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MODIFIED MINIMAX RANKED SET SAMPLING

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ABSTRACT

Based on the traditional ranked set sampling (RSS), Al-Nasser and Al-Omari [1] have recently presented a cost-effective sampling technique called MiniMax RSS (MMRSS). However, MMRSS has some drawbacks when the distribution is asymmetric. To overcome this situation, in this article, we consider developing a modified version of MMRSS (MMMRSS). Monte Carlo simulations from numerous symmetric and asymmetric distributions are employed to assess the performance of the suggested MMMRSS mean estimator. Simulation findings demonstrated that MMMRSS estimator is more efficient than their counterparts using simple random sample (SRS) and MMRSS for all distributions considered in this article. Moreover, we have constructed Quality control charts to monitor the process mean based on the suggested MMMRSS. The performance of the average run length (ARL) of these new charts was compared with the control charts based on several sampling techniques. The results, based on a simulation study, indicate that our suggested MMMRSS control charts performed the best in detecting changes in process mean in most simulated scenarios. A real-life application concerning the global temperature is also provided as an illustration of the suggested charts.

KEYWORDS

Average run length; control charts; global warming; minimax ranked set sampling; Monte Carlo simulation; mean estimation

1. INTRODUCTION

In many cases in nature, measuring experimental units can be destructive. In such situations, a need has arisen to propose a feasible and informative sampling technique that utilizes as few “actual" measurements of units as possible. Ranked set sampling (RSS) is an effective sampling technique of data collection that improves parametric estimation by employing ranking on observations. The idea of RSS was originally proposed by McIntyre [2] to estimate forage yields in pastures. Further development and statistical properties of RSS were presented by Takahasi and Wakimoto [3] and Dell and Clutter [4]. This technique is useful in the situation where the actual observations are difficult to obtain i.e. expensive or time-consuming but ranking them in a cheap way such as visual inspection or expert knowledge is applicable and relatively easy.

Over recent decades, many authors have discussed several modified forms and improvements for RSS. Samawi et al. [5] introduced extreme RSS (ERSS), Muttlak [6, 7, 8] proposed paired RSS, median RSS (MRSS) and percentile RSS (PRSS), respectively, Al-Nasser [9] investigated LRSS which is a generalized robust sampling technique for RSS, MRSS and PRSS. Also, Al-Nasser and Mustafa [10] utilized a robust ERSS (RERSS) as an alternative sampling technique. For further details, a comprehensive review on various developments on RSS and its modifications, the reader is referred to [11, 12, 13, 14] and the references therein.

In a recent interesting work, Al-Nasser and Al-Omari [1] have introduced MiniMax RSS (MMRSS). Unlike the other sampling techniques for ranked data, MMRSS used unequal set size. They showed that MMRSS is more efficient than SRS in estimating populations mean with odd set size, while it is inefficient when the set size is even under some skewed distributions such as the exponential, gamma and chi-square distributions. Due to this issue, in this article we introduced a new modified MMRSS technique, namely; MMMRSS, which is cost-effective, efficient and anti-wasting sampling technique for improved estimation. We also proposed and analyzed Shewhart control chart to monitor the process mean based on MMMRSS.

The remaining part of this article is presented as: In Section 2, a brief explanation of different ranked sampling techniques is given. The description of MMMRSS along with its mathematical properties such as the expectation and variance of the estimator of the population mean are presented in Section 3, followed by the development of new Shewhart X-bar control charts to monitor the process by utilizing the advantages of MMMRSS in Section 4. Section 5 provides the performance comparisons of control charts based on several RSS variations. Monte Carlo simulation results and discussion of the results are presented in the same Section. We considered different sample sizes and processes with different shifts from statistical control. MMMRSS control charts were compared with SRS, RSS, ERSS and MMRSS based on results already presented in the literature. A real data-based example is further presented in Section 6, and conclusion and recommendations are provided in Section 7.

2. BRIEF REVIEW OF SOME SAMPLING TECHNIQUES

The RSS is an ingenious sampling technique commonly used as a cost-efficient alternative of SRS to estimate the population parameters when it is possible to rank the values of the study variable in an inexpensive or easy way. In this section, we briefly review RSS as well as some of it’s efficient variations; ERSS and MMRSS.

2.1 Ranked Set Sampling (RSS)

The collection of m samples using the RSS technique can be described as follows:

a) Randomly select a sample of size m^2 units from the underlying population and randomly divide these units into m sets, each of size m;
b) The units within each set are ranked increasingly with respect to the variable of interest via a visual inspection or by any cost-free method;

c) From the 1^{st} set of m units, the smallest ranked unit is quantified; from the 2^{nd} set of m units, the second smallest ranked unit is quantified. The process continues in this way until the largest ranked unit is quantified from the last set.
This represents one cycle of an RSS of size m.
d) Repeat the above steps (a-c), if essential, r times to acquire the required RSS of size mr.

