Pak. J. Statist.
2022 Vol. 38(2), 197-210

STRATIFIED MIXTURE RANKED SET SAMPLING FOR ESTIMATION OF POPULATION MEAN AND MEDIAN

Zahid Khan, Muhammad Ismail and **Muhammad Noor-ul-Amin**
Department of Statistics, COMSATS University Islamabad,
Lahore Campus, Lahore, Pakistan.
Email: zahid.uom1@gmail.com
drismail39@gmail.com
nooramin.stats@gmail.com

ABSTRACT

In the present paper, stratified mixture ranked set sampling (SMIRSS) for estimation of the population mean and median is suggested. The SMIRSS provides unbiased estimator of population mean for symmetric distribution. To test the performance of estimators under suggested scheme a simulation study is conducted. The simulation results indicate that the SMIRSS is more efficient than the stratified extreme ranked set sampling (SERSS) and stratified simple random sampling (SSRS) designs. Moreover, in small sample the suggested scheme is more efficient than stratified ranked set sampling (SRSS) for some distributions.

KEYWORDS

Mixture ranked set sampling; Ranked Set Sampling; Stratified extreme ranked set sampling; Population mean; Unbiased estimator.

1. INTRODUCTION

Mclntyre (1952) suggested the ranked set sampling (RSS) design which has advantageous when ranking of sample elements has fairly easy and inexpensive. The RSS is not only more efficient design than its counterpart designs but also cost-effective. Takahasi and Wakimoto (1968) developed the statistical theory of the RSS method. Dell and Clutter (1972) investigated the effect of imperfect ranking of elements on mean estimator under RSS method. They proved that RSS provides unbiased mean estimator, whether or not ranking of elements is perfect. Stokes (1977) investigated that the study variable can be ranked on the basis of correlated concomitant variables which is relatively inexpensive than the variable of interest. For more details about RSS, see Stokes (1995), Samawi and Muttlak (1996), Samawi and Al-Sagheer (2001), Al-Saleh and Al-Omari (2002), Bai and Chen (2003), Chen et al. (2005), Mahdizadeh and Arghami (2012), Al-Omari and Bouza (2014), Strzalkowska-Kominiak and Mahdizadeh (2014), Mahdizadeh (2015), Jabrah et al. (2017), Mahdizadeh and Strzalkowska-Kominiak (2017), Mahdizadeh (2018), Khan and Ismail (2019).

Many researchers have developed and modified RSS method to obtaine more efficient and cost-effective estimators of population parameters. Samawi et al. (1996) introduced extreme ranked set sampling (ERSS) scheme for estimation of the population mean. In such a scheme, extreme elements are identified from sets. Muttalk (1997) presented median ranked set sampling (MRSS) scheme in which median elements from each set are identified for actual quantification. Jemain et al. (2008) introduced balance group ranked set sampling (BGRSS) where the n samples of size n is distributed into three groups, then, minimum elements from the sets of first group, median elements from second group, maximum elements from the third group are taken for actual quantification. Similar to this method, Sevinc et al. (2018) suggested partial balance group ranked set sampling (PBGRSS) which is more flexible sampling plan than BGRSS.

Samawi (1996) extended the ordinary RSS design to stratified ranked set sampling (SRSS) scheme for estimation of population mean. Samawai and Saeid (2004) introduced stratified extreme ranked set sampling (SERSS) design for estimation of population mean. Ibrahim et al. (2010) presented stratified median ranked set sampling (SMRSS) design in which median ranked set sampling scheme is used to identify sample from each stratum. Al-Omari et al. (2011) investigated stratified percentile ranked set sampling (SPRSS) method for estimation of population mean. Mahdizadeh and Zamanzade (2018) proposed stratified pair ranked set sampling (SPRSS) scheme which utilized minimum cost than SRSS method. For more modified schemes of RSS see, Hossain and Muttlak, (1999), Al-Saleh and Al-Kadiri, (2000), Al-Nasser and Mustafa (2009), Samawi, (2011), Al-Omari and Raqab (2013), Salehi and Ahmadi, (2014), Abbasi and Shahd, (2017), Ahmad and Shabbir, (2019), Noor-ul-Amin et al. (2019).

1.1 Ranked Set Sampling (RSS)

The RSS procedure for getting sample of size n is briefly describes as follows. Select a simple random sample of size n^2 from the target population. Distribute these elements into n sets each of size n. Rank the elements within each set with respect to the study variable visually or by any other economical method. Identify the i^{th} lowest ranked element from the i^{th} set. This procedure (cycle) can be repeated r times to get more elements say rn for actual quantification.

The mean and variance of RSS are given,

$$\bar{X}_{RSS} = \frac{1}{rn}\sum_{j=1}^{r}\sum_{i=1}^{n} X_{i(i:n)j} \ , \tag{1}$$

and

$$\operatorname{var}\left(\bar{X}_{RSS}\right) = \frac{\sigma^2}{rn} - \frac{1}{rn^2}\sum_{i=1}^{n}\left(\mu_{i(i:n)} - \mu\right)^2 . \tag{2}$$

where, $X_{i(i:n)j}$ is the i^{th} order statistics from j^{th} cycle.

1.2 Mixture Raked Set Sampling (MIRSS)

Khan et al. (2020) investigated ‘mixture ranked set sampling’ (MIRSS) method for estimation of the population mean and median. The MIRSS is suitable design when RSS method is not appropriate to each cycle of the RSS. The procedure of MIRSS to select sample of size rn is as under. Select simple random sample of size n^2 from the target population, and distribute them into n sets each of size n. Repeat this process r times, and rank the elements within each set by eye or by any other inexpensive tool. Suppose the RSS method is applicable to s cycles for $0 \le s \le r$, and ERSS is suitable method for t cycles for $t = r - s$. Thus, select sn and tn elements by RSS and ERSS methods respectively. Finally, $nr = n(s+t)$ MIRSS sample elements are selected.

