} = \rho\sqrt{\dfrac{Var\left(\bar{y}_{[EORSS]e}\right)}{Var\left(\bar{x}_{(EORSS)e}\right)}}$

The following expressions are derived for mean estimator with its variance for study variable in *EORSS* when sample observations are odd.

$$\bar{y}_{(EORSS)o} = \frac{1}{t}\left[\sum_{i=1}^{m} y_{i(2i)} + \sum_{i=1}^{m} y_{m+i(2i)} + y_{t((t+1)/2)}\right] \tag{2.19}$$

$$Var\left(\bar{y}_{(EORSS)o}\right) = \frac{2}{t}\left[\sum_{i=1}^{m} \sigma^2_{y[2i]}\right] + \frac{1}{t^2}\left[\sigma^2_{y[(t+1)/2]}\right] \tag{2.20}$$

$$COV\left(\bar{x}_{(EORSS)o}, \bar{y}_{[EORSS]o}\right) = \beta_{Eo} Var\left(\bar{x}_{(EORSS)o}\right) \tag{2.21}$$

where $\beta_{Eo} = \rho\sqrt{\frac{Var\left(\bar{y}_{[EORSS]o}\right)}{Var\left(\bar{x}_{(EORSS)o}\right)}}$

For more details, see Noor-ul-Amin et al. (2019). The mean and variance for auxiliary variable in *EORSS* can be expressed in similar manner by replacing variable *Y* with *X* in above cited equations.

3. REGRESSION-IN-RATIO ESTIMATORS IN SIMPLE RANDOM SAMPLING

Kadilar and Cingi (2004) proposed a class of regression-in-ratio estimator for population mean in *SRS* design. They concluded that this class of estimators had greater efficiency than classical ratio type estimators. Their proposed class of estimator is described as follows:

$$\bar{y}_{KCl} = \frac{\bar{y} + b_1\left(\mu_x - \bar{x}\right)}{\left(\lambda_l \bar{x} + \theta_l\right)}\left(\lambda_l \mu_x + \theta_l\right),\ l = 1,2,3,4,5 \tag{3.1}$$

where b_1 is the *OLS* estimator of regression coefficient i.e. $b_1 = s_{yx} / s_x^2$ where s_x^2 is sample variance of x and s_{yx} is sample covariance between x and y and μ_x is the population mean of auxiliary variable. We also have

$$\begin{aligned} &\lambda_1 = 1\ \&\ \theta_1 = 0,\ \lambda_2 = 1\ \&\ \theta_2 = C_x,\ \lambda_3 = 1\ \&\ \theta_3 = B_2\left(x\right), \\ &\lambda_4 = B_2\left(x\right)\ \&\ \beta_4 = C_x \text{ and } \lambda_5 = C_x\ \&\ \theta_5 = B_2\left(x\right) \end{aligned} \tag{3.2}$$

where $\beta_2\left(x\right)$ and C_x are coefficient of kurtosis and coefficient of variation of auxiliary variable respectively and $\bar{y}$ is the sample means of study variable and $\bar{x}$ is the sample mean of auxiliary variable. The *MSE* for the class of estimators given in (3.1) is described as

$$MSE\left(\bar{y}_{KCl}\right) \cong \frac{1-f}{n}\left[R^2_{KCl}\sigma_x^2 + \sigma_y^2\left(1-\rho^2\right)\right] \tag{3.3}$$

where $f=\dfrac{n}{N}$; n, N, ρ, σ_x^2 and σ_y^2 are sample size, the population size, population correlation coefficient between X and Y, population variance for X and population variance for Y respectively. For further details, see Kadilar and Cingi (2004). We have also

$$R_{KC1}=R=\frac{\mu_y}{\mu_x}, \quad R_{KC2}=\frac{\mu_y}{\mu_x+C_x}, \; R_{KC3}=\frac{\mu_y}{\mu_x+B_2(x)}$$

$$R_{KC4}=\frac{\mu_y B_2(x)}{\mu_y B_2(x)+C_x} \text{ and } R_{KC5}=\frac{\bar{Y}\,C_x}{\bar{X}\,C_x+B_2(x)} \tag{3.4}$$

where μ_y is the population mean of study variable.

4. REGRESSION-IN-RATIO ESTIMATORS IN RANKED SET SAMPLING

Al-Hadhrami (2009) proposed a new class of ratio-in-regression estimators in *RSS* by adopting the technique given by Kadilar and Cingi (2004) to estimate the population mean with greater efficiency. He suggested the following class of regression-in-ratio estimator for population mean.

$$\bar{y}_{AHl}=\frac{\bar{y}_{RSS}+b_1\left(\mu_x-\bar{x}_{RSS}\right)}{\left(\lambda_l\bar{x}_{RSS}+\theta_l\right)}\left(\lambda_l\mu_x+\theta_l\right), \; l=1,2,3,4,5, \tag{4.1}$$

where $\bar{y}_{RSS}$, $\bar{x}_{RSS}$, $Var(\bar{x}_{RSS})$ and $COV(\bar{x}_{RSS},\bar{y}_{RSS})$ are described in section *2.1* and b_1 is the *OLS* estimator of regression coefficient between x_{RSS} and y_{RSS}, i.e. $b_1=\text{cov}(\bar{x}_{RSS},\bar{y}_{RSS})/\text{var}(\bar{x}_{RSS})$.

