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STRATIFIED EXTREME-CUM-MEDIAN RANKED SET SAMPLING

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ABSTRACT

Ranked set sampling is one of the traditional sampling procedures used for cost reduction and obtaining more representative sample selection in surveys. When the problem of heterogeneity occurs in population, use of stratification is recommended in literature for increasing efficiency of estimators. In the presence of heterogeneity and outliers (extreme values), Stratified Median Ranked Set Sampling (StMRSS) and Stratified Extreme Ranked Set Sampling (StERSS) have been suggested by researchers. In this article, we propose a new ranked set sampling procedure to estimate the population mean by combining the aforementioned techniques. The new procedure is named as Stratified Extreme-cum-Median Ranked Set Sampling (StEMRSS). Efficiency comparison of this procedure with its competitor procedures has been made theoretically and conditions of efficiency have been derived. Simulation study comparisons for symmetric as well as asymmetric probability distributions have also been conducted. The results for small and medium sample size show that StEMRSS outperforms its competitor procedures for both perfect as well as imperfect rankings. For large sample size the newly proposed sampling procedure and StMRSS are found to be almost equally efficient. A real-life data set (Andersen, 1993) is also considered to study the empirical significance of the newly proposed sampling procedure. It is observed that the results of real-life application are compatible and conformable to aforementioned Monte Carlo simulation outcomes. The newly proposed technique is highly recommended for heterogeneous and Non-normal populations providing option of flexible rankings either on study or auxiliary variable.

KEYWORDS

Ranked set sampling, Relative efficiency, Stratification, Estimation, Symmetric distribution, Asymmetric distribution, Unbiased, Monte Carlo Simulation.

MSC 2020 Classifications: 62D05, 62F07, 62F10.

1. INTRODUCTION

Ranked Set Sampling (RSS) has become popular as an alternative sampling procedure of Simple Random Sampling (SRS). As it enhances the efficiency of estimators by providing true subset of population as a sample, in which the quantification of variable of interest is costly, time consuming or difficult. Moreover, RSS does not require a full quantification, it only use ranks of the units based on visual inspection or cheaply available auxiliary variables. Mclntyre (1952) introduced ranked set sampling for estimating average pasture yields, whereas its mathematical theory under perfect ranking was developed by Takahashi and Wakimoto (1968). Further, they examined that regardless of ranking error (imperfect ranking), RSS provides unbiased and efficient estimator for population mean as compare to SRS estimator of the same sample size. The error in ranking occurs when visual inspection or auxiliary variables cannot be assigned the correct rank to the observations in the set. Dell and Clutter (1972) as well as David and Levine (1972) reported similar results under imperfect ranking, which means there is a ranking error. Stokes (1972) introduced auxiliary variable in RSS, for ranking purpose to reduce cost of study, when ranking on study variable is expensive. Al-Nasser and Mustafa (2009) introduced robust extreme ranked set sampling for estimating population mean more efficiently. Zamanzade and Mahdizadeh (2017) proposed more efficient proportion estimator with auxiliary variable under ranked set sampling. Al-Omari and Raqab (2013) introduced truncation based ranked set sampling to estimate population mean and median. Hybrid ranked set sampling was introduced as cost effective sampling design for estimating population parameters (Haq et al., 2016). Ahmed and Shabbir (2017) utilized extreme ranked set sampling and median ranked set sampling to estimate population mean when non response occur. Ranked set sampling, median ranked set sampling, extreme ranked set sampling, double ranked set sampling, double median ranked set sampling and double extreme ranked set sampling schemes have been utilized for linear profile monitoring under EWMA control charts (Riaz et al., 2017). When exact measurement of observation was difficult, (Sevinç et al., 2018) introduced partial groups ranked set sampling for obtaining a more flexible sample. Robust extreme double ranked set sampling was suggested by (Hashemi Majd and Saba, 2018) for estimating of population mean. Mahdizadeh and Zamanzade (Mahdizadeh and Zamanzade, 2018) proposed kernel-based estimator of a reliability function in ranked set sampling. A class of ratio type estimators was proposed by (Sohail et al., 2018) for estimating population mean for imputing the missing values under ranked set sampling. Mahdizadeh and Zamanzade (2018) constructed a confidence interval for the reliability parameters by utilizing ranked set sampling. Even order ranked set sampling has been introduced by Noor-ul-Amin et al. (2919) for estimation of population mean. Zamanzade and Mahdizadeh (2018) proposed estimator for population proportion of air quality monitoring under pair ranked set sampling. When the ordinary RSS cannot be fully conducted in all cycles of the experiment, mixture ranked set sampling was suggested by Khan et al. (2020). Mahdizadeh and Zamanzade (2020) developed a nonparametric estimator for symmetric distribution function under multistage ranked set sampling.

1.1 Ranked Set Sampling

To select a ranked set sample using set size m, the researcher identifies m simple random samples of size m from the target population. Then he/she ranks each sample of size m in an increasing scale. The ranking is performed using a low-cost method such as visual examination or personal judgment which does not involve actual measurement of sample unit. The sample unit with judgment rank $i\left(i=1,2,\ldots,m\right)$ from the ith sample is selected for actual quantification. The entire procedure may be reiterated r times (cycles) to obtain a ranked set sample of size $n = rm$. It is worth mentioning that the set size m should be kept small to ease the essential informal rankings, see (Zamanzade and Mahdizadeh, 2020). The RSS estimator for population mean of variable of interest Y is given by

$$\hat{\mu}_{y,RSS} = \frac{1}{mr} \sum_{j=1}^{r} \sum_{i=1}^{m} Y_{i(i)j}. \tag{1}$$

$\hat{\mu}_{y,RSS}$ is an unbiased estimator of μ_y. The variance of $\hat{\mu}_{y,RSS}$ is

$$\operatorname{var}\left(\hat{\mu}_{y,RSS}\right) = \frac{1}{mr}\left[\sigma^2 - \frac{1}{m}\sum_{i=1}^{m} \varphi_{(i)}^2\right], \tag{2}$$

where, $\varphi_{(i)}^2 = \left(\mu_{y(i)} - \mu_y\right)^2$.

Ranked set sampling is revised to new structures in order to get more appropriate and efficient approaches, see Al-Saleh and Al-Omari (2002), Al-Odat and Al-Saleh (2001), Al-Omari and Haq (2019), Arnab et al. (2019), Haq et al. (2014), Hossain and Muttlak (1999), Iqbal et al. (2020), Muttlak (2003), Sevinç et al. (2018), Samawi, et al. (1996) and Yang et al. (2020)..

1.2 Stratified Ranked Set Sampling

Samawi (1996) introduced StRSS for obtaining unbiased and efficient estimates of population mean. In StRSS, we first divide our population of size N into L mutually exclusive and collectively exhaustive groups (strata) of $N_1, N_2, N_3, ..., N_L$ units. Such that $N_1 + N_2 + N_3 + ... + N_L = N$. After stratification, a sample is drawn from each stratum. The sample sizes within the strata are denoted by $n_1, n_2, n_3, ..., n_L$ respectively and over all sample size is $n = \sum_{h=1}^{L} n_h = \sum_{h=1}^{L} rm_h = rm$, where r is number of cycles, m_h is number of selected units in each stratum and cycle, and m is number of selected units in each cycle. If ranked set sample is taken from each stratum, the whole process is described as StRSS. The unbiased StRSS estimator for population mean of variable of interest Y and its variance are given by

$$\hat{\mu}_{y,StRSS} = \sum_{h=1}^{L} W_h \frac{\sum_{j=1}^{r}\sum_{i=1}^{m_h} Y_{i(i)jh}}{rm_h}, \tag{3}$$

$$\operatorname{var}\left(\hat{\mu}_{y,StRSS}\right) = \sum_{h=1}^{L} W_h^2 \frac{1}{rm_h}\left[\sigma_h^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} \varphi_{(i)h}^2\right]. \tag{4}$$

where, $W_h = \frac{N_h}{N}$ and $\varphi_{(i)h}^2 = \left(\mu_{y(i)h} - \mu_{(y)h}\right)^2$. For more details, see Samawi (1996).