Let $X_{1(1:n)k}, \ldots, X_{n(n:n)k}, X_{1(1:n)l}, X_{2(1:n)l}, \ldots, X_{n-1(n:n)l}, X_{n(n:n)l}$ for $k \ne l$ represent k^{th} and l^{th} cycle $(k = 0, \ldots s; l = 0, \ldots t)$, be the independent random samples. The mean of MIRSS for even n is as under,

$$\bar{X}_{MIRSS} = \frac{1}{nr}\left\{ \sum_{k=0}^{s}\left(\sum_{i=1}^{n} X_{i(i:n)k} \right) + \sum_{l \ne k=0}^{t}\left(\sum_{i=1}^{\frac{n}{2}} X_{i(1:n)l} + \sum_{i=\frac{n}{2}+1}^{n} X_{i(n:n)l} \right) \right\} \tag{3}$$

The variance of $\bar{X}_{MIRSS}$ is as under,

$$\operatorname{var}\left(\bar{X}_{MIRSS}\right) = \frac{1}{2n^2 r^2}\left(2s\sum_{i=1}^{n} \sigma^2_{(i:n)} + tn(\sigma^2_{(1:n)} + \sigma^2_{(n:n)} \right) \tag{4}$$

In case of odd n, the mean is defined as,

$$\bar{X}_{MIRSSa} = \frac{1}{nr}\left\{ \begin{array}{l} \sum_{k=0}^{s}\left(\sum_{i=1}^{n} X_{i(i:n)k} \right) \\ + \sum_{l \ne k=0}^{t}\left(\sum_{i=1}^{\frac{n-1}{2}} X_{i(1:n)l} + X_{i\left(\frac{n+1}{2}:n\right)l} + \sum_{i=\frac{n+1}{2}+1}^{n} X_{i(n:n)l} \right) \end{array} \right\} \tag{5}$$

The variance of $\bar{X}_{MIRSSa}$ is as under,

$$\operatorname{var}\left(\bar{X}_{MIRSSa}\right) = \frac{1}{n^2 r^2}\left(\begin{array}{l} s\sum_{i=1}^{n} \sigma^2_{(i:n)} \\ + \frac{t}{2}\left(2\sigma^2_{\left(\frac{n+1}{2}:n\right)} + (n-1)\left(\sigma^2_{(1:n)} + \sigma^2_{(n:n)} \right) \right) \end{array} \right) \tag{6}$$

1.3 Stratified Ranked Set Sampling (SRSS)

The procedure SRSS for getting sample of size n is to divide the populaiton into H mutually exclusive and exhaustive strata. Then, select independently ranked set sample of size $r_h n_h$ elements from stratum h in r_h cycle. Let $X^{h}_{[i:n]j}$ be the i^{th} judgment order statistic in the j^{th} cycle of the ranked set sample collected from h^{th} stratum. The elements

$X^{h}_{[i:n]j}$ $(h=1,....,H; i=1,..,n_h; j=1,...r_h)$ are independent, but not identically distributed. For fixed h and i, $X^{h}_{[i]j}$'s $(j=1,...r_h)$ are identically distributed, where the common mean and variance are denoted by $\mu_{[i:n]h}$ and $\sigma_{[i:n]h}$ respectively. The mean estimator of SRSS is as under,

$$\bar{X}_{(SRSS)} = \sum_{j=1}^{r} \frac{N_h}{N} \bar{X}_{(SRSS,h)}, \tag{7}$$

where, $\bar{X}_{(SRSS,h)} = \frac{1}{n_h r_h} \sum_{i=1}^{n_h} \sum_{j=1}^{r_h} X^{h}_{[i:n]j}$ is the SRSS mean estimator of h^{th} stratum.

The variance is given by,

$$\text{var}(\bar{X}_{(SRSS)}) = \sum_{h=1}^{H} \left(\frac{N_h}{N n_h} \right)^2 \frac{1}{r_h} \sum_{i=1}^{n_h} \sigma^2_{[i:n]h},$$

where,

$$\sum_{i=1}^{n_h} \sigma^2_{[i:n]h} = n_h \sigma^2_h - \sum_{i=1}^{n_h} (\bar{X}_{(i,n)h} - \mu_h)^2$$

Combining we get,

$$\text{var}(\bar{X}_{(SRSS)}) = \text{var}(\bar{X}_{(SSRS)}) - \sum_{h=1}^{H} \left(\frac{N_h}{N} \right)^2 \frac{1}{n_h^2 r_h} \sum_{i=1}^{n_h} (\bar{X}_{(i,n_h)h} - \mu_h)^2 . \tag{8}$$

2. PROPOSED SAMPLING SCHEME

In this section, we suggest stratified mixture ranked Set Sampling (SMIRSS) which is stratified version of MIRSS, suggested by Khan et al. (2020). Suppose a SMIRSS samples of size $n_h r_h$ is drawn from h^{th} stratum. Let the SRSS method is suitable for p cycle, and SERSS is applicable to q cycles, out of r cycle, thus $r = p + q$. Let $n_h p_h$ sample are drawn from stratum h using set size n_h in p_h cycles, and samples of total size $n_h q_h$ sample from q_h cycles. Thus, a total sample size of $n_h r_h = n_h \left(p_h + q_h \right)$ are identified from population in h^{th} stratum.