It is remarkable to mention that from the m^2 original sample units, only m are effectively quantified for the variable of interest (one from each set). Hence, making a comparison of RSS with SRS of the same size is meaningful. RSS becomes more efficient than SRS as long as a more accurate and accessible ranking criterion is available.

2.2 Extreme Ranked Set Sampling (ERSS)

The main steps involved in selecting an ERSS of size mr are as follows:

a) Randomly choose a sample of size m^2 units from the target population and randomly divide these units into m sets, each of size m;
b) Rank the units within each set by expert knowledge or by any cost-free method with respect to the study variable;
c) If the sample size m is even, from each of the 1^{st} set of $\frac{m}{2}$ units, the smallest ranked unit is quantified; from each of the last set of $\frac{m}{2}$ units, the largest ranked unit is quantified. If the sample size m is odd, the unit with rank $\frac{m+1}{2}$ is also quantified.
One cycle of an ERSS is done after completing all the above steps; i.e, a sample of size m units is obtained.
d) The steps (a) through (c) can be repeated, if needed, r times to draw a total sample of size mr units.

2.3 MiniMax Ranked Set Sampling (MMRSS)

The collection of m samples using the MMRSS technique can be described as follows:

a) Draw m SRS of size $i = 1,2,3, \dots, m$;
b) Arrange the sampling units within each SRS in ascending order;
c) From the odd SRS of size $m = 2i - 1$, measure the minimum; while from the even SRS of size $m = 2i$, measure the maximum;
This completes one cycle of an MMRSS of size m.
d) Repeat the process r times, if necessary, to obtain an MMRSS of size rm.

3. PROPOSED SAMPLING TECHNIQUE AND BASIC NOTATIONS

In this section, we explain MMMRSS technique and its related mathematical setups and notations. In order to clarify this procedure, it is helpful to refer to some illustrations. First let us assume that X is the study variable with probability density function (pdf) $f(x)$ and cumulative distribution function (cdf) $F(x)$. Second, let μ_x and σ_x^2 denote the mean and variance of X, respectively. Finally, suppose that $x_1, x_2, \dots, x_m$ is a SRS of size m drawn from $f(x)$ while $x_{[1:m]}, x_{[2:m]}, \dots x_{[m:m]}$ represents the order statistics of this SRS.

3.1 Modified Mini-Max RSS (MMMRSS)

The MMMRSS technique can be illustrated as follows:

Step I: Select m samples randomly from the underlying population each of size $i = 1,2,3,\dots,m$, respectively.

$$\begin{bmatrix} x_1 & & & & \\ x_1 & x_2 & & & \\ x_1 & x_2 & x_3 & & \\ \vdots & \vdots & \vdots & \ddots & \\ x_1 & x_2 & x_3 & \cdots & x_m \end{bmatrix}$$

If m is odd, then we keep the procedure as what is described in MMRSS. However, if m is even then we duplicate the last raw as follows:

$$\begin{bmatrix} x_1 & & & & \\ x_1 & x_2 & & & \\ x_1 & x_2 & x_3 & & \\ \vdots & \vdots & \vdots & \ddots & \\ x_1 & x_2 & x_3 & \cdots & x_m \\ x_1 & x_2 & x_3 & \cdots & x_m \end{bmatrix}$$

In this article, we consider the case of even samples only since, as we mentioned before, the procedure for odd samples is exactly as what is given in the MMRSS.

Step II: The units within each sample are ranked in ascending order by personal judgment or by any other inexpensive way.

$$\begin{bmatrix} \boldsymbol{x_{[1:1]}} & & & & & & \\ x_{[1:2]} & \boldsymbol{x_{[2:2]}} & & & & & \\ \boldsymbol{x_{[1:3]}} & x_{[2:3]} & \cdots & & & & \\ \vdots & \vdots & \vdots & & \ddots & & \\ x_{[1:m]} & x_{[2:m]} & \cdots & \boldsymbol{x_{[\frac{m}{2}:m]}} & x_{[\frac{m}{2}+1:m]} & \cdots & x_{[m:m]} \\ x_{[1:m]} & x_{[2:m]} & \cdots & x_{[\frac{m}{2}:m]} & \boldsymbol{x_{[\frac{m}{2}+1:m]}} & \cdots & x_{[m:m]} \end{bmatrix}$$

Step III: Select the smallest ranked unit from the 1^{st} SRS of size $i = 1$; $x_{[1:1]}$.

Step IV: Select the largest ranked unit from the 2^{nd} SRS of size $i = 2$; $x_{[2:2]}$.