1.3 Stratified Extreme Ranked Set Sampling

Samawi and Saeid (2004) suggested following procedure for StERSS, suppose our variable of interest Y belongs to population of size N. We will divide our population into L mutually exclusive and collectively exhaustive strata with size $N_1, N_2, N_3, ..., N_L$. Further we will proceed as follows:

(1) Select m_h^2 units from h^{th} stratum and divide them into m_h sets each of size m_h. Rank the units within each set in increasing scale.

(2) When ‘ m_h ’ is even, we start the selection of smallest ranked unit from each of first $m_h/2$ sets and largest unit from each of next $m_h/2$ sets. When ‘ m_h ’ is odd, we start the selection of smallest unit from each of first $\{(m_h-1)/2\}$ sets, largest unit from each of next $\{(m_h-1)/2\}$ sets and $\{(m_h+1)/2\}^{th}$ unit from $m_h{}^{th}$ set.

(3) Repeat steps (1) and (2) in each stratum to obtain $m_1+m_2+\ldots+m_L=m$ units.

(4) Repeat steps (1), (2) and (3) r times to obtain sample of size $n=rm$.

The sample means under StERSS for even (E) and odd (O) sample sizes are respectively given by

$$\hat{\mu}_{y,StERSS}^{(E)}=\sum_{h=1}^{L}W_h\frac{1}{rm_h}\left[\sum_{j=1}^{r}\left\{\sum_{i=1}^{m_h/2}Y_{i(1)jh}+\sum_{i=\frac{m_h}{2}+1}^{m_h}Y_{i(m_h)jh}\right\}\right], \quad (5)$$

and

$$\hat{\mu}_{y,StERSS}^{(O)}=\sum_{h=1}^{L}W_h\frac{1}{rm_h}\left[\sum_{j=1}^{r}\left\{\sum_{i=1}^{(m_h-1)/2}Y_{i(1)jh}+\sum_{i=\frac{(m_h-1)}{2}+1}^{m_h-1}Y_{i(m_h)jh}+Y_{m_h\left(\frac{m_h+1}{2}\right)jh}\right\}\right]. \quad (6)$$

Both $\hat{\mu}_{y,StERSS}^{(E)}$ and $\hat{\mu}_{y,StERSS}^{(O)}$ are unbiased estimators of μ_y when underlying distribution is symmetric about mean. The variances of $\hat{\mu}_{y,StERSS}^{(E)}$ and $\hat{\mu}_{y,StERSS}^{(O)}$, respectively, are

$$\mathrm{var}\left(\hat{\mu}_{y,StERSS}^{(E)}\right)=\sum_{h=1}^{L}W_h^2\frac{1}{rm_h}\left[\sigma_h^2-\frac{1}{2}\left\{\varphi_{(1)h}^2+\varphi_{(m_h)h}^2\right\}\right], \quad (7)$$

and

$$\mathrm{var}\left(\hat{\mu}_{y,StERSS}^{(O)}\right)=\sum_{h=1}^{L}W_h^2\frac{1}{rm_h^2}\left[(m_h-1)\left\{\sigma_h^2-\frac{1}{2}\left(\varphi_{(1)h}^2+\varphi_{(m_h)h}^2\right)\right\}+\left(\sigma_h^2-\varphi_{\left(\frac{m_h+1}{2}\right)h}^2\right)\right]. \quad (8)$$

where, $\varphi_{(1)h}^2=\left(\mu_{y(1)h}-\mu_{(y)h}\right)^2$, $\varphi_{(m_h)h}^2=\left(\mu_{y(m_h)h}-\mu_{(y)h}\right)^2$

and $\varphi^2_{\left(\frac{m_h+1}{2}\right)h} = \left(\mu_{y\left(\frac{m_h+1}{2}\right)h} - \mu_{(y)h}\right)^2$. For more details, see Samawi and Saeid (2004).

1.4 Stratified Median Ranked Set Sampling

Ibrahim et al. (2010) suggest StMRSS procedure for estimating population mean, this procedure is as follows: suppose our variable of interest Y belongs to population of size N. We will divide our population into L mutually exclusive and collectively exhaustive strata with size $N_1, N_2, N_3, ..., N_L$. Further we will proceed as follows:

(1) Select m_h^2 units from h^{th} stratum and divide them into m_h sets each of size m_h. Rank the units within each set in increasing scale.

(2) When 'm_h' is even, we start the selection of $(m_h/2)^{th}$ ranked unit from each of first $m_h/2$ sets and $\{(m_h+2)/2\}^{th}$ ranked unit from each of next $m_h/2$ sets. When 'm_h' is odd, we select $\{(m_h+1)/2\}^{th}$ ranked unit from all sets.

(3) Repeat steps (1) and (2) in each stratum to obtain $m_1 + m_2 + \ldots + m_L = m$ units.

(4) Repeat steps (1), (2) and (3) r times to obtain sample of size $n = rm$.

The sample means under StMRSS for even (E) and odd (O) sample sizes are respectively given by

$$\widehat{\mu}^{(E)}_{y,StMRSS} = \sum_{h=1}^{L} W_h \frac{1}{rm_h}\left[\sum_{j=1}^{r}\left\{\sum_{i=1}^{m_h/2} Y_{i\left(\frac{m_h}{2}\right)jh} + \sum_{i=\frac{m_h}{2}+1}^{m_h} Y_{i\left(\frac{m_h+2}{2}\right)jh}\right\}\right], \tag{9}$$

and

$$\widehat{\mu}^{(O)}_{y,StMRSS} = \sum_{h=1}^{L} W_h \frac{1}{rm_h}\left[\sum_{j=1}^{r}\left\{\sum_{i=1}^{m_h} Y_{i\left(\frac{m_h+1}{2}\right)jh}\right\}\right]. \tag{10}$$

Both $\widehat{\mu}^{(E)}_{y,StMRSS}$ and $\widehat{\mu}^{(O)}_{y,StMRSS}$ are unbiased estimators of μ_y when underlying distribution is symmetric about mean. The variances of $\widehat{\mu}^{(E)}_{y,StMRSS}$ and $\widehat{\mu}^{(O)}_{y,StMRSS}$, respectively, are

$$\mathrm{var}\left(\widehat{\mu}^{(E)}_{y,StMRSS}\right) = \sum_{h=1}^{L} W_h^2 \frac{1}{rm_h}\left[\sigma_h^2 - \frac{1}{2}\left\{\varphi^2_{\left(\frac{m_h}{2}\right)h} + \varphi^2_{\left(\frac{m_h+2}{2}\right)h}\right\}\right], \tag{11}$$

and

$$\operatorname{var}\left(\hat{\mu}_{y,StMRSS}^{(O)}\right)=\sum_{h=1}^{L} W_h^2 \frac{1}{rm_h}\left[\sigma_h^2-\varphi_{\left(\frac{m_h+1}{2}\right)h}^2\right]. \quad (12)$$

where, $\varphi_{\left(\frac{m_h}{2}\right)h}^2=\left(\mu_{y\left(\frac{m_h}{2}\right)h}-\mu_{(y)h}\right)^2$, $\varphi_{\left(\frac{m_h+2}{2}\right)h}^2=\left(\mu_{y\left(\frac{m_h+2}{2}\right)h}-\mu_{(y)h}\right)^2$

and $\varphi_{\left(\frac{m_h+1}{2}\right)h}^2=\left(\mu_{y\left(\frac{m_h+1}{2}\right)h}-\mu_{(y)h}\right)^2$. For more details, see Ibrahim et al. (2010).