The SMIRSS mean estimator for even sample size is given by,

$$\bar{X}_{(SMIRSS1)} = \sum_{h=1}^{H} \frac{N_h}{N} \bar{X}_{(SMIRSS1,h)}, \tag{9}$$

where,

$$\bar{X}_{(SMIRSS1,h)} = \frac{1}{n_h r_h}\left\{ \begin{array}{l} \sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h} X_{i(i:n_h)k}^{h}\right) \\ \qquad + \sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{i(1:n_h)l}^{h} + \sum_{i=\frac{n_h}{2}+1}^{n_h} X_{i(n_h:n_h)l}^{h}\right) \end{array} \right\}, \tag{10}$$

If n_h is odd then mean of $\bar{X}_{(SMIRSS2,h)}$ is as under,

$$\bar{X}_{(SMIRSS2,h)} = \frac{1}{n_h r_h}\left\{ \begin{array}{l} \sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h} X_{i(i:n_h)k}^{h}\right) \\ + \sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{i(1:n_h)l}^{h} + X_{\left(\frac{n_h+1}{2}:n_h\right)l}^{h} + \sum_{i=\frac{n_h+1}{2}+1}^{n_h} X_{i(n_h:n_h)l}^{h}\right) \end{array} \right\}. \tag{11}$$

To derive variance of $\bar{X}_{(SMIRSS1,h)}$, for even sample size, applying variance both sides to equation (10) we get,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS1)}\right) = \sum_{h=1}^{H}\left(\frac{N_h}{N}\right)^2 \operatorname{var}\left(\bar{X}_{(SMIRSS1,h)}\right). \tag{12}$$

Since the sample drawn from stratum are independent, therefore, covariance term is vanished. Now,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS1,h)}\right) = \frac{1}{n_h^2 r_h^2}\left\{ \begin{array}{l} \sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\operatorname{var}\left(X_{(i:n_h)k}^{h}\right)\right) \\ \quad + \sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h}{2}}\operatorname{var}\left(X_{(1:n_h)l}^{h}\right) + \sum_{i=\frac{n_h}{2}+1}^{n_h}\operatorname{var}\left(X_{(n_h:n_h)l}^{h}\right)\right) \end{array} \right\},$$

$$= \frac{1}{n_h^2 r_h^2}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\sigma_{(i:n_h)}^2\right) + \sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h}{2}}\sigma_{(1:n_h)}^2 + \sum_{i=\frac{n_h}{2}+1}^{n_h}\sigma_{(n_h:n_h)}^2\right)\right\},$$

$$= \frac{1}{n_h^2 r_h^2}\left\{p_h\sum_{i=1}^{n_h}\sigma_{(i:n_h)}^2 + \frac{q_h n_h}{2}\left(\sigma_{(1:n_h)}^2 + \sigma_{(n_h:n_h)}^2\right)\right\},$$

$$= \frac{1}{2n_h^2 r_h^2}\left\{2p_h\sum_{i=1}^{n_h}\sigma_{(i:n_h)}^2 + q_h n_h\left(\sigma_{(1:n_h)}^2 + \sigma_{(n_h:n_h)}^2\right)\right\}.$$

Putting the value of $\operatorname{var}\left(\bar{X}_{(SMIRSS1,h)}\right)$ in equation (12) we get,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS1)}\right)=\sum_{h=1}^{H}\left(\frac{N_h}{N}\right)^2\frac{1}{2n_h^2r_h^2}\left\{2p_h\sum_{i=1}^{n_h}\sigma^2_{(i:n_h)}+q_hn_h\left(\sigma^2_{(1:n_h)}+\sigma^2_{(n_h:n_h)}\right)\right\}. \tag{13}$$

To derive variance of $\bar{X}_{(SMIRSS2,h)}$, for odd sample size, applying variance both sides to equation (11) we get,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS2)}\right)=\sum_{h=1}^{H}\left(\frac{N_h}{N}\right)^2\operatorname{var}\left(\bar{X}_{(SMIRSS2,h)}\right). \tag{14}$$

From equation (11) we get,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS2,h)}\right)=\frac{1}{n_h^2r_h^2}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\operatorname{var}\left(X^h_{(i:n_h)k}\right)\right)\right.$$

$$\left.+\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}\operatorname{var}\left(X^h_{(1:n_h)l}\right)+\operatorname{var}\left(X^h_{\left(\frac{n_h+1}{2}:n_h\right)l}\right)+\sum_{i=\frac{n_h+1}{2}+1}^{n_h}\operatorname{var}\left(X^h_{(n_h:n_h)l}\right)\right)\right\},$$

$$=\frac{1}{n_h^2r_h^2}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\sigma^2_{(i:n_h)}\right)+\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}\sigma^2_{(1:n_h)}+\sigma^2_{\left(\frac{n_h+1}{2}:n_h\right)}+\sum_{i=\frac{n_h+1}{2}+1}^{n_h}\sigma^2_{(n_h:n_h)}\right)\right\},$$

$$=\frac{1}{n_h^2r_h^2}\left\{p_h\sum_{i=1}^{n_h}\sigma^2_{(i:n_h)}+\frac{q_h}{2}\left(2\sigma^2_{\left(\frac{n_h+1}{2}:n_h\right)}+\left(n_h-1\right)\left(\sigma^2_{(1:n_h)}+\sigma^2_{(n_h:n_h)}\right)\right)\right\},$$

$$\operatorname{var}\left(\bar{X}_{(SMIRSS2,h)}\right)=\frac{1}{2n_h^2r_h^2}$$

$$\left\{2p_h\sum_{i=1}^{n_h}\sigma^2_{(i:n_h)}+q_h\left(2\sigma^2_{\left(\frac{n_h+1}{2}:n_h\right)}+\left(n_h-1\right)\left(\sigma^2_{(1:n_h)}+\sigma^2_{(n_h:n_h)}\right)\right)\right\}.$$