Step V: Select the smallest ranked unit from the 3^{rd} SRS of size $i = 3$; $x_{[1:3]}$.

Step VI: Continue the above process till in the last sample, if m is odd; select the minimum, otherwise if m is even; draw another SRS from the underlying population of size m, rank it, and select $x_{[\frac{m}{2}:m]}$ from the 1^{st} sample and $x_{[\frac{m}{2}+1:m]}$ from the $(m+1)^{th}$ sample. Then the MMMRSS samples will be $\{x_{[1:1]}, x_{[2:2]}, x_{[1:3]}, \dots, \frac{x_{[\frac{m}{2}:m]}+x_{[\frac{m}{2}+1:m]}}{2}\}$

This completes one cycle of an MMMRSS of size m.

Step VII: The entire process (Steps I through VI) can be repeated r times (cycles), if essential, to acquire the desired sample size rm.

It follows that the form of the MMMRSS samples can be represented as:

$$\begin{cases} \left\{x_{[1:1]k}, x_{[2:2]k}, x_{[1:3]k}, \ldots, x_{[1:m]k}; k = 1,2,\ldots,r\right\}; & \text{if } m \text{ is odd} \\ \left\{x_{[1:1]k}, x_{[2:2]k} x_{[1:3]k}, \ldots, \frac{x_{\left[\frac{m}{2}:m\right]k} + x_{\left[\frac{m}{2}+1:m\right]k}}{2}; k = 1,2,\ldots,r\right\}; & \text{if } m \text{ is even} \end{cases}$$

3.2 Population Mean Estimation based on MMMRSS

Without loss of generality, assume that $r = 1$, it follows that the mean based on MMMRSS is identified as:

$$\bar{X}_{(MMMRSS)} = \frac{1}{m}\left\{\sum_{i=1}^{\frac{m}{2}} x_{[1:2i-1]} + \sum_{i=1}^{\left(\frac{m}{2}\right)-1} x_{[2i:2i]} + \frac{x_{\left[\frac{m}{2}:m\right]} + x_{\left[\frac{m}{2}+1:m\right]}}{2}\right\}$$

Then expected value of the sample mean from MMMRSS is given by:

$$E\left(\bar{X}_{(MMMRSS)}\right) = \frac{1}{m}\left\{\sum_{i=1}^{\frac{m}{2}} \mu_{[1:2i-1]} + \sum_{i=1}^{\left(\frac{m}{2}\right)-1} \mu_{[2i:2i]} + \frac{\mu_{\left[\frac{m}{2}:m\right]} + \mu_{\left[\frac{m}{2}+1:m\right]}}{2}\right\}$$

with respective variances

$$Var\left(\bar{X}_{(MMMRSS)}\right) = \frac{1}{m^2}\left\{\sum_{i=1}^{\frac{m}{2}} Var\left(x_{[1:2i-1]}\right) + \sum_{i=1}^{\left(\frac{m}{2}\right)-1} Var\left(x_{[2i:2i]}\right) + Var\left(\frac{x_{\left[\frac{m}{2}:m\right]} + x_{\left[\frac{m}{2}+1:m\right]}}{2}\right)\right\}$$

Moreover, the relative efficiency (RE) of the estimator of the population mean based on MMMRSS technique with respect to the traditional SRS can be defined by:

$$RE = \frac{MSE(\bar{X}_{SRS})}{MSE\left(\bar{X}_{(MMMRSS)}\right)}$$

Now, to illustrate the RE of the suggested sampling technique in estimating the population mean of the standard uniform distribution i.e., $U(0,1)$. The expected value and variance of the i^{th} order statistics from $U(0,1)$ are:

$$\mu_{(i:m)} = \frac{i}{m+1}; \sigma^2_{(i:m)} = \frac{i(m-i+1)}{(m+1)^2(m+2)}$$

Consequently, the expected value of the sample mean using MMMRSS technique is equal to:

$$E(\bar{X}_{(MMMRSS)}) = \frac{-H_{\frac{m-1}{2}} + H_{\frac{m}{2}} + m + 1 - \log(4)}{2m}$$

Noting that, $H_m = \sum_{i=1}^{m} \frac{1}{i}$ is denoted by the m^{th} harmonic number and it is equal to $\gamma + \psi_0(m+1)$, where γ is the Euler-Mascheroni constant and ψ_0 is the digamma function. Figure 1 shows a comparison between the mean values of MMMRSS technique and SRS technique. It is very clear that based on the suggested technique, the mean value is always less than the actual mean using SRS.