To calculate the *MSE* for the class of estimator given in (4.1), expanding $\bar{y}_{AHi}$ up to order one for $\bar{y}_{RSS}$ and $\bar{x}_{RSS}$ using Taylor series, then

$$\bar{y}_{AHl}\cong\mu_y+(\bar{y}_{RSS}-\mu_y)-G_l\left(\bar{x}_{RSS}-\mu_x\right) \tag{4.2}$$

The variance of $\bar{y}_{AHl}$ is expressed as

$$Var\left(\bar{y}_{AHl}\right)\cong Var(\bar{y}_{RSS})+G^2Var\left(\bar{x}_{RSS}\right)-2G.COV(\bar{x}_{RSS},\bar{y}_{RSS}) \tag{4.3}$$

as bias is zero in this expression, so $MSE\left(\bar{y}_{AHl}\right)\cong Var\left(\bar{y}_{AHl}\right)$. By putting the values of variances and covariance from section *2.1*, we obtain the following *MSE* expression of $\bar{y}_{AHl}$ in *RSS* design.

$$\begin{aligned}MSE\left(\bar{y}_{AHl}\right)\cong&\left\{\frac{\sigma_y^2}{t}-\frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]}-\mu_y\right)^2\right\}+G_l^2\left\{\frac{\sigma_x^2}{t}-\frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{x(i)}-\mu_x\right)^2\right\}\\&-2G_l\left\{\frac{\sigma_{xy}}{t}-\frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]}-\mu_y\right)\left(\mu_{x(i)}-\mu_x\right)\right\}\end{aligned} \tag{4.4}$$

where $G_l = \beta_1 + \lambda_l \mu_x / \left(\lambda_l \mu_x + \theta_l\right)$ and its values depend on λ_l and θ_l which are given in (3.2).

For more details, consult Al-Hadhrami (2009).

5. PROPOSED REGRESSION-IN-RATIO ESTIMATORS IN RANKED SET SAMPLING SCHEMES

The *RSS* design is used to obtain efficient estimators of population mean but performance of *RSS* estimators can be worse due to presence of outliers. In this situation, special types of robust estimators called M-estimators (discussed in section *1*) can be used as an alternative of *OLS* estimators in *RSS* design. We have proposed an efficient class of robust regression-in-ratio estimators for population mean using objective functions given in (1.1) and (1.2) in *RSS* schemes described earlier.

5.1 Proposed Class of Regression-in-Ratio Estimators in Ranked Set Sampling

The performance of regression-in-ratio estimators for population mean in *RSS* proposed by Al-Hadhrami (2009) can be affected due to presence of outliers. An efficient class of regression-in-ratio estimators for population mean to cope the effects of outliers using robust regression is proposed in *RSS* schemes which is give as

$$\bar{y}_{RSSjl} = \frac{\bar{y}_{RSS} + b_j\left(\mu_x - \bar{x}_{RSS}\right)}{\left(\lambda_l \bar{x}_{RSS} + \theta_l\right)}\left(\lambda_l \mu_x + \theta_l\right), l = 1,2,3,4,5, \tag{5.1}$$

where $j = 1,2,3$. Al-Hadhrami (2009) proposed estimators are special case of our proposed class of regression-in-ratio estimators for $j = 1$. For $j = 2$ estimate of b_2 is obtained by minimizing $\sum \rho_2\left(y_{RSS} - a_0 - bx_{RSS}\right)$ using objective function given in (1.1) and for $j = 3,$ estimate of b_3 is obtained by minimizing $\sum \rho_3\left(y_{RSS} - a_0 - bx_{RSS}\right)$ using objective function given in (1.2) and remaining terms are same as described in section *4*.

The following expression of *MSE* of proposed class of estimators is obtained by adopting the methodology used by Al-Hadhrami (2009)

$$\begin{aligned} MSE\left(\bar{y}_{RSSjl}\right) \cong & \left\{\frac{\sigma_y^2}{t} - \frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]} - \mu_y\right)^2\right\} + G_l^2\left\{\frac{\sigma_x^2}{t} - \frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{x(i)} - \mu_x\right)^2\right\} \\ & - 2G_l\left\{\frac{\sigma_{xy}}{t} - \frac{1}{t^2}\sum_{i=1}^{t}\left(\mu_{y[i]} - \mu_y\right)\left(\mu_{x(i)} - \mu_x\right)\right\} \end{aligned} \tag{5.2}$$

where $G_l = \beta_j + \lambda_l \bar{X} / \left(\lambda_l \bar{X} + \theta_l\right), j = 1,2,3$.

5.2 Proposed Class of Regression-in-Ratio Estimators in Median Ranked Set Sampling

The *MRSS* procedure produced more efficient estimators of population mean but their performance can also be affected due to atypical observations in the data. Moreover robust regression-in-ratio estimators using methodology given by Raza et al. (2019) are not constructed in *MRSS* scheme. So an efficient class of regression-in-ratio estimator using robust regression is developed in *MRSS* design to estimate population mean. The proposed class of ratio estimator is described as follows:

$$\bar{y}_{MRSSjl} = \frac{\bar{y}_{MRSS} + b_j\left(\mu_x - \bar{x}_{MRSS}\right)}{\left(\lambda_l \bar{x}_{MRSS} + \theta_l\right)}\left(\lambda_l \mu_x + \theta_l\right), l = 1,2,3,4,5\,, \tag{5.3}$$

where $\bar{y}_{MRSS}$, $\bar{x}_{MRSS}$ are described in section *2.2*. Estimate of b_j for $j = 1,2,3$ are similar as defined in session 5.1. Following expression is obtain by expanding $\bar{y}_{MRSSjl}$ up to order one for $\bar{y}_{MRSS}$ and $\bar{x}_{MRSS}$ using Taylor series

$$\bar{y}_{MRSSjl} \cong \mu_y + (\bar{y}_{MRSS} - \mu_y) - G_l\left(\bar{x}_{MRSS} - \mu_x\right) \tag{5.4}$$

We can write (5.4) in following form

$$\bar{y}_{MRSSjl} - \mu_y \cong (\bar{y}_{MRSS} - \mu_y) - G_l\left(\bar{x}_{MRSS} - \mu_x\right) \tag{5.5}$$

Taking square and applying expectation on both side of (5.5)

$$E\left(\bar{y}_{MRSSjl} - \mu_y\right)^2 \cong E(\bar{y}_{MRSS} - \mu_y)^2 - G_l^2 E\left(\bar{x}_{MRSS} - \mu_x\right)^2 - 2G_l E\left(\bar{y}_{MRSS} - \mu_y\right)\left(\bar{x}_{MRSS} - \mu_x\right)$$

$$MSE\left(\bar{y}_{MRSSjl}\right) \cong Var(\bar{y}_{MRSS}) + G^2 Var\left(\bar{x}_{MRSS}\right) - 2G.\text{cov}(\bar{x}_{MRSS}, \bar{y}_{MRSS}) \tag{5.6}$$