Put in equation (14) finally we get,

$$\operatorname{var}\left(\bar{X}_{(SMIRSS2)}\right)=\sum_{h=1}^{H}\left(\frac{N_h}{N}\right)^2\frac{1}{2n_h^2r_h^2}$$

$$\left\{2p_h\sum_{i=1}^{n_h}\sigma^2_{(i:n_h)}+q_h\left(2\sigma^2_{\left(\frac{n_h+1}{2}:n_h\right)}+\left(n_h-1\right)\left(\sigma^2_{(1:n_h)}+\sigma^2_{(n_h:n_h)}\right)\right)\right\}, \tag{15}$$

Theorem 2.1

1) The SMIRSS provides unbiased estimator of population mean for even sample size under symmetric distributions.
2) The SMIRSS provides unbiased estimator of population mean for odd sample size under symmetric distributions.

Proof:

By applying expectation both sides of equation (10) we get,

$$E\left(\bar{X}_{(SMIRSS1,h)}\right)=\frac{1}{n_h r_h}\left\{\begin{array}{l}\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}E\left(X^h_{(i:n_h)k}\right)\right)\\ +\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h}{2}}E\left(X^h_{(1:n_h)l}\right)+\sum_{i=\frac{n_h}{2}+1}^{n_h}E\left(X^h_{(n_h:n_h)l}\right)\right)\end{array}\right\},$$

$$=\frac{1}{n_h r_h}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\mu^h_{(i:n_h)}\right)+\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h}{2}}\mu^h_{(1:n_h)}+\sum_{i=\frac{n_h}{2}+1}^{n_h}\mu^h_{(n_h:n_h)}\right)\right\},$$

$$=\frac{1}{n_h r_h}\left\{p_h n_h \mu^h_{(i:n_h)}+q_h\frac{n_h}{2}(\mu^h_{(1:n_h)}+\mu^h_{(n_h:n_h)}\right\},$$

For symmetric distribution $\mu^h_{(i:n_h)}=\mu^h_{(1:n_h)}=\mu^h_{(n_h:n_h)}=\mu$, (David and Nagaraja, 2003),

$$E\left(\bar{X}_{(SMIRSS1,h)}\right)=\frac{1}{2r_h}\left\{p_h 2\mu+2q_h\mu\right\},$$

Since $p_h+q_h=r_h$,

$$E\left(\bar{X}_{(SMIRSS1,h)}\right)=\mu .$$

3) By taking expectation of equation (11) both sides we get,

$$E\left(\bar{X}_{(SMIRSS\,2,h)}\right)=\frac{1}{n_h r_h}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}E\left(X^h_{i(i:n_h)k}\right)\right)+\right.$$

$$\left.\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}E\left(X^h_{i(1:n_h)l}\right)+E\left(X^h_{\left(\frac{n_h+1}{2}:n_h\right)l}\right)+\sum_{i=\frac{n_h+1}{2}+1}^{n_h}E\left(X^h_{i(n_h:n_h)l}\right)\right)\right\},$$

$$=\frac{1}{n_h r_h}\left\{\sum_{k=1}^{p_h}\left(\sum_{i=1}^{n_h}\mu^h_{(i:n_h)}\right)+\sum_{l\neq k=0}^{q_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}\mu^h_{(1:n_h)}+E\left(\mu^h_{\left(\frac{n_h+1}{2}:n_h\right)}\right)+\sum_{i=\frac{n_h+1}{2}+1}^{n_h}\mu^h_{(n_h:n_h)}\right)\right\},$$

$$= \frac{1}{n_h r_h}\left\{ p_h n_h \mu_{(i:n_h)}^h + \frac{n_h - 1}{2} q_h \mu_{(1:n_h)}^h + q_h \mu_{\left(\frac{n_h+1}{2}:n_h\right)}^h + \frac{n_h - 1}{2} q_h \mu_{(n_h:n_h)}^h \right\},$$

Since $\mu_{(i:n_h)}^h = \mu_{(1:n_h)}^h = \mu_{\left(\frac{n_h+1}{2}:n_h\right)}^h = \mu_{(n_h:n_h)}^h = \mu$,

$$E\left(\bar{X}_{(SMIRSS2,h)}\right) = \frac{1}{n_h r_h}\left\{ p_h n_h \mu + q_h \mu + n_h q_h \mu - q_h \mu \right\},$$

$$E\left(\bar{X}_{(SMIRSS2,h)}\right) = \frac{\mu}{r_h}\left\{ p_h + q_h \right\},$$

Since $p_h + q_h = r_h$, we get,

$$E\left(\bar{X}_{(SMIRSS2,h)}\right) = \mu .$$

Hence proved.

3. SIMULATION STUDY

In this section we conducted a simulation study in order to test the performance of mean estimator under proposed sampling scheme. The equation used for such comparison for symmetric distributions are given below,

$$RE\left(\bar{X}_v, \bar{X}_{SSRS}\right) = \frac{Var\left(\bar{X}_{SSRS}\right)}{Var\left(\bar{X}_v\right)}$$

where, v denotes SMIRSS, SRSS and SERSS designs.

The equation of RE for asymmetric distributions is as under,

$$RE\left(\bar{X}_v, \bar{X}_{SSRS}\right) = \frac{Var\left(\bar{X}_{SSRS}\right)}{MSE\left(\bar{X}_v\right)}.$$

Each variance/MSE are simulated based on 50,000 samples using R software. Four distributions are considered for generating data, Normal (0,1), Logistic (0,1), Gamma (1,4) and Weibull (2,4).