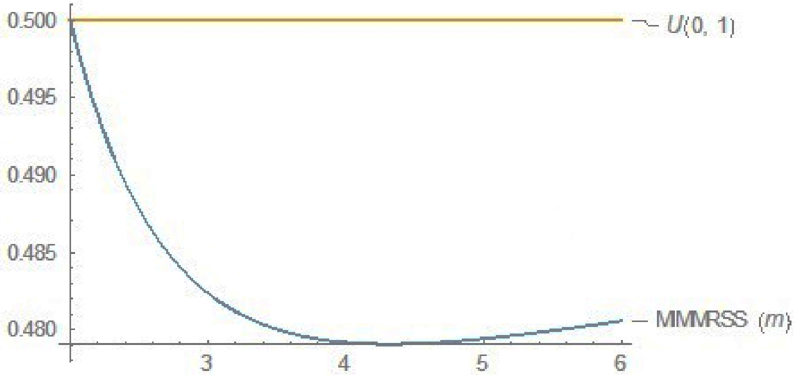


Figure 1: A Comparison between the Mean Values of the MMMRSS and the SRS

In addition, the variance of the sample mean using MMMRSS technique is given by:

$$Var(\bar{X}_{(MMMRSS)}) = \frac{-4\pi^2(m+1)^2 + 3m(16m+17) + 24(m+1)^2\psi^{(1)}(m+1)}{24m^2(m+1)^2}$$

where $\psi^{(1)}(m+1)$ is the first derivative of the digamma function. Figure 2 shows that the variance under the MMMRSS technique is always lower than the one under the SRS for the standard uniform distribution.

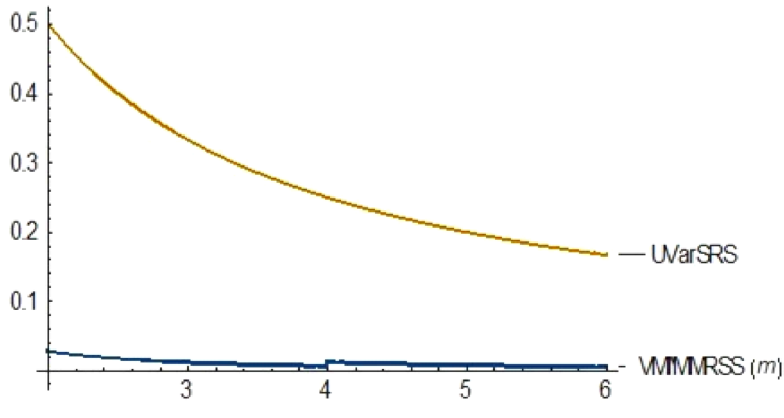


Figure 2: A Comparison between the Variance Values of the MMMRSS and the SRS.

Moreover, the results in terms of REs of the mean estimators are given in Figure 3. It is demonstrated that the efficiency of MMMRSS technique is higher than SRS in estimating the population mean when the underlying distribution is the standard uniform distribution.

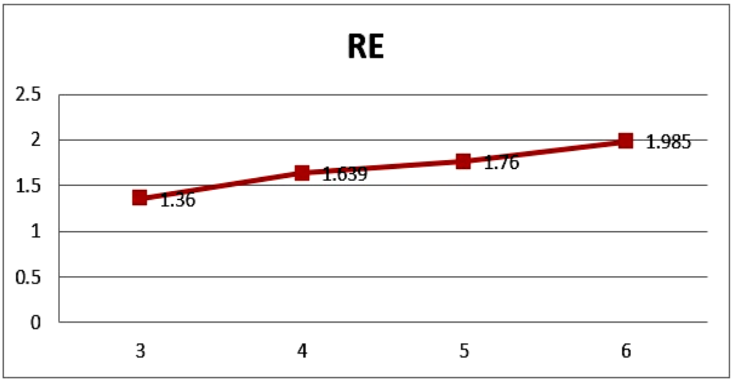


Figure 3: RE for Estimation the Population Mean using MMMRSS

3.3 Improvement Percentage in Estimating the Population Mean

To illustrate the benefit of the modifications that we suggest to the original MMRSS sampling technique in estimating the population mean, we computed the bias, mean squared error (MSE) and the RE of the proposed sampling technique with respect to SRS. Several symmetric and asymmetric distributions are utilized to assess the performance. The results are compared with the MMRSS sampling technique using a set of size $m = 4$ and 6. Moreover, for each evaluation statistic we compute the improvement percentage using the following formula:

$$Improvment\ Percentage = \frac{New - Old}{Old} \times 100\%$$

The results are presented in Tables 1 and 2 for set size 4 and 6, respectively. Based on Tables 1 and 2, a list of main points for the suggested sampling technique is as follows:

- The MMMRSS mean estimators are superior to the traditional SRS estimators in all cases.
- Unlike the MMRSS sampling technique, the mean value based on the suggested technique is always less than the actual mean for all distributions.
- The suggested technique is more accurate and more efficient than the SRS and MMRSS sampling techniques.
- Based on the improvement percentage, the bias and MSE are highly reduced, while the RE is highly increased.