Putting the values of variances and covariance of $\bar{y}_{MRSS}$ and $\bar{x}_{MRSS}$, from (2.5) and (2.6), we obtain the following *MSE* of $\bar{y}_{MRSSjl}$ for even sample size *t* with single cycle.

$$MSE\left(\bar{y}_{MRSSjl}\right)_e \cong \left\{\frac{1}{2t}\left[\sigma^2_{y\left[\frac{t}{2}\right]} + \sigma^2_{y\left[\frac{t+2}{2}\right]}\right]\right\} + G_l\left(G_l - 2\beta_{Me}\right)\left\{\frac{1}{2t}\left[\sigma^2_{x\left(\frac{t}{2}\right)} + \sigma^2_{x\left(\frac{t+2}{2}\right)}\right]\right\} \tag{5.7}$$

For odd sample size *t* with single cycle, $MSE\left(\bar{y}_{MRSSjl}\right)$ is calculated using (2.8) and (2.9), and expressed as follows

$$MSE\left(\bar{y}_{MRSSjl}\right)_o \cong \left\{\frac{1}{t}\left[\sigma^2_{y\left[\frac{t+1}{2}\right]}\right]\right\} + G_l\left(G_l - 2\beta_{Mo}\right)\left\{\frac{1}{t}\left[\sigma^2_{x\left(\frac{t+1}{2}\right)}\right]\right\} \tag{5.8}$$

values of β_{Me} and β_{Mo} are described in section *2.2*.

5.3 Proposed Class of Regression-in-Ratio Estimators in Quartile Ranked Set Sampling

Efficiency of regression-in-ratio estimators for population mean in *QRSS* can be affected due to extreme values in the data, so an efficient class of regression-in-ratio estimator is suggested for estimation of population mean in *QRSS* scheme to enhance the performance of estimators in the presence of outliers. The proposed class of regression-in-ratio estimators is defined as

$$\bar{y}_{QRSSjl} = \frac{\bar{y}_{QRSS} + b_j\left(\mu_x - \bar{x}_{QRSS}\right)}{\left(\lambda_l \bar{x}_{QRSS} + \theta_l\right)}\left(\lambda_l \mu_x + \theta_l\right), l = 1,2,3,4,5 \; \& \; j = 1,2,3 , \qquad (5.9)$$

where $\bar{y}_{QRSS}$, $\bar{x}_{QRSS}$ are means of study variable and auxiliary variable in QRSS design respectively and described in section *2.3*. The *MSE* of $\bar{y}_{QRSSjl}$ is derived as follows

$$MSE\left(\bar{y}_{QRSSjl}\right) \cong Var(\bar{y}_{QRSS}) + G^2 Var\left(\bar{x}_{QRSS}\right) - 2G.\mathrm{cov}(\bar{x}_{QRSS}, \bar{y}_{QRSS}) \qquad (5.10)$$

The values of $Var(\bar{y}_{QRSS})$, $Var\left(\bar{x}_{QRSS}\right)$ and $COV(\bar{x}_{QRSS}, \bar{y}_{QRSS})$ are incorporated in (5.10) from (2.11) and (2.12) to obtain the MSE expression of $\bar{y}_{QRSSjl}$ for even sample size t with single cycle.

$$MSE\left(\bar{y}_{QRSSjl}\right)_e \cong \left\{\frac{1}{2t}\left[\sigma^2_{y\left[\frac{t+1}{4}\right]} + \sigma^2_{y\left[\frac{3(t+1)}{4}\right]}\right]\right\} + G_l\left(G_l - 2\beta_{Qe}\right)\left\{\frac{1}{2t}\left[\sigma^2_{x\left(\frac{t+1}{4}\right)} + \sigma^2_{x\left(\frac{3(t+1)}{4}\right)}\right]\right\} \qquad (5.11)$$

For odd sample size t, $MSE\left(\bar{y}_{QRSSjl}\right)$ is calculated using (2.14), (2.15) and give as

$$MSE\left(\bar{y}_{QRSSjl}\right)_o \cong \left\{\frac{t-1}{2t^2}\left[\sigma^2_{y\left[\frac{t+1}{4}\right]} + \sigma^2_{y\left[\frac{3(t+1)}{4}\right]}\right] + \frac{1}{t^2}\left[+\sigma^2_{y\left[\frac{t+1}{2}\right]}\right]\right\} + G_l\left(G_l - 2\beta_{Qe}\right)\left\{\frac{t-1}{2t^2}\left[\sigma^2_{x\left(\frac{t+1}{4}\right)} + \sigma^2_{x\left(\frac{3(t+1)}{4}\right)}\right] + \frac{1}{t^2}\left[+\sigma^2_{x\left(\frac{t+1}{2}\right)}\right]\right\} \qquad (5.12)$$

values of β_{Qe} and β_{Qo} are described in section *2.3*.

5.4 Proposed Class of Regression-in-Ratio Estimators in Even Order Ranked Set Sampling

The following class of regression-in-ratio estimators is proposed in *EORSS* to estimate the population mean using robust regression to minimize the effects the outliers.

$$\bar{y}_{EORSSjl} = \frac{\bar{y}_{EORSS} + b_j\left(\mu_x - \bar{x}_{EORSS}\right)}{\left(\lambda_l \bar{x}_{EORSS} + \theta_l\right)}\left(\lambda_l \mu_x + \theta_l\right), l = 1,2,3,4,5 \tag{5.13}$$

where $\bar{y}_{EORSS}$, $\bar{x}_{EORSS}$ are the means of study variable and auxiliary variable in *EORSS* scheme respectively and their values are discussed in section *2.4*. Here b_1 is the regression coefficient for regression line $\bar{y}_{EORSS}$ on $\bar{x}_{EORSS}$. Estimates of b_j , for $j = 2,3$ are obtain by minimizing $\sum \rho_j\left(y_{EORSS} - a_0 - bx_{EORSS}\right)$ using (1.1) and (1.2).