In this article, we suggest a new sampling procedure named stratified extreme-cum-median ranked set sampling (StEMRSS). This procedure will provide more representative sample and efficient estimate of population mean than its competitors.

2. PROPOSED SAMPLING SCHEME

Taking motivation from Samawi and Saeid (2004), Ibrahim et al. (2010) and Ahmed and Shabbir (2019), we propose a new sampling procedure named as stratified extreme-cum-median ranked set sampling (StEMRSS). This procedure is the combination of StERSS and StMRSS. Therefore, the sample selected in this procedure will be more representative of population as compare to above mentioned procedures. The proposed sampling procedure will provide more efficient and representative sample when data contains heterogeneity and outliers. The general procedure of StEMRSS is as follows: suppose our variable of interest Y belongs to population of size N. We will divide our population into L mutually exclusive and collectively exhaustive strata with size $N_1, N_2, N_3, ..., N_L$. Further we will proceed as follows:

(1) Select $2m_h$ independent random sets from each stratum, each of size $2m_h$. Rank the units within each set in increasing scale by any economical method.

(2) When m_h is even, select lowest rank unit from each of first $m_h/2$ sets, and highest rank unit from each of next $m_h/2$ sets, it completes StERSS procedure. Further, select $(m_h)^{th}$ unit from each of (m_h+1) to $(3m_h/2)^{th}$ sets and $(m_h+1)^{th}$ unit from each of last $m_h/2$ sets to complete StEMRSS procedure. When m_h is odd, select lowest rank unit from each of first $\{(m_h-1)/2\}$ sets, highest rank unit from each of next $\{(m_h-1)/2\}$ sets, and $(m_h)^{th}$ unit from $(m_h)^{th}$ set, it completes StERSS procedure. Further, select $(m_h+1)^{th}$ unit from $(m_h+1)^{th}$ set, $(m_h)^{th}$ unit from each of $(m_h+2)^{th}$ to $\{((m_h+1)/2)+m_h\}^{th}$ sets and $(m_h+1)^{th}$ unit from each of last $\{(m_h-1)/2\}$ sets to complete StEMRSS procedure.

(3) Repeat steps (1) and (2) in each stratum to obtain $2m_1 + 2m_2 + \ldots + 2m_L = 2m$ units.

(4) Repeat steps (1), (2) and (3) r times to obtain sample of size $n = 2rm$.

The newly proposed selection procedure can be well understood from the following two layouts:

2.1a. Layout when m is even:

To explain StEMRSS with even size, we take $m_h = 4,\ h = 2,\ r = 1$ and $n = 2rm = 16$ for simplicity. The sampling layout is given below

$$
\textit{Stratum 1},\ \begin{bmatrix}
\underline{Y_{1(1)1}} & Y_{1(2)1} & Y_{1(3)1} & Y_{1(4)1} & Y_{1(5)1} & Y_{1(6)1} & Y_{1(7)1} & Y_{1(8)1} \\
\underline{Y_{2(1)1}} & Y_{2(2)1} & Y_{2(3)1} & Y_{2(4)1} & Y_{2(5)1} & Y_{2(6)1} & Y_{2(7)1} & Y_{2(8)1} \\
Y_{3(1)1} & Y_{3(2)1} & Y_{3(3)1} & Y_{3(4)1} & Y_{3(5)1} & Y_{3(6)1} & Y_{3(7)1} & \underline{Y_{3(8)1}} \\
Y_{4(1)1} & Y_{4(2)1} & Y_{4(3)1} & Y_{4(4)1} & Y_{4(5)1} & Y_{4(6)1} & Y_{4(7)1} & \underline{Y_{4(8)1}} \\
Y_{5(1)1} & Y_{5(2)1} & Y_{5(3)1} & \underline{Y_{5(4)1}} & Y_{5(5)1} & Y_{5(6)1} & Y_{5(7)1} & Y_{5(8)1} \\
Y_{6(1)1} & Y_{6(2)1} & Y_{6(3)1} & \underline{Y_{6(4)1}} & Y_{6(5)1} & Y_{6(6)1} & Y_{6(7)1} & Y_{6(8)1} \\
Y_{7(1)1} & Y_{7(2)1} & Y_{7(3)1} & Y_{7(4)1} & \underline{Y_{7(5)1}} & Y_{7(6)1} & Y_{7(7)1} & Y_{7(8)1} \\
Y_{8(1)1} & Y_{8(2)1} & Y_{8(3)1} & Y_{8(4)1} & \underline{Y_{8(5)1}} & Y_{8(6)1} & Y_{8(7)1} & Y_{8(8)1}
\end{bmatrix}
$$

$$
\textit{Stratum 2},\ \begin{bmatrix}
\underline{Y_{1(1)2}} & Y_{1(2)2} & Y_{1(3)2} & Y_{1(4)2} & Y_{1(5)2} & Y_{1(6)2} & Y_{1(7)2} & Y_{1(8)2} \\
\underline{Y_{2(1)2}} & Y_{2(2)2} & Y_{2(3)2} & Y_{2(4)2} & Y_{2(5)2} & Y_{2(6)2} & Y_{2(7)2} & Y_{2(8)2} \\
Y_{3(1)2} & Y_{3(2)2} & Y_{3(3)2} & Y_{3(4)2} & Y_{3(5)2} & Y_{3(6)2} & Y_{3(7)2} & \underline{Y_{3(8)2}} \\
Y_{4(1)2} & Y_{4(2)2} & Y_{4(3)2} & Y_{4(4)2} & Y_{4(5)2} & Y_{4(6)2} & Y_{4(7)2} & \underline{Y_{4(8)2}} \\
Y_{5(1)2} & Y_{5(2)2} & Y_{5(3)2} & \underline{Y_{5(4)2}} & Y_{5(5)2} & Y_{5(6)2} & Y_{5(7)2} & Y_{5(8)2} \\
Y_{6(1)2} & Y_{6(2)2} & Y_{6(3)2} & \underline{Y_{6(4)2}} & Y_{6(5)2} & Y_{6(6)2} & Y_{6(7)2} & Y_{6(8)2} \\
Y_{7(1)2} & Y_{7(2)2} & Y_{7(3)2} & Y_{7(4)2} & \underline{Y_{7(5)2}} & Y_{7(6)2} & Y_{7(7)2} & Y_{7(8)2} \\
Y_{8(1)2} & Y_{8(2)2} & Y_{8(3)2} & Y_{8(4)2} & \underline{Y_{8(5)2}} & Y_{8(6)2} & Y_{8(7)2} & Y_{8(8)2}
\end{bmatrix}
$$

The layout 2.1a shows 2 strata each having 8 ranked sets of size 8. From each set of both strata the underlined units are selected by utilizing the StEMRSS procedure.