Thus, we can observe that the new modification on the MMRSS sampling technique is very important to give an accurate and efficient estimator whatever the distribution is.

Table 1
Efficiency Performance and Improvement Percentage when $m = 4$

$m = 4$	Efficiency Performance						Improvement Percentage		
Distribution	Modified MiniMax			Minimax					
	Bias	MSE	RE	Bias	MSE	RE	Bias	MSE	RE
U(0,1)	-0.021	0.013	1.639	0.054	0.016	1.330	-138.89%	-18.75%	23.23%
N(0,1)	-0.071	0.156	1.699	0.186	0.207	1.215	-138.17%	-24.64%	39.84%
Logistic(5,2)	-0.25	2.097	1.568	0.667	2.836	1.160	-137.48%	-26.06%	35.17%
Student t(4)	-0.092	0.332	1.505	0.247	0.464	1.080	-137.25%	-28.45%	39.35%
Beta(5,2)	-0.009	0.004	1.589	0.034	0.006	1.114	-126.47%	-33.33%	42.64%
Rayleigh(1)	-0.051	0.067	1.607	0.137	0.097	1.102	-137.23%	-30.93%	45.83%
HalfNormal(2)	-0.031	0.022	1.589	0.084	0.035	1.025	-136.90%	-37.14%	55.02%
Exponential(1)	-0.083	0.164	1.526	0.229	0.289	**0.865**	-136.24%	-43.25%	76.42%
Gamma(2,3)	-0.347	2.785	1.565	0.945	4.729	**0.952**	-136.72%	-41.11%	64.39%
ChiSquare(3)	-0.202	0.968	1.551	0.553	1.636	**0.917**	-136.53%	-40.83%	69.14%

Table 2
Efficiency Performance and Improvement Percentage when $m = 6$

$m = 6$	Efficiency Performance						Improvement Percentage		
Distribution	Modified MiniMax			Minimax					
	Bias	MSE	RE	Bias	MSE	RE	Bias	MSE	RE
U(0,1)	-0.019	0.007	1.985	0.040	0.008	1.689	-147.50%	-12.50%	17.53%
N(0,1)	-0.069	0.097	1.727	0.142	0.119	1.390	-148.59%	-18.49%	24.24%
Logistic(5,2)	-0.25	1.368	1.603	0.510	1.741	1.260	-149.02%	-21.42%	27.22%
Student t(4)	-0.094	0.237	1.406	0.191	0.307	1.090	-149.21%	-22.80%	28.99%
Beta(5,2)	-0.014	0.002	1.701	0.031	0.004	1.199	-145.16%	-50.00%	41.87%
Rayleigh(1)	-0.033	0.039	1.790	0.125	0.061	1.178	-126.40%	-36.07%	51.95%
HalfNormal(2)	-0.014	0.013	1.767	0.084	0.023	1.029	-116.67%	-43.48%	71.72%
Exponential(1)	-0.0167	0.109	1.527	0.261	0.216	**0.772**	-106.40%	-49.54%	97.80%
Gamma(2,3)	-0.133	1.823	1.645	0.992	3.299	**0.909**	-113.41%	-44.74%	80.97%
ChiSquare(3)	-0.063	0.623	1.604	0.600	1.173	**0.853**	-110.50%	-46.89%	88.04%

3.4 Comparisons with other Ranked Data Techniques

To illustrate the robustness of the suggested sampling technique, we compare the RE of the MMMRSS with several sampling techniques including the novel RSS, ERSS and MMRSS when the set size is 4 or 6. The results are given in Table 3 for set size 4 and Table 4 for set size 6. The results indicated the following:

- The suggested MMMRSS is a comparable sampling technique with RSS and ERSS, and more efficient than the MMRSS and SRS under all distributions.
- The benefit of the suggested MMMRSS is more clear with asymmetric distribution. There are many cases when the underlying distribution is Gamma, Weibull, Pareto, Log Normal; the ERSS and MMRSS are less efficient than the SRS, while the RSS and the proposed MMMRSS are more efficient than all other sampling techniques.
- The most interesting result is that the suggested MMMRSS is more efficient than the Novel RSS technique under the Weibull and Pareto distributions when the set size is 4.