The *MSE* of proposed estimator for even sample size *t* with single cycle is derived using (2.17) and (2.18) and expressed as

$$MSE\left(\bar{y}_{EORSSjl}\right)_e \cong \left\{\frac{2}{t}\left[\sum_{i=1}^{k} \sigma^2_{y_{[2i]}}\right]\right\} + G_l\ \left(G_l - 2\beta_{Ee}\right)\left\{\frac{2}{t}\left[\sum_{i=1}^{k} \sigma^2_{y_{(2i)}}\right]\right\} \tag{5.14}$$

For odd sample size t, the $MSE\left(\bar{y}_{EORSSjl}\right)$ using (2.20) and (2.21) is give as:

$$MSE\left(\bar{y}_{EORSSjl}\right)_o \cong \left\{\frac{2}{t}\left[\sum_{i=1}^{m} \sigma^2_{y[2i]}\right] + \frac{1}{t^2}\left[\sigma^2_{y\left[(t+1)/2\right]}\right]\right\}$$

$$+G_l\ \left(G_l - 2\beta_{Eo}\right)\left\{\frac{2}{t}\left[\sum_{i=1}^{m} \sigma^2_{x(2i)}\right] + \frac{1}{t^2}\left[\sigma^2_{x\left((t+1)/2\right)}\right]\right\} \tag{5.15}$$

values of β_{Ee} and β_{Eo} are described in section *2.4*.

6. COMPARATIVE ANALYSIS

In this section, a comparative analysis based on *MSEs* of proposed class of regression-in-ratio estimators for population mean given in section *5*, with previously existing estimators of population mean in considered sampling schemes is presented. Superiority of proposed class of regression-in-ratio estimator is supported by using a real life example and extensive simulation study.

6.1 Real life data Example

Data used in example is collected by Kadilar in 1999 for 104 villages in the region DOĞU ANADOLU, Turkey. The level of apple production in tons is taken as study variable (*Y*) and numbers of apple tress (*X*, 1 unit =10) is taken as auxiliary variable. The scatter diagram of this data is shown in Figure 1. Statistics regarding example 6.1 are presented in Table 1.

Table 1
Statistics for Data Used in Example 1

$N=104$	$\sigma_{yx}=2377821$	$C_x=1.6123$
$t=9$	$\mu_x=1427.495$	$R_{KC1}=0.43808$
$\rho=0.8853$	$\mu_y=625.3654$	$R_{KC2}=0.43220$
$B_1=0.4489$	$\sigma_x=2301.615$	$R_{KC3}=0.43759$
$B_2=0.3332$	$\sigma_y=1167.008$	$R_{KC4}=0.43442$
$B_3=0.3006$	$B_2(x)=19.4377$	$R_{KC5}=0.43806$

A sample of size 9 is taken from the population to calculate estimates using *SRS* design and *RSS* schemes to obtain the estimates. The *MSE* of regression-in-ratio estimators are calculated using (3.3), (4.4), (5.2), (5.8), (5.12) and (5.15) and their relative efficiencies (*R.E*) in *SRS* and *RSS* designs are calculated using (6.1) and (6.2) and presented in Table 2 and Table 3.

$$R.E\left(\bar{y}_{Pujl}\right)=\frac{MSE\left(\bar{y}_{KCl}\right)}{MSE\left(\bar{y}_{Pujl}\right)}, l=1,2,..5,\ j=1,2,3, u=1,2,3,4 \tag{6.1}$$

where $\bar{y}_{P1jl}=\bar{y}_{RSSjl}$, $\bar{y}_{P2jl}=\bar{y}_{MRSSjl}$, $\bar{y}_{P3jl}=\bar{y}_{QRSSjl}$ and $\bar{y}_{P4jl}=\bar{y}_{EOSSjl}$

$$R.E\left(\bar{y}_{Pu1l}\right)=\frac{MSE\left(\bar{y}_{Pu1l}\right)}{MSE\left(\bar{y}_{Pujl}\right)}, l=1,2,..5,\ j=2,3, u=1,2,3,4 \tag{6.2}$$

$\bar{y}_{KCl}=$ Represents the estimators using Kadilar and Cingi (2004).

$\bar{y}_{Pujl}=$ Represents robust estimators of population mean in *RSS* Schemes.

Table 2 describes that performance of ratio-in-regression estimators gain significant adeptness due to change of sampling design as well as due to change the methodology to estimate the regression coefficient. There is significant improvement in the efficiencies of estimators in RSS schemes relative to *OLS* based estimators in *SRS* design.

Table 2
***R.E* of Proposed Estimators with Estimators Proposed by Kadilar and Cingi (2004) for $t = 9$ and $r = 1$**