2.1b. Layout when m is odd:

To explain StEMRSS with odd size, we take $m_h = 5,\ h = 2,\ r = 1$ and $n = 2rm = 20$ for simplicity. The sampling layout is given below

Stratum 1,

$$\begin{bmatrix}
\underline{Y_{1(1)1}} & Y_{1(2)1} & Y_{1(3)1} & Y_{1(4)1} & Y_{1(5)1} & Y_{1(6)1} & Y_{1(7)1} & Y_{1(8)1} & Y_{1(9)1} & Y_{1(10)1} \\
\underline{Y_{2(1)1}} & Y_{2(2)1} & Y_{2(3)1} & Y_{2(4)1} & Y_{2(5)1} & Y_{2(6)1} & Y_{2(7)1} & Y_{2(8)1} & Y_{2(9)1} & Y_{2(10)1} \\
Y_{3(1)1} & Y_{3(2)1} & Y_{3(3)1} & Y_{3(4)1} & Y_{3(5)1} & Y_{3(6)1} & Y_{3(7)1} & Y_{3(8)1} & Y_{3(9)1} & \underline{Y_{3(10)1}} \\
Y_{4(1)1} & Y_{4(2)1} & Y_{4(3)1} & Y_{4(4)1} & Y_{4(5)1} & Y_{4(6)1} & Y_{4(7)1} & Y_{4(8)1} & Y_{4(9)1} & \underline{Y_{4(10)1}} \\
Y_{5(1)1} & Y_{5(2)1} & Y_{5(3)1} & Y_{5(4)1} & \underline{Y_{5(5)1}} & Y_{5(6)1} & Y_{5(7)1} & Y_{5(8)1} & Y_{5(9)1} & Y_{5(10)1} \\
Y_{6(1)1} & Y_{6(2)1} & Y_{6(3)1} & Y_{6(4)1} & Y_{6(5)1} & \underline{Y_{6(6)1}} & Y_{6(7)1} & Y_{6(8)1} & Y_{6(9)1} & Y_{6(10)1} \\
Y_{7(1)1} & Y_{7(2)1} & Y_{7(3)1} & Y_{7(4)1} & \underline{Y_{7(5)1}} & Y_{7(6)1} & Y_{7(7)1} & Y_{7(8)1} & Y_{7(9)1} & Y_{7(10)1} \\
Y_{8(1)1} & Y_{8(2)1} & Y_{8(3)1} & Y_{8(4)1} & \underline{Y_{8(5)1}} & Y_{8(6)1} & Y_{8(7)1} & Y_{8(8)1} & Y_{8(9)1} & Y_{8(10)1} \\
Y_{9(1)1} & Y_{9(2)1} & Y_{9(3)1} & Y_{9(4)1} & Y_{9(5)1} & \underline{Y_{9(6)1}} & Y_{9(7)1} & Y_{9(8)1} & Y_{9(9)1} & Y_{9(10)1} \\
Y_{10(1)1} & Y_{10(2)1} & Y_{10(3)1} & Y_{10(4)1} & Y_{10(5)1} & \underline{Y_{10(6)1}} & Y_{10(7)1} & Y_{10(8)1} & Y_{10(9)1} & Y_{10(10)1}
\end{bmatrix}$$

Stratum 2,

$$\begin{bmatrix}
\underline{Y_{1(1)2}} & Y_{1(2)2} & Y_{1(3)2} & Y_{1(4)2} & Y_{1(5)2} & Y_{1(6)2} & Y_{1(7)2} & Y_{1(8)2} & Y_{1(9)2} & Y_{1(10)2} \\
\underline{Y_{2(1)2}} & Y_{2(2)2} & Y_{2(3)2} & Y_{2(4)2} & Y_{2(5)2} & Y_{2(6)2} & Y_{2(7)2} & Y_{2(8)2} & Y_{2(9)2} & Y_{2(10)2} \\
Y_{3(1)2} & Y_{3(2)2} & Y_{3(3)2} & Y_{3(4)2} & Y_{3(5)2} & Y_{3(6)2} & Y_{3(7)2} & Y_{3(8)2} & Y_{3(9)2} & \underline{Y_{3(10)2}} \\
Y_{4(1)2} & Y_{4(2)2} & Y_{4(3)2} & Y_{4(4)2} & Y_{4(5)2} & Y_{4(6)2} & Y_{4(7)2} & Y_{4(8)2} & Y_{4(9)2} & \underline{Y_{4(10)2}} \\
Y_{5(1)2} & Y_{5(2)2} & Y_{5(3)2} & Y_{5(4)2} & \underline{Y_{5(5)2}} & Y_{5(6)2} & Y_{5(7)2} & Y_{5(8)2} & Y_{5(9)2} & Y_{5(10)2} \\
Y_{6(1)2} & Y_{6(2)2} & Y_{6(3)2} & Y_{6(4)2} & Y_{6(5)2} & \underline{Y_{6(6)2}} & Y_{6(7)2} & Y_{6(8)2} & Y_{6(9)2} & Y_{6(10)2} \\
Y_{7(1)2} & Y_{7(2)2} & Y_{7(3)2} & Y_{7(4)2} & \underline{Y_{7(5)2}} & Y_{7(6)2} & Y_{7(7)2} & Y_{7(8)2} & Y_{7(9)2} & Y_{7(10)2} \\
Y_{8(1)2} & Y_{8(2)2} & Y_{8(3)2} & Y_{8(4)2} & \underline{Y_{8(5)2}} & Y_{8(6)2} & Y_{8(7)2} & Y_{8(8)2} & Y_{8(9)2} & Y_{8(10)2} \\
Y_{9(1)2} & Y_{9(2)2} & Y_{9(3)2} & Y_{9(4)2} & Y_{9(5)2} & \underline{Y_{9(6)2}} & Y_{9(7)2} & Y_{9(8)2} & Y_{9(9)2} & Y_{9(10)2} \\
Y_{10(1)2} & Y_{10(2)2} & Y_{10(3)2} & Y_{10(4)2} & Y_{10(5)2} & \underline{Y_{10(6)2}} & Y_{10(7)2} & Y_{10(8)2} & Y_{10(9)2} & Y_{10(10)2}
\end{bmatrix}$$

The layout 2.1b shows 2 strata each having 10 ranked sets of size 10. From each set of both strata the underlined units are selected by utilizing the StEMRSS procedure.

The mean estimators under StEMRSS for even (E) and odd (O) sample sizes are given by

$$\hat{\mu}_{y,StEMRSS}^{(E)} = \sum_{h=1}^{L} W_h \frac{1}{2m_h r} \sum_{j=1}^{r} \left[\begin{array}{l} \left(\sum_{i=1}^{m_h/2} Y_{i(1)jh} + \sum_{i=\frac{m_h}{2}+1}^{m_h} Y_{i(2m_h)jh} \right) \\ + \left(\sum_{i=m_h+1}^{\frac{3m_h}{2}} Y_{i(m_h)jh} + \sum_{i=\frac{3m_h}{2}+1}^{2m_h} Y_{i(m_h+1)jh} \right) \end{array} \right], \quad (13)$$

$$\hat{\mu}_{y,StEMRSS}^{(O)} = \sum_{h=1}^{L} W_h \frac{1}{2m_h r} \sum_{j=1}^{r} \left[\begin{array}{l} \left(\sum_{i=1}^{(m_h-1)/2} Y_{i(1)jh} + \sum_{i=\{(m_h-1)/2\}+1}^{m_h-1} Y_{i(2m_h)jh} + Y_{m_h(m_h)jh} \right) \\ + \left(Y_{m_h+1(m_h+1)jh} + \sum_{i=m_h+2}^{(3m_h+1)/2} Y_{i(m_h)jh} + \sum_{i=\frac{3(m_h+1)}{2}}^{2m_h} Y_{i(m_h+1)jh} \right) \end{array} \right]. \quad (14)$$

With

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(E)}\right) = \sum_{h=1}^{L} W_h \frac{1}{2m_h} \left[\begin{array}{l} \left(\frac{m_h}{2} \varphi_{(1)h} + \frac{m_h}{2} \varphi_{(2m_h)h} \right) \\ + \left(\frac{m_h}{2} \varphi_{(m_h)h} + \frac{m_h}{2} \varphi_{(m_h+1)h} \right) \end{array} \right],$$

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(O)}\right) = \sum_{h=1}^{L} W_h \frac{1}{2m_h} \left[\begin{array}{l} \left(\frac{m_h-1}{2} \varphi_{(1)h} + \frac{m_h-1}{2} \varphi_{(2m_h)h} + \varphi_{(m_h)h} \right) \\ + \left(\varphi_{(m_h+1)h} + \frac{m_h-1}{2} \varphi_{(m_h)h} + \frac{m_h-1}{2} \varphi_{(m_h+1)h} \right) \end{array} \right].$$