Table 3
RE Comparisons between RSS, ERSS, MMRSS and MMMRSS: $m = 4$

$m = 4$	Distribution	RE			
		RSS	ERSS	MMRSS	MMMRSS
Symmetric	U(0,1)	2.5	3.125	1.330	1.639
	N(0,1)	2.34695	2.0337	1.210	1.699
	Logistic (5,2)	2.2164	1.7056	1.160	1.568
	Student t(4)	1.9626	1.3078	1.080	1.505
	Beta(3,3)	2.4432	2.4227	1.265	1.622
	ArcSin(0,1)	2.4493	3.8271	1.386	1.635
Asymmetric	Beta(5,2)	2.3565	2.0936	1.114	1.589
	Rayleigh(1)	2.3251	1.9867	1.102	1.607
	HalfNormal(2)	2.2393	1.7676	1.025	1.589
	Exponential(1)	1.920	1.1707	**0.865**	1.526
	Gamma(2,3)	2.0957	1.4533	**0.952**	1.565
	ChiSquare(3)	2.0304	1.3389	**0.917**	1.551
	LogNormal(0,1)	1.4711	**0.7515**	0.7085	1.4398
	Pareto(1,3)	1.3305	**0.6659**	0.6674	1.4109
	Weibull (0.5,1)	1.3345	**0.6443**	0.6583	1.3976
	Gamma(0.5,1)	1.6963	**0.9059**	0.7742	1.4715

Table 4
RE Comparisons between RSS, ERSS, MMRSS and MMMRSS: $m = 6$

$m = 6$	Distribution	RE			
		RSS	**ERSS**	**MMRSS**	**MMMRSS**
Symmetric	**U(0,1)**	3.5	5.4444	1.689	1.985
	N(0,1)	3.1856	2.4042	1.390	1.727
	Logistic (5,2)	2.9275	1.8014	1.260	1.603
	Student t(4)	2.4483	1.1903	1.090	1.406
	Beta(3,3)	3.3828	3.2881	1.510	1.8411
	ArcSin(0,1)	3.3975	8.9319	1.840	2.0755
Asymmetric	**Beta(5,2)**	3.2226	2.1721	1.199	1.701
	Rayleigh(1)	3.1551	2.0379	1.178	1.790
	HalfNormal(2)	3.010	1.4562	1.029	1.767
	Exponential(1)	2.449	**0.752**	**0.772**	1.527
	Gamma(2,3)	2.742	1.089	**0.909**	1.645
	ChiSquare(3)	2.632	**0.9387**	**0.853**	1.604
	LogNormal(0,1)	1.6971	**0.4541**	0.5832	1.1912
	Pareto(1,3)	1.4754	**0.4062**	0.5422	1.0917
	Weibull (0.5,1)	1.5094	**0.3708**	0.5232	1.0903
	Gamma(0.5,1)	2.0908	**0.5218**	0.6463	1.3629

4. SHEWHART CONTROL CHART USING MMMRSS TECHNIQUE

Shewhart's X-bar control charts have been most commonly used in industries to monitor the mean and variation of a process based on samples taken from the process at given times. The control charts are identified via the upper and lower control limits as well as the central limit term. If both population mean (μ) and variance (σ^2) are known, then the MMMRSS based Shewhart X-bar control limits are given by:

$$LCL = \mu - 3\sigma_{\bar{X}_{MMMRSS}}$$

$$CL = \mu$$

$$UCL = \mu + 3\sigma_{\bar{X}_{MMMRSS}}$$

where UCL, CL and LCL represent the upper control limit, central limit and lower control limit for the X-bar charts respectively. In practice, mostly μ and σ^2 are unknown, so estimated values of the parameters are considered. Consequently, the X-bar MMMRSS control chart can be constructed as:

$$LCL = \bar{X}_{(MMRSS)} - 3\hat{\sigma}_{\bar{X}_{MMMRSS}}$$

$$CL = \bar{X}_{(MMRSS)}$$

$$UCL = \bar{X}_{(MMRSS)} + 3\hat{\sigma}_{\bar{X}_{MMMRSS}}$$

where $\bar{X}_{(MMRSS)}$ is a sample mean based on MMMRSS technique and $\hat{\sigma}_{\bar{X}_{MMMRSS}} = \sqrt{Var(\bar{X}_{(MMRSS)})}$.

5. ARL COMPARISON FOR MMMRSS AND OTHER TECHNIQUES

One of the most popular procedures used to assess the performance of control chart is Average Run Length (ARL). In this section, the ARL is utilized to study the performance of the suggested X-bar MMMRSS control charts against the existing SRS, RSS, ERSS and MMRSS mean charts measured through comprehensive simulation study.

Now, the in-control ARL $ARL_0 = \frac{1}{\alpha}$, with α represents the probability of type I error, is the average number of plotted samples before a signal indicates an out-of-control while the process is in-control. The out-of-control ARL, $ARL_1 = \frac{1}{1-\beta}$ with β represents the probability of type II error, is the average number of plotted samples before a signal indicates an out-of-control while the process is out-of-control [15].