R.E	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	*R.E*	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$
$\bar{y}_{RSS11}$	3.11	3.09	2.86	3.11	2.95	$\bar{y}_{MRSS11}$	2.98	2.95	2.72	2.98	2.81
$\bar{y}_{RSS12}$	3.12	3.09	2.87	3.11	2.96	$\bar{y}_{MRSS12}$	3.00	2.97	2.74	3.00	2.83
$\bar{y}_{RSS13}$	3.18	3.16	2.93	3.18	3.02	$\bar{y}_{MRSS13}$	3.21	3.18	2.93	3.2	3.03
$\bar{y}_{RSS14}$	3.11	3.09	2.86	3.11	2.95	$\bar{y}_{MRSS14}$	2.98	2.96	2.72	2.98	2.81
$\bar{y}_{RSS15}$	3.16	3.13	2.90	3.16	2.99	$\bar{y}_{MRSS15}$	3.12	3.09	2.85	3.12	2.94
$\bar{y}_{RSS21}$	4.35	4.31	4.00	4.34	4.12	$\bar{y}_{MRSS21}$	4.78	4.74	4.37	4.77	4.51
$\bar{y}_{RSS22}$	4.35	4.32	4.01	4.35	4.13	$\bar{y}_{MRSS22}$	4.81	4.77	4.39	4.80	4.54
$\bar{y}_{RSS23}$	4.45	4.41	4.09	4.44	4.22	$\bar{y}_{MRSS23}$	5.14	5.1	4.7	5.14	4.85
$\bar{y}_{RSS24}$	4.35	4.31	4.00	4.34	4.12	$\bar{y}_{MRSS24}$	4.78	4.74	4.37	4.78	4.51
$\bar{y}_{RSS25}$	4.41	4.38	4.06	4.41	4.18	$\bar{y}_{MRSS25}$	5.00	4.96	4.57	5.00	4.72
$\bar{y}_{RSS21}$	4.77	4.73	4.39	4.77	4.52	$\bar{y}_{MRSS31}$	5.54	5.49	5.06	5.54	5.23
$\bar{y}_{RSS32}$	4.78	4.74	4.40	4.78	4.53	$\bar{y}_{MRSS32}$	5.57	5.53	5.09	5.57	5.26
$\bar{y}_{RSS33}$	4.88	4.84	4.49	4.87	4.63	$\bar{y}_{MRSS33}$	5.96	5.91	5.45	5.96	5.63
$\bar{y}_{RSS34}$	4.77	4.73	4.39	4.77	4.52	$\bar{y}_{MRSS34}$	5.54	5.50	5.06	5.54	5.23
$\bar{y}_{RSS35}$	4.84	4.8	4.45	4.83	4.59	$\bar{y}_{MRSS35}$	5.80	5.75	5.30	5.8	5.47
$\bar{y}_{QRSS11}$	3.54	3.51	3.26	3.54	3.36	$\bar{y}_{EORSS11}$	3.80	3.77	3.50	3.80	3.61
$\bar{y}_{QRSS12}$	3.55	3.52	3.27	3.55	3.37	$\bar{y}_{EORSS12}$	3.81	3.79	3.51	3.81	3.62
$\bar{y}_{QRSS13}$	3.69	3.66	3.40	3.69	3.5	$\bar{y}_{EORSS13}$	3.93	3.90	3.62	3.93	3.73
$\bar{y}_{QRSS14}$	3.54	3.51	3.26	3.54	3.36	$\bar{y}_{EORSS14}$	3.80	3.78	3.50	3.80	3.61
$\bar{y}_{QRSS15}$	3.63	3.61	3.35	3.63	3.45	$\bar{y}_{EORSS15}$	3.88	3.85	3.57	3.88	3.68
$\bar{y}_{QRSS21}$	4.81	4.77	4.43	4.81	4.57	$\bar{y}_{EORSS21}$	5.11	5.07	4.70	5.11	4.85
$\bar{y}_{QRSS22}$	4.82	4.79	4.45	4.82	4.58	$\bar{y}_{EORSS22}$	5.12	5.09	4.72	5.12	4.86
$\bar{y}_{QRSS23}$	4.99	4.96	4.60	4.99	4.74	$\bar{y}_{EORSS23}$	5.27	5.23	4.85	5.26	5.00
$\bar{y}_{QRSS24}$	4.81	4.78	4.43	4.81	4.57	$\bar{y}_{EORSS24}$	5.11	5.07	4.70	5.11	4.85
$\bar{y}_{QRSS25}$	4.92	4.89	4.54	4.92	4.68	$\bar{y}_{EORSS25}$	5.21	5.17	4.79	5.21	4.94
$\bar{y}_{QRSS31}$	5.23	5.19	4.82	5.22	4.96	$\bar{y}_{EORSS31}$	5.53	5.49	5.09	5.53	5.25
$\bar{y}_{QRSS32}$	5.24	5.21	4.83	5.24	4.98	$\bar{y}_{EORSS32}$	5.55	5.51	5.10	5.54	5.26
$\bar{y}_{QRSS33}$	5.42	5.38	4.99	5.41	5.14	$\bar{y}_{EORSS33}$	5.69	5.65	5.24	5.69	5.40
$\bar{y}_{QRSS34}$	5.23	5.19	4.82	5.23	4.96	$\bar{y}_{EORSS34}$	5.53	5.49	5.09	5.53	5.25
$\bar{y}_{QRSS35}$	5.35	5.31	4.93	5.34	5.08	$\bar{y}_{EORSS35}$	5.63	5.59	5.18	5.63	5.34

For example, relative efficiency of $\bar{y}_{QRSS11}$ with respect to $\bar{y}_{KC1}$ is 3.54, it means that *OLS* estimator in *QRSS* scheme is 354% better than *OLS* estimator in *SRS* design for $l=1$. Similarly estimator $\bar{y}_{QRSS21}$ is 481% efficient than $\bar{y}_{KC1}$ and $\bar{y}_{QRSS31}$ is 523% efficient than $\bar{y}_{KC1}$. From above table it is also depicted that *R.E* for third class of ratio-in-regression estimator i.e. $j=3$ is the highest for all *RSS* schemes. For instance, $\bar{y}_{RSS3l}$ produced the most efficient estimates than $\bar{y}_{RSS2l}$ and $\bar{y}_{RSS1l}$ for all values of *l*, it means redescending M-estimator given in (1.2) has greater tendency to tackle the effects of outliers than the consider estimators.

Table 3
***R.E* of Proposed Estimators using *OLS* Algorithm with Proposed Estimators based on Robust Regression in *RSS* Schemes for $t=9$ and $r=1$**