Figures 1 and 2 presents the relative efficiencies of mean estimator based on SMIRSS, SRSS, and SERSS relative to mean estimator of SSRS for two and three strata respectively. In Figure 1, different set sizes i.e. (3,3), (4,4), (4,6), (7,5) in two strata using $r = 2$, is considered. In SRSS and SERSS the same procedure is repeated two times while in the suggested SMIRSS scheme, the SRSS and SERSS methods are applied in first cycle and second cycle respectively.

The figure reveals that the relative efficiencies (REs) of proposed estimator is greater than one which shows that the SMIRSS, SRSS and SERSS are more efficient than the SSRS. The REs of estimator under SMIRSS is higher than estimator under SERSS which indicates that SMIRSS provides more efficient mean estimator than SERSS. On the contrary, in large samples sizes, the REs of SRSS mean estimator is greater than REs of SMIRSS mean estimator. While in small sample size the suggested scheme performs better than in some distributions. Thus, SMIRSS should be used when the SRSS is not appropriate to all cycles. For more details about this result, see Khan et al. (2020).

Figure 2 shows the REs of mean estimators under SMIRSS, SSRS, SERSS relative to SSRS design for three strata having set sizes (3,3,4), (4,5,3), (4,6,4), (7,5,4) for $r = 2$. It is clear from Figure 2 that the suggested design is more efficient than SERSS and SSRS for small sample sizes in given distributions except normal. Thus, the suggested design is not only more practical but also more efficient for small sample sizes in some distributions.

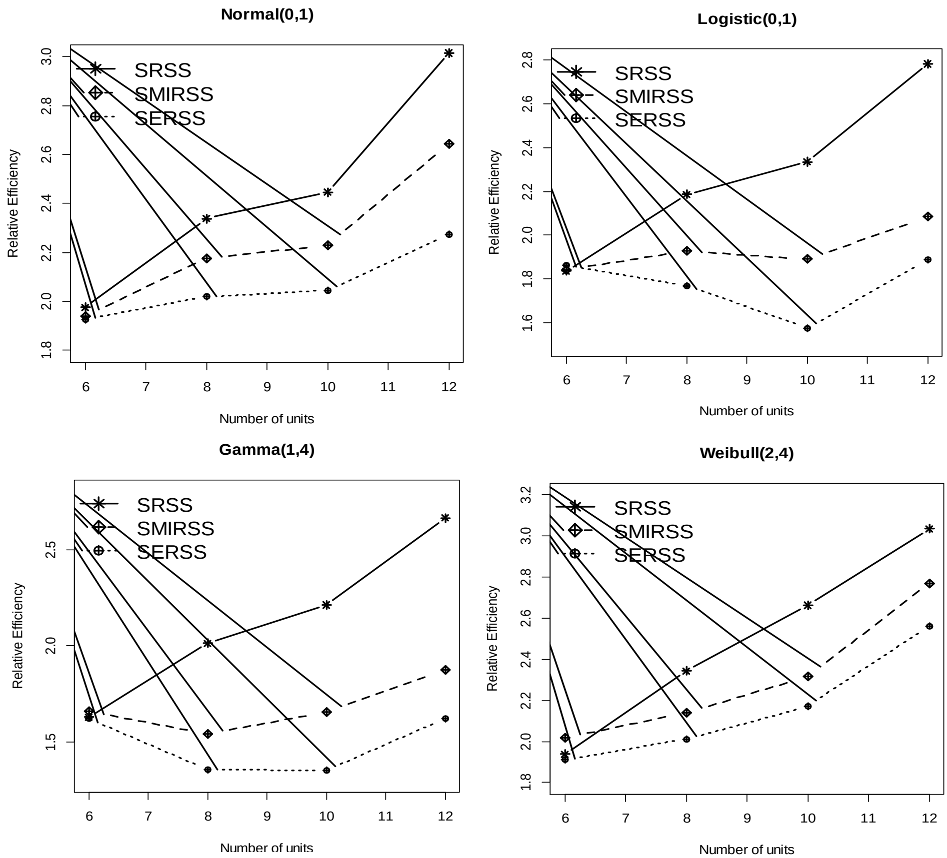

Figure 1: REs of Mean Estimator based on SMIRSS, SRSS, SERSS relative to SSRS Mean Estimator for Set Sizes (3, 3), (4,4), (4,6), (7,5) in Two Strata for $r = 2$