Generally speaking, for any control-chart setup, it is desirable to have large values of in-control and small values of out-of-control which indicates that the control chart is performing good in detecting random shifts in the process. The ARL value of the suggested X-bar MMMRSS and the other control charts for different values of set size (m), shifts (δ) and in-control ARL are evaluated. The set size is taken to be $m = 4{,}6$.

Based on the ARL method, the process remains in control with mean μ_0 and standard deviation σ_0, otherwise, it goes out of control in terms of a mean shift of the amount $\delta \frac{\sigma_0}{\sqrt{m}}$, i.e., a shift in the mean from μ_0 to $\mu_1 = \mu_0 + \delta \frac{\sigma_0}{\sqrt{m}}$ where δ is a nonnegative and selected to dominate shift in the mean.

Monte Carlo simulation studies were conducted to evaluate the performance of the suggested control chart in comparison with the other considered charts along with sample size $m = 4{,}6$ from different distributions and shift in mean δ vary between (0 to 3.4) to cover the in- and out-of-control process. According to Lee et al. [16], the simulation steps based on Mean-MMMRSS chart is illustrated as follows:

Step 1: Evaluating sample mean and sample variance

I. Draw a MMMRSS sample of size $m = 4$ and 6 from the standard normal distribution. Note that the exact value of variance under normality assumption using MMMRSS can be computed and are given in the following table:

Table 5
The Exact Value of Variance under Normality Assumption using MMMRSS for Different Values of m

Sample size	Variance based on MMMRSS
3	0.258
4	0.150
5	0.134
6	0.0915

II. Evaluate $\bar{X}_{MMMRSS}$ of each sample.

Step 2: Setting up the control limits

I. Select an initial value of k for a fixed $ARL_1 = 370$ (here $k = 3$).

II. Evaluate the control chart limits (LCL, UCL).

Step 3: Evaluating the out-of-control ARL

I. Check the mean for out-of-control process. If the process is declared as in-control, go **Step 1**. If the process is declared to be out-of-control, record it

$$\begin{cases} \text{out}-\text{of}-\text{control}; & \bar{X}_{MMMRSS} > UCL \text{ } or \text{ } \bar{X}_{MMMRSS} < LCL \\ \text{in}-\text{control}; & LCL < \bar{X}_{MMMRSS} < UCL \end{cases}$$

Step 4: **I.** Repeat steps 1 and 2 1000000 times to compute out-of-control ARL.

II. Assume that the number the out-of-control run length is R. Then the $ARL = R/1000000$

III. Evaluate ARL for $\delta = 0.1, 0.2, \ldots, 3.4$.

The comparisons between the three sampling techniques are given in Tables 6-7 and Figures 4-5 (see [17]):

Table 6
ARL using Several Ranked Data Techniques when $m = 4$

δ	SRS	RSS	ERSS	MMRSS	MMMRSS
0	369.4126	349.0401	331.785	329.8277	306.7485
0.1	337.7238	312.3048	304.5995	263.1914	230.9469
0.2	312.9890	229.4104	243.2498	178.5714	167.9699
0.3	266.0990	166.7500	179.6945	140.8451	130.3781
0.4	200.7226	115.9420	126.3584	95.2371	94.2507
0.5	158.1778	76.7048	88.1213	74.7269	63.6132
0.6	119.2890	52.7816	60.2882	56.1698	47.1476
1.0	43.7101	14.1495	17.4028	18.2148	15.8328
1.4	18.3006	5.1341	6.3553	7.4139	6.4362
1.8	8.6781	2.4803	3.0136	3.8332	3.2311
2.2	4.7293	1.5504	1.8029	2.2627	1.9721
2.6	2.9022	1.1932	1.3136	1.5776	1.4269
3.0	1.9999	1.0584	1.1109	1.2586	1.1741
3.4	1.5244	1.0138	1.0332	1.1021	1.0648