R.E	$\bar{y}_{RSS11}$	$\bar{y}_{RSS12}$	$\bar{y}_{RSS13}$	$\bar{y}_{RSS14}$	$\bar{y}_{RSS15}$	R.E	$\bar{y}_{MRSS11}$	$\bar{y}_{MRSS12}$	$\bar{y}_{MRSS13}$	$\bar{y}_{MRSS14}$	$\bar{y}_{MRSS15}$
$\bar{y}_{RSS21}$	1.40	1.39	1.36	1.40	1.38	$\bar{y}_{MRSS21}$	1.60	1.59	1.49	1.60	1.53
$\bar{y}_{RSS22}$	1.40	1.40	1.37	1.40	1.38	$\bar{y}_{MRSS22}$	1.61	1.60	1.5	1.61	1.54
$\bar{y}_{RSS23}$	1.43	1.43	1.40	1.43	1.41	$\bar{y}_{MRSS23}$	1.73	1.72	1.6	1.73	1.65
$\bar{y}_{RSS24}$	1.40	1.39	1.36	1.40	1.38	$\bar{y}_{MRSS24}$	1.60	1.59	1.49	1.60	1.53
$\bar{y}_{RSS25}$	1.42	1.41	1.38	1.42	1.40	$\bar{y}_{MRSS25}$	1.68	1.67	1.56	1.68	1.60
$\bar{y}_{RSS31}$	1.53	1.53	1.50	1.53	1.51	$\bar{y}_{MRSS31}$	1.86	1.85	1.73	1.86	1.78
$\bar{y}_{RSS32}$	1.54	1.53	1.50	1.54	1.51	$\bar{y}_{MRSS32}$	1.87	1.86	1.74	1.87	1.79
$\bar{y}_{RSS33}$	1.57	1.57	1.53	1.57	1.55	$\bar{y}_{MRSS33}$	2.00	1.99	1.86	2.00	1.91
$\bar{y}_{RSS34}$	1.53	1.53	1.50	1.53	1.51	$\bar{y}_{MRSS34}$	1.86	1.85	1.73	1.86	1.78
$\bar{y}_{RSS35}$	1.56	1.55	1.52	1.55	1.53	$\bar{y}_{MRSS35}$	1.95	1.93	1.81	1.95	1.86
R.E.	$\bar{y}_{QRSS11}$	$\bar{y}_{QRSS12}$	$\bar{y}_{QRSS13}$	$\bar{y}_{QRSS14}$	$\bar{y}_{QRSS15}$	R.E.	$\bar{y}_{EORSS11}$	$\bar{y}_{EORSS12}$	$\bar{y}_{EORSS13}$	$\bar{y}_{EORSS14}$	$\bar{y}_{EORSS15}$
$\bar{y}_{QRSS21}$	1.36	1.35	1.30	1.36	1.32	$\bar{y}_{EORSS21}$	1.34	1.34	1.3	1.34	1.32
$\bar{y}_{QRSS22}$	1.36	1.36	1.31	1.36	1.33	$\bar{y}_{EORSS22}$	1.35	1.34	1.3	1.35	1.32
$\bar{y}_{QRSS23}$	1.41	1.41	1.35	1.41	1.37	$\bar{y}_{EORSS23}$	1.38	1.38	1.34	1.38	1.36
$\bar{y}_{QRSS24}$	1.36	1.35	1.3	1.36	1.32	$\bar{y}_{EORSS24}$	1.34	1.34	1.3	1.34	1.32
$\bar{y}_{QRSS25}$	1.39	1.39	1.33	1.39	1.36	$\bar{y}_{EORSS25}$	1.37	1.37	1.33	1.37	1.34
$\bar{y}_{QRSS31}$	1.48	1.47	1.42	1.48	1.44	$\bar{y}_{EORSS31}$	1.45	1.45	1.41	1.45	1.43
$\bar{y}_{QRSS32}$	1.48	1.48	1.42	1.48	1.44	$\bar{y}_{EORSS32}$	1.46	1.45	1.41	1.46	1.43
$\bar{y}_{QRSS33}$	1.53	1.53	1.47	1.53	1.49	$\bar{y}_{EORSS33}$	1.50	1.49	1.45	1.50	1.47
$\bar{y}_{QRSS34}$	1.48	1.47	1.42	1.48	1.44	$\bar{y}_{EORSS34}$	1.46	1.45	1.41	1.45	1.43
$\bar{y}_{QRSS35}$	1.51	1.51	1.45	1.51	1.47	$\bar{y}_{EORSS35}$	1.48	1.48	1.43	1.48	1.45

Table 3 provides the comparative analysis of ratio-in-regression estimators using *OLS* and robust regression techniques within *RSS* Schemes. A close look on the Table 6, it is described that robust ratio-in-regression estimators using objective function in (1.2) have higher *R.E* as compare to ratio-in-regression estimators using objective function given in (1.1). For example, $\bar{y}_{MRSS31}$ is 186 % efficient and $\bar{y}_{MRSS21}$ is 160% efficient than $\bar{y}_{MRSS11}$. This table also depicts that estimator based on Raza et al. (2019) produces more efficient results than estimators based on objective function given by Huber (1964). For instance, estimator $\bar{y}_{MRSS31}$ has 16% higher relative efficiency than $\bar{y}_{MRSS21}$.

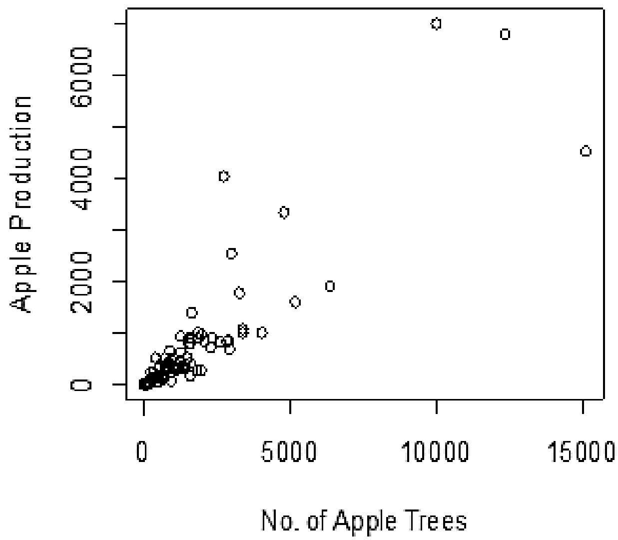

Figure 1: Scatter Diagram of Data used in Real Life Data Example

6.2 Simulation Studies

To validate the performance of proposed class of robust regression-in-ratio estimators of population mean, an extensive simulation study is carried out. For this purpose, R-program is used to take a sample of size $t = 9$ from the population given in session 6.1, using sampling techniques described in section *2*. Estimates of B_j 's are obtained from this sample using prescribe techniques and this procedure is repeated 50000 time to obtain the *MSE* of the ratio-in-regression estimators discussed in section *4* and in section *5* using (6.3). The R.E of proposed estimators to the *OLS* estimators in *SRS* and *RSS* schemes are obtained using (6.1) and (6.2) and presented in Table 4 and Table 5.

$$MSE(\bar{y}_{Ph}) = \frac{1}{50000}\sum_{i=1}^{50000}\left(\bar{y}_{hi} - \mu_y\right)^2 \qquad h = 1,2 \qquad (6.3)$$

where $\bar{y}_1 = \bar{y}_{KCl}, \bar{y}_2 = \bar{y}_{RPjl}, j = 1,2,3 \,\&\, l = 1,2,3,4,5$, and μ_y is population mean of study variable.