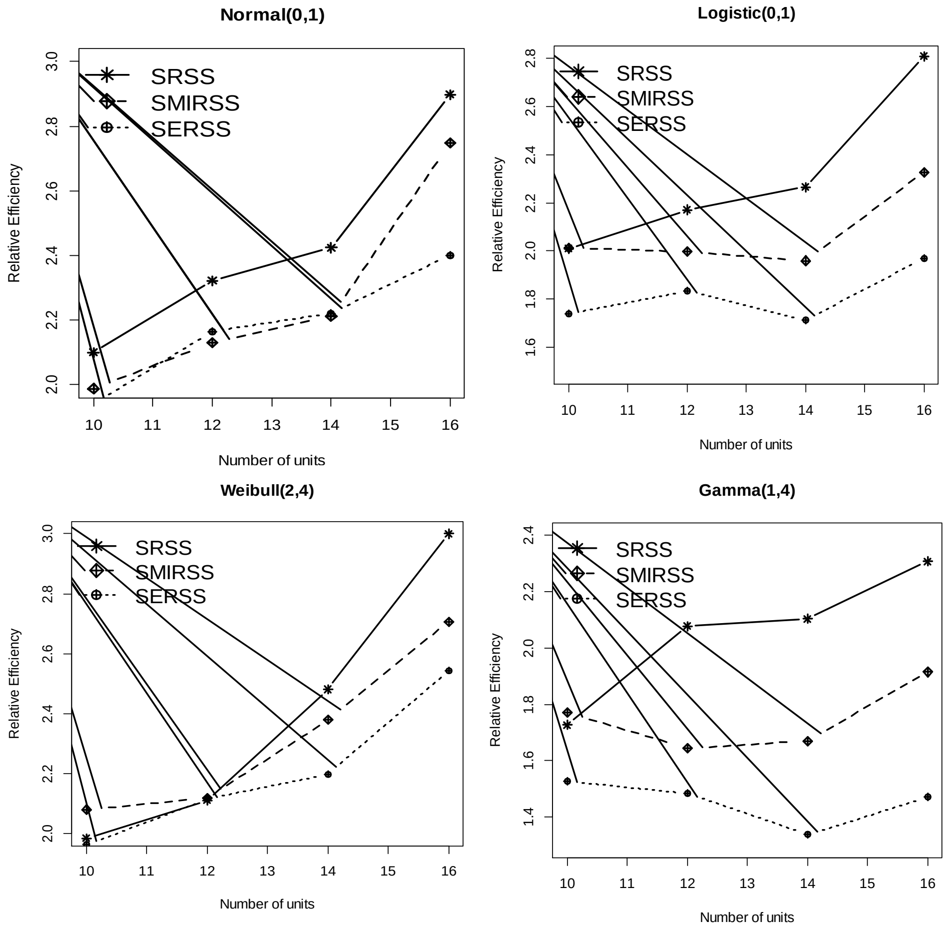

Figure 2: REs of Mean Estimator based on SMIRSS, SRSS, SERSS relative to SSRS Mean Estimator for Set Sizes (3,3,4), (4,5,3), (4,6,4), (7,5,4) in Three Strata for $r = 2$

4. ESTIMATION OF MEDIAN

Sometimes the underlying population is highly skewed such as production, expenditure and income. For such distributions the median is often considered the more suitable measure of location than mean. The median estimator based on SMIRSS for h^{th} stratum is as follows,

$$\hat{\theta}_{SMIRSS} = Median\left\{X_{1(1:n)p_h},\ldots,X_{n(n:n)p_h},X_{1(1:n)q_h},X_{2(1:n)q_h},\ldots,X_{n-1(n:n)q_h},X_{n(n:n)q_h}\right\}$$

The equation for RE of median estimator for h^{th} stratum is given as,

$$RE\left(\bar{X}_v, \bar{X}_{SSRS}\right) = \frac{MSE\left(\hat{\theta}_{SSRS}\right)}{MSE\left(\hat{\theta}_v\right)}$$

where, v denotes SRSS, SMIRSS and SERSS.

Figures 3 and 4 show the REs of median estimator based on SMIRSS, SRSS and SERSS relative to median estimator of SSRS for two and three strata respectively. Both figures reveal that SRSS performs better in estimation of median than proposed scheme except for small sample size in normal and gamma distributions in Figures 4 (three strata case). Moreover, the suggested scheme is not superior than SERSS in small sample size in Figures 3 except normal distribution. These results suggest that the SMIRSS is efficient design than SERSS in estimation of population median in large sample sizes. While in normal distribution the SMIRSS is efficient than SRSS and SERSS in small sample size in two strata case.