Simplifying and rewriting the bias of estimators as

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(E)}\right) = \sum_{h=1}^{L} W_h \frac{1}{2} \left[\left(\varphi_{(1)h} + \varphi_{(2m_h)h} \right) + \left(\varphi_{(m_h)h} + \varphi_{(m_h+1)h} \right) \right], \quad (15)$$

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(O)}\right) = \sum_{h=1}^{L} W_h \frac{1}{2m_h} \left[\frac{m_h-1}{2} \left(\varphi_{(1)h} + \varphi_{(2m_h)h} \right) + \frac{m_h+1}{2} \left(\varphi_{(m_h)h} + \varphi_{(m_h+1)h} \right) \right]. \quad (16)$$

where, $\varphi_{(i)}^2 = \left(\mu_{y(i)h} - \mu_{(y)}\right)^2$ and $\mu_{y(i)h}$ represents population mean of i^{th} order statistic in h^{th} stratum for $\left(i = 1, 2, 3, \ldots, 2m_h\right)$. Further, we derive the variances of our estimators by assuming that Y follows symmetric distribution about mean $\left(\mu_{(y)}\right)$.

$$\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StEMRSS}\right)=\sum_{h=1}^{L}W_h^2\frac{1}{\left(2rm_h\right)^2}\sum_{j=1}^{r}\left[\begin{array}{r}\left(\sum_{i=1}^{m_h/2}\sigma^2_{(1)h}+\sum_{i=\frac{m_h}{2}+1}^{m_h}\sigma^2_{(2m_h)h}\right)\\+\left(\sum_{i=m_h+1}^{\frac{3m_h}{2}}\sigma^2_{(m_h)h}+\sum_{i=\frac{3m_h}{2}+1}^{2m_h}\sigma^2_{(m_h+1)h}\right)\end{array}\right],$$

$$=\sum_{h=1}^{L}W_h^2\frac{1}{\left(2rm_h\right)^2}\sum_{j=1}^{r}\left[\begin{array}{l}\left(\sum_{i=1}^{m_h/2}\left(\sigma_h^2-\varphi^2_{(1)h}\right)+\sum_{i=\frac{m_h}{2}+1}^{m_h}\left(\sigma_h^2-\varphi^2_{(2m_h)h}\right)\right)\\+\left(\sum_{i=m_h+1}^{\frac{3m_h}{2}}\left(\sigma_h^2-\varphi^2_{(m_h)h}\right)+\sum_{i=\frac{3m_h}{2}+1}^{2m_h}\left(\sigma_h^2-\varphi^2_{(m_h+1)h}\right)\right)\end{array}\right],$$

$$\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StEMRSS}\right)=\sum_{h=1}^{L}W_h^2\left[\frac{\sigma_h^2}{2rm_h}-\frac{1}{4rm_h^2}\left\{\begin{array}{r}\frac{m_h}{2}\left(\varphi^2_{(1)h}+\varphi^2_{(2m_h)h}\right)\\+\frac{m_h}{2}\left(\varphi^2_{(m_h)h}+\varphi^2_{(m_h+1)h}\right)\end{array}\right\}\right]. \qquad (17)$$

For odd sample size, the variance will be

$$\operatorname{var}\left(\hat{\mu}^{(O)}_{y,StEMRSS}\right)=\sum_{h=1}^{L}W_h^2\frac{1}{\left(2rm_h\right)^2}\left[\begin{array}{l}\left(\sum_{i=1}^{\frac{m_h-1}{2}}\sigma^2_{(1)h}+\sum_{i=\frac{m_h-1}{2}+1}^{m_h-1}\sigma^2_{(2m_h)h}+\sigma^2_{(m_h)h}\right)\\+\left(\sigma^2_{(m_h+1)h}+\sum_{i=m_h+2}^{\frac{3m_h+1}{2}}\sigma^2_{(m_h)h}+\sum_{i=\frac{3(m_h+1)}{2}}^{2m_h}\sigma^2_{(m_h+1)h}\right)\end{array}\right],$$

$$=\sum_{h=1}^{L}W_h^2\frac{1}{\left(2rm_h\right)^2}\left[\begin{array}{l}\left(\sum_{i=1}^{\frac{m_h-1}{2}}\left(\sigma_h^2-\varphi^2_{(1)h}\right)+\sum_{i=\frac{m_h-1}{2}+1}^{m_h-1}\left(\sigma_h^2-\varphi^2_{(2m_h)h}\right)+\left(\sigma_h^2-\varphi^2_{(m_h)h}\right)\right)\\+\left(\left(\sigma_h^2-\varphi^2_{(m_h+1)h}\right)+\sum_{i=m_h+2}^{\frac{3m_h+1}{2}}\left(\sigma_h^2-\varphi^2_{(m_h)h}\right)+\sum_{i=\frac{3(m_h+1)}{2}}^{2m_h}\left(\sigma_h^2-\varphi^2_{(m_h+1)h}\right)\right)\end{array}\right],$$

$$\text{var}\left(\hat{\mu}_{y,StEMRSS}^{(O)}\right)=\sum_{h=1}^{L}W_h^2\left[\frac{\sigma_h^2}{2rm_h}-\frac{1}{4rm_h^2}\left\{\begin{matrix}\frac{m_h-1}{2}\left(\varphi_{(1)h}^2+\varphi_{(2m_h)h}^2\right)\\+\frac{m_h+1}{2}\left(\varphi_{(m_h)h}^2+\varphi_{(m_h+1)h}^2\right)\end{matrix}\right\}\right]. \quad (18)$$

The following theorem is stated to discuss the properties of mean estimator under new sampling procedure.

Theorem 2.1:

(i) $Bias\left(\hat{\mu}_{y,StEMRSS}^{(K)}\right)=0$ for symmetric distributions, K = E, O.

(ii) $\text{var}\left(\hat{\mu}_{y,StERSS}^{(K)}\right)\geq \text{var}\left(\hat{\mu}_{y,StEMRSS}^{(K)}\right)$.

(iii) $\text{var}\left(\hat{\mu}_{y,StMRSS}^{(K)}\right)\geq \text{var}\left(\hat{\mu}_{y,StEMRSS}^{(K)}\right)$.

Proof:

(i) Bias of $\hat{\mu}_{y,StEMRSS}^{(K)}$ for $K=E,O$ are given in (15) and (16). If study variable Y follows symmetric distribution then, $\varphi_{(m_h+1)h}=-\varphi_{(m_h)h}$ and $\varphi_{(2m_h)h}=-\varphi_{(1)h}$, see Ahmed and Shabbir (2019). By Putting this in (15), We have

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(E)}\right)=\sum_{h=1}^{L}W_h\frac{1}{2}\left[\left(\varphi_{(1)h}-\varphi_{(1)h}\right)+\left(\varphi_{(m_h)h}-\varphi_{(m_h)h}\right)\right],$$

$$Bias\left(\hat{\mu}_{y,StEMRSS}^{(E)}\right)=0. \quad (19)$$

It shows that when study variable Y follows symmetric distribution, $\hat{\mu}_{y,StEMRSS}^{(E)}$ will be unbiased estimator of population mean $\mu_{(y)}$. Result in (19) can be proved for odd sample size as well.