Table 4
***R.E* of Proposed Estimators with Estimators Proposed by Kadilar and Cingi (2004) for $t = 9$ and $r = 1$**

R.E	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	R.E	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$
$\bar{y}_{RSS11}$	4.45	4.39	3.87	4.45	4.06	$\bar{y}_{MRSS11}$	1.75	1.74	1.64	1.75	1.68
$\bar{y}_{RSS12}$	4.46	4.40	3.87	4.45	4.07	$\bar{y}_{MRSS12}$	1.76	1.75	1.65	1.76	1.69
$\bar{y}_{RSS13}$	4.55	4.49	3.96	4.55	4.15	$\bar{y}_{MRSS13}$	1.85	1.84	1.74	1.85	1.78
$\bar{y}_{RSS14}$	4.45	4.39	3.87	4.45	4.06	$\bar{y}_{MRSS14}$	1.75	1.74	1.64	1.75	1.68
$\bar{y}_{RSS15}$	4.51	4.45	3.92	4.51	4.12	$\bar{y}_{MRSS15}$	1.81	1.80	1.70	1.81	1.74
$\bar{y}_{RSS21}$	5.25	5.18	4.56	5.25	4.79	$\bar{y}_{MRSS21}$	2.05	2.04	1.92	2.05	1.97
$\bar{y}_{RSS22}$	5.26	5.19	4.57	5.26	4.8	$\bar{y}_{MRSS22}$	2.06	2.05	1.93	2.06	1.98
$\bar{y}_{RSS23}$	5.37	5.3	4.67	5.37	4.9	$\bar{y}_{MRSS23}$	2.17	2.16	2.04	2.17	2.09
$\bar{y}_{RSS24}$	5.25	5.18	4.56	5.25	4.79	$\bar{y}_{MRSS24}$	2.06	2.04	1.93	2.05	1.97
$\bar{y}_{RSS25}$	5.33	5.26	4.63	5.32	4.86	$\bar{y}_{MRSS25}$	2.13	2.12	1.99	2.13	2.04
$\bar{y}_{RSS21}$	5.77	5.70	5.02	5.77	5.27	$\bar{y}_{MRSS31}$	2.45	2.43	2.29	2.45	2.35
$\bar{y}_{RSS32}$	5.78	5.71	5.03	5.78	5.28	$\bar{y}_{MRSS32}$	2.46	2.45	2.30	2.46	2.36
$\bar{y}_{RSS33}$	5.91	5.83	5.13	5.9	5.39	$\bar{y}_{MRSS33}$	2.59	2.58	2.43	2.59	2.49
$\bar{y}_{RSS34}$	5.77	5.70	5.02	5.77	5.27	$\bar{y}_{MRSS34}$	2.45	2.43	2.29	2.45	2.35
$\bar{y}_{RSS35}$	5.86	5.78	5.09	5.85	5.34	$\bar{y}_{MRSS35}$	2.54	2.52	2.38	2.54	2.44
$\bar{y}_{QRSS11}$	2.86	2.84	2.64	2.86	2.72	$\bar{y}_{EORSS11}$	3.54	3.52	3.31	3.54	3.39
$\bar{y}_{QRSS12}$	2.87	2.85	2.65	2.87	2.73	$\bar{y}_{EORSS12}$	3.55	3.53	3.32	3.55	3.40
$\bar{y}_{QRSS13}$	2.96	2.94	2.73	2.96	2.82	$\bar{y}_{EORSS13}$	3.65	3.63	3.42	3.65	3.5
$\bar{y}_{QRSS14}$	2.86	2.84	2.64	2.86	2.72	$\bar{y}_{EORSS14}$	3.54	3.52	3.31	3.54	3.39
$\bar{y}_{QRSS15}$	2.93	2.90	2.70	2.92	2.78	$\bar{y}_{EORSS15}$	3.61	3.59	3.38	3.61	3.46
$\bar{y}_{QRSS21}$	3.23	3.21	2.98	3.23	3.07	$\bar{y}_{EORSS21}$	3.90	3.87	3.65	3.89	3.74
$\bar{y}_{QRSS22}$	3.24	3.22	2.99	3.24	3.08	$\bar{y}_{EORSS22}$	3.91	3.88	3.66	3.91	3.75
$\bar{y}_{QRSS23}$	3.35	3.32	3.09	3.35	3.18	$\bar{y}_{EORSS23}$	4.02	4.00	3.76	4.02	3.85
$\bar{y}_{QRSS24}$	3.23	3.21	2.98	3.23	3.07	$\bar{y}_{EORSS24}$	3.90	3.87	3.65	3.90	3.74
$\bar{y}_{QRSS25}$	3.3	3.28	3.05	3.3	3.14	$\bar{y}_{EORSS25}$	3.97	3.95	3.72	3.97	3.81
$\bar{y}_{QRSS31}$	3.40	3.38	3.14	3.40	3.23	$\bar{y}_{EORSS31}$	4.24	4.22	3.97	4.24	4.07
$\bar{y}_{QRSS32}$	3.41	3.39	3.15	3.41	3.24	$\bar{y}_{EORSS32}$	4.25	4.23	3.98	4.25	4.08
$\bar{y}_{QRSS33}$	3.52	3.50	3.25	3.52	3.35	$\bar{y}_{EORSS33}$	4.37	4.35	4.09	4.37	4.19
$\bar{y}_{QRSS34}$	3.40	3.38	3.14	3.40	3.23	$\bar{y}_{EORSS34}$	4.24	4.22	3.97	4.24	4.07
$\bar{y}_{QRSS35}$	3.48	3.45	3.21	3.48	3.30	$\bar{y}_{EORSS35}$	4.32	4.30	4.05	4.32	4.15

The preeminence of our proposed class of robust regression-in-ratio estimators in *RSS* scheme over estimators given by Kadilar and Cingi (2004) also verified from simulation results presented in Table 4. For instance considering the performance of estimators in RSS designs, the estimators $\bar{y}_{RSS15}$, $\bar{y}_{RSS25}$ and $\bar{y}_{RSS35}$ have 412%, 486 and 585% higher *R.E* as compare to $\bar{y}_{KC5}$ respectively. These results are also certified from the theoretical results obtained in section *6.1*, given in Table 2.