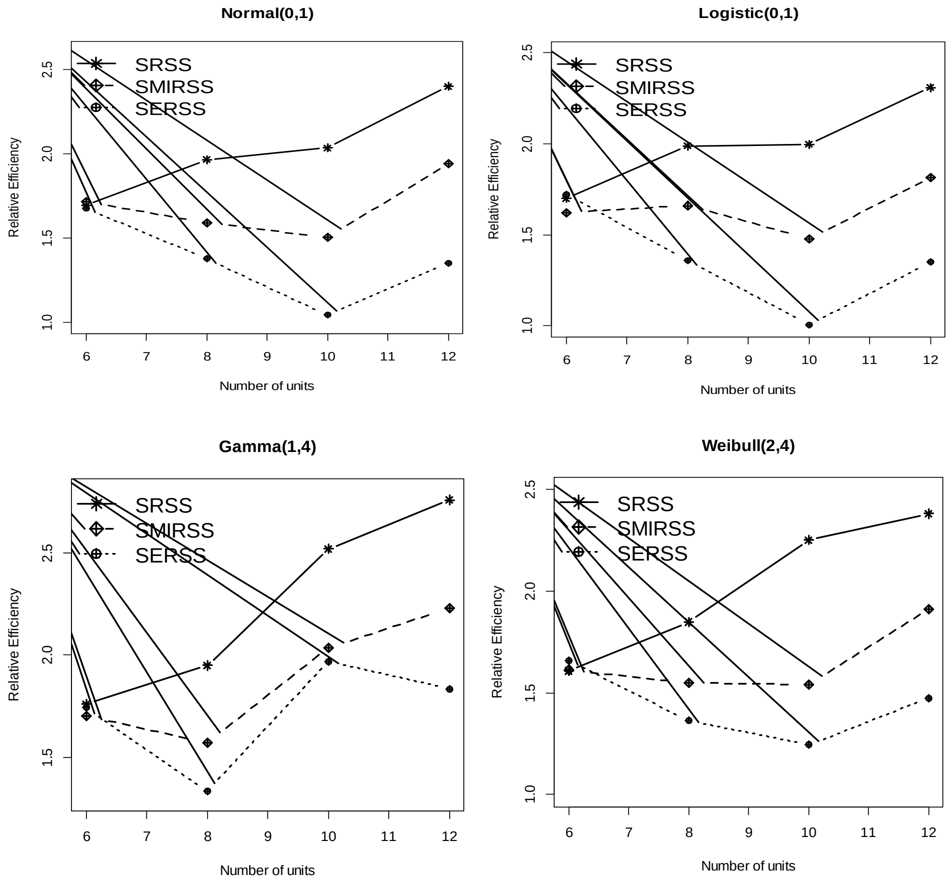

Figure 3: REs of Median Estimator based on SMIRSS, SRSS, SERSS relative to SSRS Median Estimator for Set Sizes (3,3), (4,4), (4,6), (7,5) in Two Strata for $r = 2$

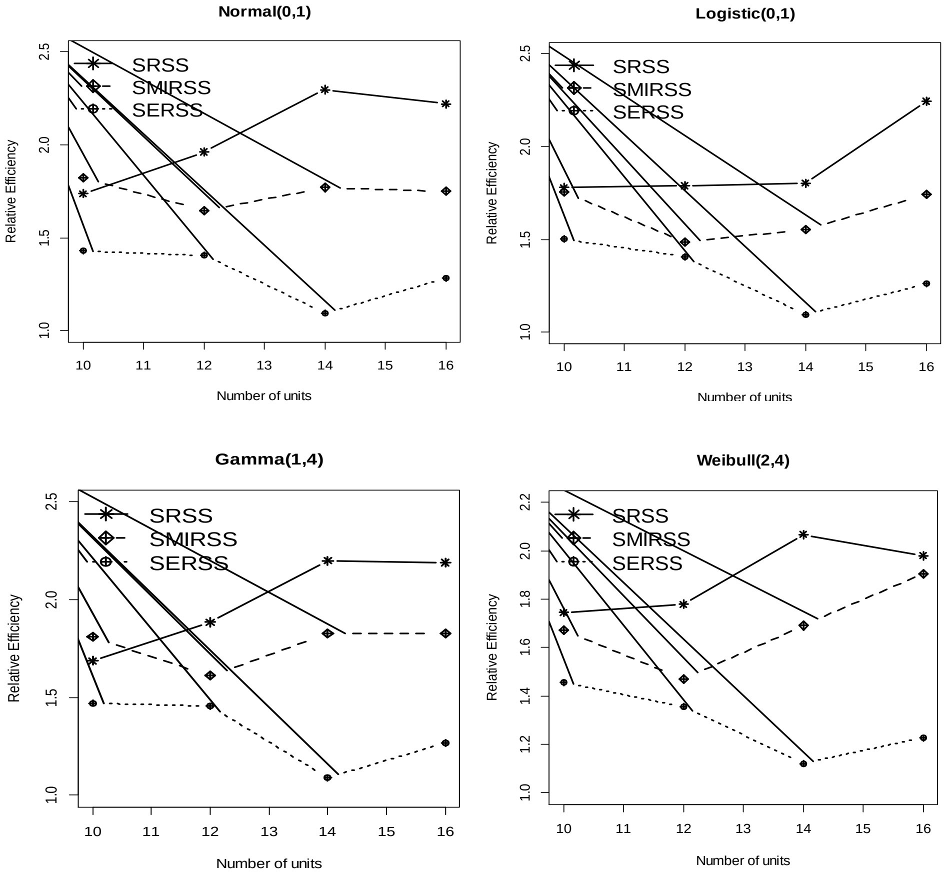

Figure 4: REs of Median Estimator based on SMIRSS, SRSS, SERSS relative to SSRS Median Estimator for Set Sizes (3,3,4), (4,5,3), (4,6,4), (7,5,4) in Three Strata for $r = 2$

5. CONCLUSION

In this article, the MIRSS method is extended to 'stratified mixture ranked set sampling' method. The scheme is effective when SRSS method is not appropriate for all cycles of the experiments. In such a situation, the SERSS method is applied to those cycle(s) where SRSS cannot be adopted with full confidence. The SMIRSS provided unbiased estimator of population mean when the underlying distribution is symmetric. Moreover, the suggested design is more efficient than the SERSS and SSRS methods. The present study can be extended to estimation of ratio and regression estimators.