(ii) From (7) and (17), We see that

$$\text{var}\left(\hat{\mu}_{y,StERSS}^{(E)}\right)-\text{var}\left(\hat{\mu}_{y,StEMRSS}^{(E)}\right)\geq 0,$$

$$\sum_{h=1}^{L}W_h^2\frac{1}{rm_h}\left[\sigma_h^2-\frac{1}{2}\left\{\varphi_{(1)h}^2+\varphi_{(m_h)h}^2\right\}\right]$$
$$-\sum_{h=1}^{L}W_h^2\left[\frac{\sigma_h^2}{2rm_h}-\frac{1}{4rm_h^2}\left\{\begin{matrix}\frac{m_h}{2}\left(\varphi_{(1)h}^2+\varphi_{(2m_h)h}^2\right)\\+\frac{m_h}{2}\left(\varphi_{(m_h)h}^2+\varphi_{(m_h+1)h}^2\right)\end{matrix}\right\}\right]\geq 0.$$

On simplification for equal sample size, we have

$$\left(\varphi^2_{(m_h)h}+\varphi^2_{(m_h+1)h}\right)\geq\left(\varphi^2_{(1)h}+\varphi^2_{(2m_h)h}\right). \tag{20}$$

Therefore, we can say that when inequality of (20) satisfied, then $\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StERSS}\right)\geq\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StEMRSS}\right)$. Result in (20) can be proved easily for odd sample size as well.

(iii) From (11) and (17). We see that

$$\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StMRSS}\right)-\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StEMRSS}\right)\geq 0,$$

$$\sum_{h=1}^{L}W_h^2\frac{1}{rm_h}\left[\sigma_h^2-\frac{1}{2}\left\{\varphi^2_{\left(\frac{m_h}{2}\right)h}+\varphi^2_{\left(\frac{m_h+2}{2}\right)h}\right\}\right]$$

$$-\sum_{h=1}^{L}W_h^2\left[\frac{\sigma_h^2}{2rm_h}-\frac{1}{4rm_h^2}\left\{\begin{array}{l}\frac{m_h}{2}\left(\varphi^2_{(1)h}+\varphi^2_{(2m_h)h}\right)\\+\frac{m_h}{2}\left(\varphi^2_{(m_h)h}+\varphi^2_{(m_h+1)h}\right)\end{array}\right\}\right]\geq 0.$$

On simplification for equal sample size, we have

$$\left(\varphi^2_{(1)h}+\varphi^2_{(2m_h)h}\right)\geq\left(\varphi^2_{(m_h)h}+\varphi^2_{(m_h+1)h}\right). \tag{21}$$

Therefore, we can say that when inequality of (21) satisfied, then $\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StMRSS}\right)\geq\operatorname{var}\left(\hat{\mu}^{(E)}_{y,StEMRSS}\right)$. Result in (21) can be proved easily for odd sample size as well.

3. SIMULATION STUDY

In this section, we conducted a Monte Carlo Simulation for the efficiency comparison of sample mean based on SRS, RSS, StRSS, StERSS, StMRSS and StEMRSS. Relative efficiency (RE) is used as performance criterion for estimators. Monte Carlo Simulation has been carried out through following steps:

(1) Generating a hypothetical population of variable X (Concomitant variable) of size 1000 from symmetric distributions (Normal (5, 1) and Uniform (0, 1)) and asymmetric distributions (Gamma (4, 3) and Weibull (1.5, 5)).

(2) Study variable Y is computed using following regression model

$$Y=X+e.$$

where 'e' is normally distributed error term having mean zero and variance one.

(3) Number of iteration is one million.

(4) The performance of estimators has been computed by taking number of cycles $r=5$, set size $m=$ 3, 4, 5, 6, 7, 8, 9 and 10, and number of strata $h=2$.

(5) Both perfect (ranking with respect to study variable Y) and imperfect (ranking with respect to concomitant variable X) rankings have been utilized for comparison purpose.

Table 1
REs of Mean Estimators with respect to SRS under Perfect Ranking for Symmetric Distributions

Distribution	*m*	*n*	RSS	StRSS	StERSS	StMRSS	StEMRSS
Uniform (0,1)	3	15	1.2622	1.4425	1.3763	1.6033	**1.7953**
	4	20	1.2803	1.4604	1.4288	1.7268	**1.8524**
	5	25	1.3343	1.5206	1.4817	1.7700	**1.9061**
	6	30	1.5094	1.6227	1.5394	1.9707	**2.1472**
	7	35	1.5988	1.9232	1.7590	2.1404	**2.3162**
	8	40	1.7752	2.0927	1.9812	2.1642	**2.3403**
	9	45	1.8737	2.1186	2.0087	***2.4638***	**2.4347**
	10	50	1.9933	2.2048	2.1220	***2.7073***	**2.6869**
Normal (5,1)	3	15	1.2458	1.5668	1.3137	1.7977	**2.0937**
	4	20	1.2900	1.5920	1.4620	1.9858	**2.1869**
	5	25	1.3040	1.7832	1.5058	2.0607	**2.2357**
	6	30	1.4313	1.8352	1.6686	2.1491	**2.4047**
	7	35	1.5033	2.3287	2.0529	2.9708	**3.0810**
	8	40	1.5041	2.6591	2.3598	***3.2042***	**3.1681**
	9	45	1.6694	2.7173	2.5067	***3.4620***	**3.3938**
	10	50	1.7817	2.8993	2.6178	***4.0982***	**4.0426**

Table 2
REs of Mean Estimators with respect to SRS under Perfect Ranking for Asymmetric Distributions

Distribution	*m*	*n*	RSS	StRSS	StERSS	StMRSS	StEMRSS
Gamma (4,3)	3	15	1.1741	1.3733	1.3352	1.5003	**1.6462**
	4	20	1.2407	1.3876	1.3428	1.5947	**1.6883**
	5	25	1.2846	1.4886	1.3617	1.6252	**1.7252**
	6	30	1.3810	1.5125	1.4480	1.7697	**1.8930**
	7	35	1.4766	1.7339	1.5227	1.8860	**2.0041**
	8	40	1.5646	1.8479	1.7706	***2.0239***	**2.0132**
	9	45	1.6930	1.8639	1.8389	***2.0901***	**2.0717**
	10	50	1.7642	1.9204	1.8514	***2.2376***	**2.2253**
Weibull (1.5,5)	3	15	1.1866	1.6135	1.3350	1.8740	**2.2175**
	4	20	1.2842	1.6416	1.3457	2.0912	**2.3281**
	5	25	1.3526	1.8592	1.5467	2.1815	**2.4501**
	6	30	1.4648	2.1336	1.6274	2.8701	**3.0049**
	7	35	1.5452	2.5083	2.1764	3.1990	**3.4699**
	8	40	2.3409	2.9302	2.6653	***3.6401***	**3.6040**
	9	45	2.4368	3.0086	2.9164	***4.0202***	**3.9234**
	10	50	2.7128	3.2508	3.1920	***4.9760***	**4.8898**

(6) Relative efficiencies (REs) of estimators for symmetric distributions have been calculated by using following equation

$$R.E = \frac{\operatorname{var}\left(\hat{\mu}_{y,SRS}\right)}{\operatorname{var}\left(\hat{\mu}_{y,i}\right)}.$$

where $\operatorname{var}\left(\hat{\mu}_{y,SRS}\right) = \sigma^2/n$, and '$i$' is RSS, StRSS, StERSS, StMRSS and StEMRSS. For asymmetric distributions, the equation for relative efficiency is

$$R.E = \frac{\operatorname{var}\left(\hat{\mu}_{y,SRS}\right)}{MSE\left(\hat{\mu}_{y,i}\right)}.$$

where, $MSE\left(\hat{\mu}_{y,i}\right)$ is the mean square error of estimators with $i =$ RSS, StRSS, StERSS, StMRSS and StEMRSS.

The simulation results for estimation of mean under perfect ranking have been presented in Table 1 and 2. Results show that REs of all estimators increases with increase in sample size. Results also revealed that REs of StEMRSS are greater than its competitor estimators for all distributions at small and moderate sample sizes. But at large sample size (i-e 40, 45, and 50) StEMRSS and StMRSS are almost equally efficient for both symmetric and asymmetric distributions. StEMRSS provides more efficient results under Weibull (1.5,5) distribution in which its maximum RE is 4.8898. Under Uniform (0,1) distribution, maximum RE of StEMRSS is 2.6869. While under Normal (5,1) and Gamma (4,3) distributions, maximum RE of StEMRSS is 4.0426 and 2.2253 respectively.