Table 5
***R.E* of Proposed Estimators using *OLS* Algorithm with Proposed Estimators based Robust Regression in *RSS* Schemes for $t = 9$ and $r = 1$**

R.E	$\bar{y}_{RSS11}$	$\bar{y}_{RSS12}$	$\bar{y}_{RSS13}$	$\bar{y}_{RSS14}$	$\bar{y}_{RSS15}$	R.E	$\bar{y}_{MRSS11}$	$\bar{y}_{MRSS12}$	$\bar{y}_{MRSS13}$	$\bar{y}_{MRSS14}$	$\bar{y}_{MRSS15}$
$\bar{y}_{RSS21}$	1.18	1.18	1.15	1.18	1.16	$\bar{y}_{MRSS21}$	1.17	1.17	1.11	1.17	1.13
$\bar{y}_{RSS22}$	1.18	1.18	1.16	1.18	1.17	$\bar{y}_{MRSS22}$	1.18	1.17	1.11	1.18	1.14
$\bar{y}_{RSS23}$	1.21	1.21	1.18	1.21	1.19	$\bar{y}_{MRSS23}$	1.24	1.23	1.17	1.24	1.2
$\bar{y}_{RSS24}$	1.18	1.18	1.15	1.18	1.16	$\bar{y}_{MRSS24}$	1.17	1.17	1.11	1.17	1.13
$\bar{y}_{RSS25}$	1.20	1.2	1.17	1.20	1.18	$\bar{y}_{MRSS25}$	1.21	1.21	1.15	1.21	1.17
$\bar{y}_{RSS31}$	1.30	1.30	1.27	1.30	1.28	$\bar{y}_{MRSS31}$	1.40	1.39	1.32	1.40	1.35
$\bar{y}_{RSS32}$	1.30	1.30	1.27	1.30	1.28	$\bar{y}_{MRSS32}$	1.40	1.4	1.33	1.40	1.36
$\bar{y}_{RSS33}$	1.33	1.33	1.30	1.33	1.31	$\bar{y}_{MRSS33}$	1.48	1.47	1.4	1.48	1.43
$\bar{y}_{RSS34}$	1.30	1.30	1.27	1.30	1.28	$\bar{y}_{MRSS34}$	1.40	1.39	1.32	1.40	1.35
$\bar{y}_{RSS35}$	1.32	1.31	1.29	1.32	1.3	$\bar{y}_{MRSS35}$	1.45	1.44	1.37	1.45	1.40

R.E.	$\bar{y}_{QRSS11}$	$\bar{y}_{QRSS12}$	$\bar{y}_{QRSS13}$	$\bar{y}_{QRSS14}$	$\bar{y}_{QRSS15}$	R.E.	$\bar{y}_{EORSS11}$	$\bar{y}_{EORSS12}$	$\bar{y}_{EORSS13}$	$\bar{y}_{EOSS14}$	$\bar{y}_{EORSS15}$
$\bar{y}_{QRSS21}$	1.13	1.13	1.09	1.13	1.10	$\bar{y}_{EORSS21}$	1.10	1.10	1.07	1.10	1.08
$\bar{y}_{QRSS22}$	1.13	1.13	1.09	1.13	1.11	$\bar{y}_{EORSS22}$	1.10	1.10	1.07	1.10	1.08
$\bar{y}_{QRSS23}$	1.17	1.17	1.13	1.17	1.14	$\bar{y}_{EORSS23}$	1.14	1.13	1.10	1.14	1.11
$\bar{y}_{QRSS24}$	1.13	1.13	1.09	1.13	1.10	$\bar{y}_{EORSS24}$	1.10	1.10	1.07	1.10	1.08
$\bar{y}_{QRSS25}$	1.15	1.15	1.11	1.15	1.13	$\bar{y}_{EORSS25}$	1.12	1.12	1.09	1.12	1.10
$\bar{y}_{QRSS31}$	1.19	1.18	1.15	1.19	1.16	$\bar{y}_{EORSS31}$	1.20	1.20	1.16	1.2	1.18
$\bar{y}_{QRSS32}$	1.19	1.19	1.15	1.19	1.17	$\bar{y}_{EORSS32}$	1.20	1.2	1.17	1.2	1.18
$\bar{y}_{QRSS33}$	1.23	1.23	1.19	1.23	1.20	$\bar{y}_{EORSS33}$	1.24	1.23	1.20	1.24	1.21
$\bar{y}_{QRSS34}$	1.19	1.18	1.15	1.19	1.16	$\bar{y}_{EORSS34}$	1.20	1.20	1.16	1.20	1.18
$\bar{y}_{QRSS35}$	1.22	1.21	1.17	1.21	1.19	$\bar{y}_{EORSS35}$	1.22	1.22	1.18	1.22	1.20

The Simulation results shown in Table 5 also verifies the superiority of proposed robust regression-in-ratio estimators of population mean to the *OLS* based regression-in-ratio estimators in all *RSS* modified schemes. It means objective function given in (1.2) produced more efficient results than objective function described in (1.1) in all RSS schemes when comparisons are made with *OLS* in the presence of outliers. For example, $\bar{y}_{EORSS33}$ produced 10% more efficient estimates than $\bar{y}_{EORSS23}$ when their efficiencies are compared with $\bar{y}_{EORSS13}$.

7. CONCLUSION

The theoretical and simulation results obtained in section *6*, showed that regression-in-ratio estimators in *RSS* schemes provided more efficient results as compare to *SRS* scheme based on *OLS* technique to estimate the population mean when data contain outliers. Obtained results also depicted that in the presence of outliers, estimators based on objective functions described in section *1*, have high *R.E* as compare to estimators using *OLS* procedure in all *RSS* schemes. Moreover, objective function proposed by Raza, Noor-ul-Amin and Hanif. (2019) produced more robust and reliable estimate of population mean in the presence of outliers than objective function given by Huber (1964). So it is recommended that suggested robust regression-in-ratio estimators should be used to estimate the population mean when data contain outliers in *SRS* and *RSS* designs. In future studies, authors are trying to construct robust estimators in double ranked set sampling schemes.

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