REFERENCES

1. Abbasi, A.M. and Shahd, M.Y. (2017). Manipulation-based ranked set sampling scheme. *Pakistan Journal of Statistics and Operation Research*, 13(4), 775-798.
2. Ahmed, S. and Shabbir, J. (2019). Extreme-cum-median ranked set sampling. *Brazilian Journal of Probability and Statistics*, 33(1), 24-38.
3. Al-Nasser, A.D. and Mustafa, A.B. (2009). Robust extreme ranked set sampling. *Journal of Statistical Computation and Simulation*, 79(7), 859-867.
4. Al-Omari, A.I. and Bouza, C.N. (2014). Review of ranked set sampling: modifications and applications. *Revista Investigacion Operational*, 35(3), 215-240.
5. Al-Omari, A.I., Kamarulzaman, I. and Syam, M.I. (2011). Investigating the use of stratified percentile ranked set sampling method for estimating the population mean. *Proyecciones (Antofagasta)*, 30(3), 351-368.
6. Al-Omari, A.I. and Raqab, M.Z. (2013). Estimation of the population mean and median using truncation-based ranked set samples. *Journal of Statistical Computation and Simulation*, 83(8), 1453-1471.
7. Al-Saleh, M.F. and Al-Kadiri, M.A. (2000). Double-ranked set sampling. *Statistics and Probability Letters*, 48(2), 205-212.
8. Al-Saleh, M.F. and Al-Omari, A.I. (2002). Multistage ranked set sampling. *Journal of Statistical planning and Inference*, 102(2), 273-286.
9. Bai, Z. and Chen, Z. (2003). On the theory of ranked-set sampling and its ramifications. *Journal of Statistical Planning and Inference*, 109(1-2), 81-99.
10. Chen, Z., Zheng, G., Ghosh, K. and Li, Z. (2005). Linkage disequilibrium mapping of quantitative trait loci by selective genotyping. *Am. J. Hum. Genet*., 77(4), 661-669.
11. David, H.A. and Nagaraja, H.N. (2003). *Order Statistics*, 3rd ed. John Wiley & Sons, Inc. Hoboken, New Jersey.
12. Dell, T.R. and Clutter, J.L. (1972). Ranked set sampling theory with order statistics background. *Biometrics*, 28(2), 545-555.
13. Hossain, S.S. and Muttlak, H.A. (1999). Paired ranked set sampling a more efficient procedure. *Environmetrics*, 10(2), 195-212.
14. Ibrahim, K., Syam, M. and Al-Omari, A.I. (2010). Estimating the population mean using stratified median ranked set sampling. *Applied Mathematical Science*, 47(4), 2341-2354.
15. Jabrah, R., Samawi, H.M., Vogel, R., Rochani, H., Linder, D. and Klibert, J. (2017). Using ranked auxiliary covariate as a more efficient sampling design for ANCOVA model Analysis of a psychological intervention to buttress resilience. *Communications for Statistical Applications and Methods*, 24(3), 241-257.
16. Jemain, A.A. and Al-Omari, A. (2008). Ibrahim K. Some variations of ranked set sampling. *Electronic Journal of Applied Statistical Analysis*, 1(1), 1-15.
17. Khan, Z., Ismail M. and Samawi, H.M. (2020). Mixture ranked set sampling for estimation of population mean and median. *Journal of Statistical Computation and Simulation*, 90(4), 1-13.
18. Khan, Z. and Ismail, M. (2019). Modified Ratio Estimators of population mean based on ranked set sampling. *Pakistan Journal of Statistics and Operation Research*, 15(2), 445-9.
19. Mahdizadeh, M. and Zamanzade, E. (2018). Stratified pair ranked set sampling. *Communications in Statistics-Theory and Methods*, 47(24), 5904-5915.

20. Mahdizadeh, M. and Arghami, N.R. (2012). Quantile estimation using ranked set samples from a population with known mean. *Communications in Statistics: Simulation and Computation*, 41(10), 1872-1881.
21. Mahdizadeh, M. (2015). On entropy-based test of exponentiality in ranked set sampling. *Communications in Statistics: Simulation and Computation*, 44(4), 979-995.
22. Mahdizadeh, M. (2018). On estimating a stress-strength type reliability. *Hacettepe Journal of Mathematics and Statistics*, 47(1), 243-253.
23. Mahdizadeh, M. and Strzalkowska-Kominiak, E. (2017). Resampling based inference for a distribution function using censored ranked set samples. *Computational Statistics*, 32(4), 1285-1308.
24. McIntyre, G.A. (1952). Method for unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research*, 3, 385-90.
25. Muttlak, H.A. (1997). Median ranked set sampling. *Journal of Applied Statistical Sciences*, 6, 245-255.
26. Noor-ul-Amin, M., Tayyab, M. and Hanif, M. (2019). Mean Estimation Using Even Order Ranked Set Sampling. *Punjab University Journal of Mathematics*, 51(1), 91-99.
27. Salehi, M. and Ahmadi, J. (2014). Record ranked set sampling scheme. *Metron*, 72(3), 351-365.
28. Samawai, H.M. and Saeid, L.J. (2004). Stratified extreme ranked set sample with application to ratio estimators. *Journal of Modern Applied Statistical Methods*, 3(1), 117-19.
29. Samawi, H.M. (2011). Varied set size ranked set sampling with applications to mean and ratio estimation. *International Journal of Modelling and Simulation*, 31(1), 6-13.
30. Samawi, H.M., Ahmed, M.S. and Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrical Journal*, 38(5), 577-586.
31. Samawi, H.M. (1996). Stratified ranked set sample. *Pakistan Journal of Statistics*, 12(1), 9-16.
32. Samawi, H.M. and Muttlak, H.A. (1996). Estimation of ratio using rank set sampling. *Biometrical Journal*, 38(6), 753-764.
33. Samawi, H.M. and Al-Sagheer, O.A. (2001). On the estimation of the distribution function using extreme and median ranked set sampling. *Biometrical Journal*, 43(3), 357-373.
34. Strzalkowska-Kominiak, E. and Mahdizadeh, M. (2014). On the Kaplan-Meier estimator based on ranked set samples. *Journal of Statistical Computation and Simulation*, 84(12), 2577-2591.
35. Sevinc, B., Gurler, S. and Cetintav, B. (2018). Partial groups ranked set sampling and mean estimation. *Journal of Statistical Computation and Simulation*, 18(14), 2799-2810.
36. Stokes, L. (1995). Parametric ranked set sampling. *Annals of the Institute of Statistical Mathematics*, 47(3), 465-482.
37. Stokes, S.L. (1977). Ranked set sampling with concomitant variables. *Communications in Statistics-Theory and Methods*, 6(12), 1207-1211.
38. Takahasi, K. and Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, 20(1), 1-31.