Table 3
REs of Mean Estimators with respect to SRS under Imperfect Ranking for Symmetric Distributions

Distribution	*m*	*n*	RSS	StRSS	StERSS	StMRSS	StEMRSS
Uniform (0,1)	3	15	1.1254	1.3618	1.2256	1.5210	**1.7132**
	4	20	1.1412	1.3793	1.2715	1.6443	**1.7707**
	5	25	1.1861	1.4373	1.3149	1.7677	**1.8703**
	6	30	1.2591	1.5376	1.3631	1.8882	**2.0689**
	7	35	1.3248	1.6928	1.4461	1.9659	**2.2430**
	8	40	1.3550	1.8314	1.4865	2.0830	**2.2660**
	9	45	1.4151	2.0352	1.7575	***2.3949***	**2.3643**
	10	50	1.4872	2.1237	1.8522	***2.6532***	**2.4416**
Normal (5,1)	3	15	1.0481	1.2903	1.1378	1.4168	**1.5640**
	4	20	1.1226	1.3043	1.2288	1.5117	**1.6067**
	5	25	1.1289	1.3273	1.2510	1.5412	**1.6210**
	6	30	1.1473	1.4266	1.2688	1.5770	**1.6874**
	7	35	1.2444	1.5429	1.4358	1.7531	**1.7847**
	8	40	1.2389	1.6398	1.5395	***1.9480***	**1.9369**
	9	45	1.2543	1.7807	1.5851	***2.0717***	**1.9977**
	10	50	1.3031	1.8386	1.7401	***2.2157***	**2.0814**

Table 4
REs of Mean Estimators with respect to SRS under Imperfect Ranking for Asymmetric Distributions

Distribution	*m*	*n*	RSS	StRSS	StERSS	StMRSS	StEMRSS
Gamma (4,3)	3	15	1.0618	1.1868	1.0907	1.2608	**1.3411**
	4	20	1.1047	1.2378	1.1303	1.3131	**1.3794**
	5	25	1.1307	1.2516	1.1777	1.3581	**1.4071**
	6	30	1.1477	1.2632	1.1841	1.3988	**1.4588**
	7	35	1.1973	1.3243	1.2671	1.4160	**1.5072**
	8	40	1.1928	1.4282	1.3349	***1.5086***	**1.5039**
	9	45	1.2975	1.4338	1.4038	***1.5642***	**1.5414**
	10	50	1.3229	1.5261	1.4262	***1.5937***	**1.5888**
Weibull (1.5,5)	3	15	1.0499	1.2496	1.1050	1.4495	**1.6054**
	4	20	1.0762	1.3312	1.1632	1.5500	**1.6508**
	5	25	1.1110	1.4986	1.2757	1.6029	**1.7142**
	6	30	1.1674	1.4976	1.3161	1.7014	**1.8750**
	7	35	1.2045	1.5848	1.3848	1.7794	**1.9999**
	8	40	1.3379	1.6896	1.6159	1.8557	**2.0321**
	9	45	1.4432	1.8416	1.6797	***2.0982***	**2.0770**
	10	50	1.5106	2.0802	1.7427	***2.2703***	**2.2336**

In Table 3 and 4, results for estimation of Mean under imperfect ranking have been presented. Results demonstrate that REs of all estimators increases with increase in sample size in imperfect ranking. As we observe in the case of perfect ranking, REs of StEMRSS are also greater than its competitor estimators for all distributions at small and moderate sample sizes in imperfect ranking. At large sample size (i.e. 40, 45, and 50) StEMRSS and StMRSS are almost equally efficient for both symmetric and asymmetric distributions. StEMRSS provides more efficient results under Uniform (0,1) distribution in which its maximum RE is 2.4416. Under Weibull (1.5,5) distribution, maximum RE of StEMRSS is 2.2336. While under Normal (5,1) and Gamma (4,3) distributions, maximum RE of StEMRSS is 2.0814 and 1.588 respectively.

Results show that, REs of all estimators decreases in imperfect ranking as compare to perfect ranking in both symmetric as well as asymmetric distributions (due to errors in ranking). In Uniform (0,1) distribution, maximum RE of StEMRSS for perfect and imperfect ranking is 2.6869 and 2.4416 respectively, while in Normal (5,1) distribution, maximum RE of StEMRSS for perfect and imperfect ranking is 4.0426 and 2.0814 respectively. Moreover, in Gamma (4,3) distribution, maximum RE of StEMRSS for perfect and imperfect ranking is 2.2253 and 1.588 respectively, whereas in Weibull (1.5,5) distribution, maximum RE of StEMRSS for perfect and imperfect ranking is 4.8898 and 2.2336 respectively. Under Weibull and Normal distributions, REs of StEMRSS are approximately same for imperfect ranking but in perfect ranking, relative efficiency in Weibull distribution is slightly on upper side.

4. REAL LIFE DATA

To compare the relative efficiencies (REs) of proposed and existing sampling procedures, a real-life data set given by Andersen et al. (1993) has been utilized. Data consists of 7 variables related to patients with malignant melanoma, we select thickness of tumor (mm) as study variable Y, age of patient at the time of operation as concomitant variable X, and variable 'whether patient was ulcerated or not' has been used for stratification purpose. Stratum 1 consists of those patients, who are not ulcerated and remaining are placed in stratum 2. Summary statistics of data has been presented in Table 5.

Table 5
Summary Statistics of Study Variable Y

Strata	Size	Mean	Variance	Minimum	Maximum
1	115	1.8113	4.7364	0.1	14.66
2	90	4.3433	10.4244	0.16	17.42

Results in Table 5 show that, population of size 205 is divided into two strata of size 115 and 90 respectively. Average thickness of tumor in stratum 1 is 1.8113 where in stratum 2 its value is 4.3433. Variation in thickness of tumor in strata 1 and 2 are 4.7364 and 10.4244 respectively.

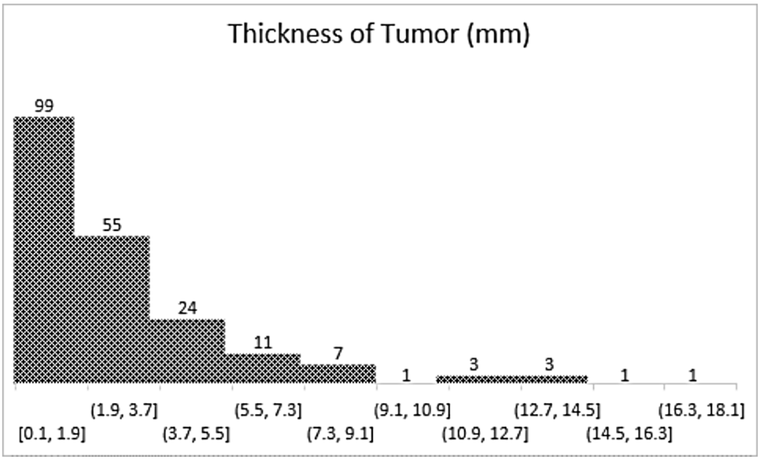

Figure 1: Histogram of Study Variable Y

Figure 1 showed the histogram of our study variable 'Thickness of Tumor'. It showed that study variable follows positive skewed asymmetrical distribution. It also showed that outliers are present in our data as showed in the right tail of histogram. From the results of Table 5 and Figure 1, it revealed that this data has heterogeneity and outliers. Therefore, in this type of data, we suggest to use our proposed stratified extreme-cum-median ranked set sampling. For simplicity of analysis, we select $r = 1$, 2, 3 and 4, $m = 3$, 4, 5, 6, 7, 8, 9 and 10.

Table 6
REs of Mean Estimators in Real Life Data with respect to SRS under Perfect Ranking

r	*m*	*n*	**RSS**	**StRSS**	**StERSS**	**StMRSS**	**StEMRSS**
1	3	3	1.0149	1.0393	1.0211	1.0452	**1.0577**
	4	4	1.0455	1.0789	1.0483	1.0892	**1.1190**
	5	5	1.0318	1.0591	1.0567	1.0945	**1.1160**
	6	6	1.0109	1.0821	1.0486	1.1024	**1.1297**
	7	7	1.0467	1.1148	1.0791	1.1519	**1.1955**
	8	8	1.0489	1.1255	1.0982	1.1687	**1.2120**
	9	9	1.0526	1.1423	1.1153	1.1930	**1.2134**
	10	10	1.0412	1.1191	1.1031	1.1895	**1.2290**
2	3	6	1.0046	1.0312	1.0235	1.0504	**1.0644**
	4	8	1.0216	1.0522	1.0483	1.0860	**1.1044**
	5	10	1.0128	1.0584	1.0369	1.1064	**1.1309**
	6	12	1.0122	1.0823	1.0548	1.1162	**1.1309**
	7	14	1.0376	1.1142	1.0742	1.1561	**1.2058**
	8	16	1.0556	1.1091	1.0956	1.1755	**1.2252**
	9	18	1.0442	1.1647	1.0800	1.2037	**1.2274**
	10	20	1.0310	1.1580	1.0830	1.2221	**1.2699**
3	3	9	1.0040	1.0154	1.0106	1.0275	**1.0286**
	4	12	1.0153	1.0284	1.0198	1.0503	**1.0536**
	5	15	1.0067	1.0191	1.0131	1.0392	**1.0612**
	6	18	1.0103	1.0689	1.0461	1.0969	**1.1089**
	7	21	1.0315	1.0947	1.0619	1.1285	**1.1682**
	8	24	1.0214	1.0630	1.0399	1.1431	**1.1765**
	9	27	1.0366	1.1335	1.0658	1.1641	**1.1825**
	10	30	1.0255	1.1273	1.0677	1.1600	**1.1954**
4	3	12	1.0040	1.0267	1.0201	1.0430	**1.0549**
	4	16	1.0185	1.0445	1.0413	1.0731	**1.0884**
	5	20	1.0109	1.0496	1.0314	1.0897	**1.1100**
	6	24	1.0103	1.0689	1.0461	1.0969	**1.1089**
	7	28	1.0315	1.1075	1.0619	1.1285	**1.1682**
	8	32	1.0462	1.1039	1.0790	***1.1905***	**1.1823**
	9	36	1.0366	1.1492	1.0658	***1.1907***	**1.1825**
	10	40	1.0255	1.1428	1.0677	***1.2171***	**1.2134**

Results of REs with perfect ranking in Table 6 illustrated that mean estimator under StEMRSS outperform it's all competitor estimators considered in this study. Results show that there is increasing trend in efficiencies of all mean estimators with increasing sample size. At very high sample size with $r = 4$, both StEMRSS and StMRSS are almost equally efficient, but for all other cases, StEMRSS performs better then StMRSS.

Table 7
REs of Mean Estimators in Real Life Data with respect to SRS under Imperfect Ranking

r	*m*	*n*	**RSS**	**StRSS**	**StERSS**	**StMRSS**	**StEMRSS**
1	3	3	1.0035	1.0133	1.0088	1.0199	**1.0239**
	4	4	1.0050	1.0128	1.0091	1.0184	**1.0238**
	5	5	1.0015	1.0187	1.0108	1.0297	**1.0399**
	6	6	1.0040	1.0281	1.0140	1.0350	**1.0502**
	7	7	1.0196	1.0419	1.0360	1.0494	**1.0593**
	8	8	1.0143	1.0459	1.0233	1.0617	**1.0777**
	9	9	1.0147	1.0217	1.0197	1.0331	**1.0457**
	10	10	1.0155	1.0291	1.0192	1.0459	**1.0649**
2	3	6	1.0059	1.0262	1.0140	1.0351	**1.0417**
	4	8	1.0074	1.0257	1.0143	1.0335	**1.0417**
	5	10	1.0039	1.0321	1.0161	1.0459	**1.0594**
	6	12	1.0065	1.0424	1.0196	1.0518	**1.0709**
	7	14	1.0229	1.0574	1.0430	1.0676	**1.0809**
	8	16	1.0174	1.0621	1.0296	1.0813	**1.1016**
	9	18	1.0179	1.0365	1.0261	1.0508	**1.0672**
	10	20	1.0188	1.0446	1.0256	1.0651	**1.0886**
3	3	9	1.0076	1.0262	1.0152	1.0355	**1.0469**
	4	12	1.0089	1.0258	1.0155	1.0342	**1.0469**
	5	15	1.0060	1.0311	1.0170	1.0443	**1.0616**
	6	18	1.0082	1.0395	1.0198	1.0491	**1.0710**
	7	21	1.0216	1.0516	1.0389	1.0619	**1.0792**
	8	24	1.0171	1.0552	1.0280	1.0728	**1.0958**
	9	27	1.0175	1.0345	1.0251	1.0482	**1.0680**
	10	30	1.0182	1.0411	1.0247	1.0596	**1.0853**
4	3	12	1.0208	1.0413	1.0319	1.0645	**1.0795**
	4	16	1.0219	1.0410	1.0321	1.0634	**1.0796**
	5	20	1.0197	1.0454	1.0335	1.0722	**1.0923**
	6	24	1.0216	1.0524	1.0360	1.0764	**1.1004**
	7	28	1.0325	1.0623	1.0516	1.0872	**1.1074**
	8	32	1.0290	1.0653	1.0430	1.0966	**1.1216**
	9	36	1.0296	1.0489	1.0409	1.0768	**1.0992**
	10	40	1.0303	1.0543	1.0407	1.0864	**1.1137**

Table 7 shows the relative efficiency of mean estimator under proposed and existing sampling procedures for imperfect ranking. Concomitant variable ‘Age’ has been utilized for ranking purpose. Results show that in imperfect ranking, mean estimator of StEMRSS outperform it’s all competitor estimators. Results also revealed that, there is decreasing trend in overall relative efficiencies of all estimators in imperfect ranking case as compare to perfect ranking. It is also presented in Figure 4 that efficiencies of all mean estimators have increasing trend with increase in sample size for imperfect ranking.

5. CONCLUSION

In this study, an efficient and easily applicable ranked set sampling procedure has been proposed for estimating population mean. The proposed procedure is the combination of StERSS and StMRSS. It is proved theoretically that under certain conditions proposed mean estimator of 'StEMRSS' has been unbiased and efficient in estimating population mean as compare to StERSS and StMRSS. Further, by utilizing extensive Monte Carlo Simulation the comparison of relative efficiencies of StEMRSS with other well-known sampling procedures have been made. Results of Monte Carlo Simulation revealed that StEMRSS will always provide more efficient estimate of population mean than SRS, RSS, StRSS, StERSS for all sample sizes while it is efficient than StMRSS usually for small and medium sample size. While, for very large sample size, StEMRSS and StMRSS are almost equally efficient. Real life data set has also been presented to illustrate the relative efficiency of StEMRSS over its competitor procedures. It is observed that the results of real-life application are compatible and conformable to aforementioned Monte Carlo simulation outcomes. The proposed ranked set sampling procedure can be utilized in research areas where ranking of all observations is tiresome but obtaining median and extreme values are easy. It is recommended for future research to proposed some ratio, product, regression and exponential type generalized estimators under StEMRSS for estimating of population parameters under heterogeneous environment. This procedure will also be recommended to use in monitoring of location and scale parameters with control charting schemes such as Shewhart, EWMA and CUSUM.

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