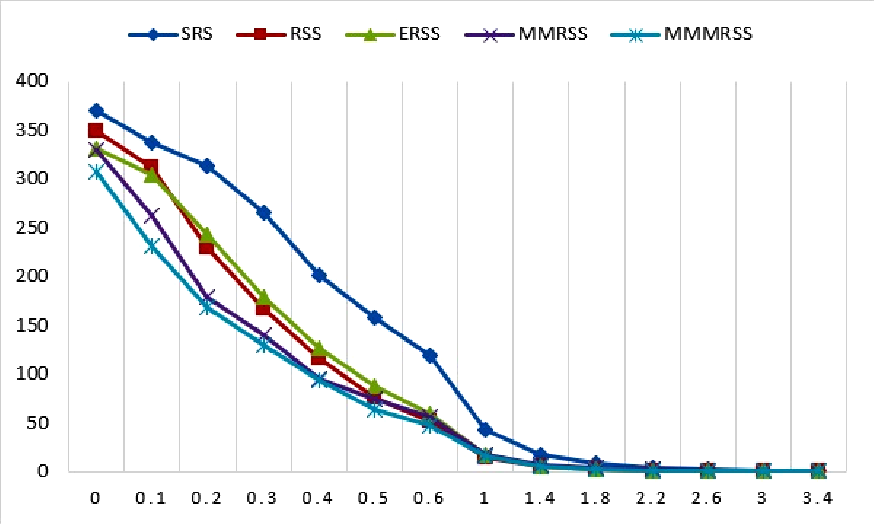


Figure 4: Comparison between ARL for Different Sampling Techniques when $m = 4$

Table 7
ARL using Several Ranked Data Techniques when $m = 6$

δ	SRS	RSS	ERSS	MMRSS	MMMRSS
0	375.5163	346.1405	331.1258	340.1361	312.5
0.1	349.406	300.8423	309.8853	279.3296	204.499
0.2	309.0235	218.7705	232.6664	207.9002	164.7446
0.3	247.0356	137.1178	158.4033	143.4720	113.5074
0.4	196.8891	87.0019	110.6072	106.1571	77.2798
0.5	154.9427	55.9503	75.0356	71.9425	54.9451
0.6	120.8021	37.0508	51.0882	52.7148	40.0481
1.0	43.5749	9.0035	13.6605	18.1258	13.0194
1.4	18.2435	3.2467	4.9405	7.3714	5.2888
1.8	8.6944	1.7118	2.3992	3.5957	2.7297
2.2	4.7149	1.2144	1.5167	2.1614	1.7361
2.6	2.9026	1.0530	1.1770	1.5121	1.2979
3.0	1.9961	1.0095	1.0521	1.2188	1.1124
3.4	1.5244	1.0012	1.0118	1.0847	1.0354

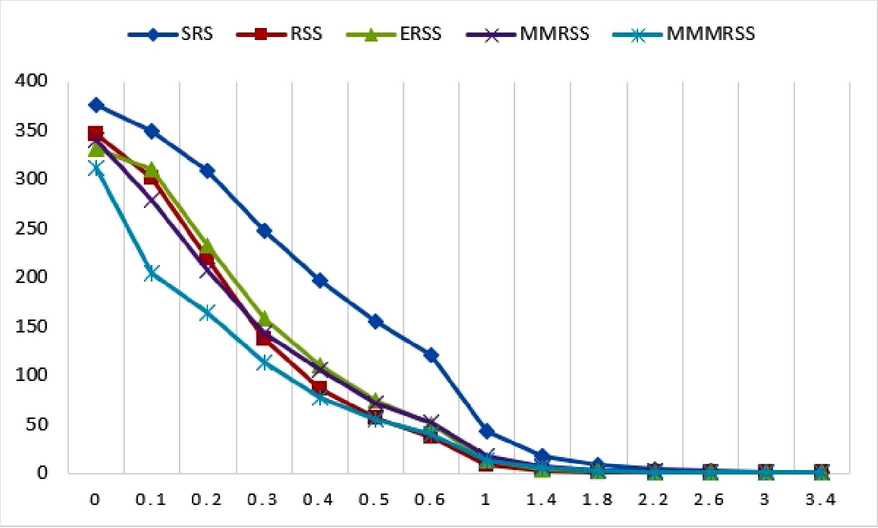


Figure 5: Comparison between ARL for Different Sampling Techniques when $m = 6$

6. APPLICATION

In this section, we used a real data set to investigate the implementation of the suggested Shewhart control charts based on ERSS, RSS, MMRSS, MMMRSS and SRS techniques. We also plot control charts based on these techniques to study the detection ability of the suggested control chart.

The results of simulation in the previous section demonstrated that Mean-MMMRSS chart performs better to detect shift in the process mean than SRS, RSS, ERSS and MMRSS mean control charts. In order to clarify the implementation and performance of the suggested MMMRSS-type charts in real-life situation, an example of dataset is considered. The data used in this analysis was downloaded from the Goddard Institute for Space Studies (GISS) website at this linkhttp://data.giss.nasa.gov/gistemp/. GISS is part of the National Aeronautics and Space Administration (NASA).

Discussions about global warming often give the impression that the phenomenon is exclusively about projections of the future. Indeed, the rising global average temperature has already had vital impacts on infrastructure and economies. Therefore studying such data is important for our life. The data covered the years from 1880- 2017. Descriptive statistics of normalized global temperatures are presented in Table 8.

Table 8
Descriptive Statistics of Global Temperatures

Min	Q_1	Median	Mean	Q_1	Max	σ
-0.80000	-0.23000	-0.05000	0.02612	0.23000	1.34000	0.3479158

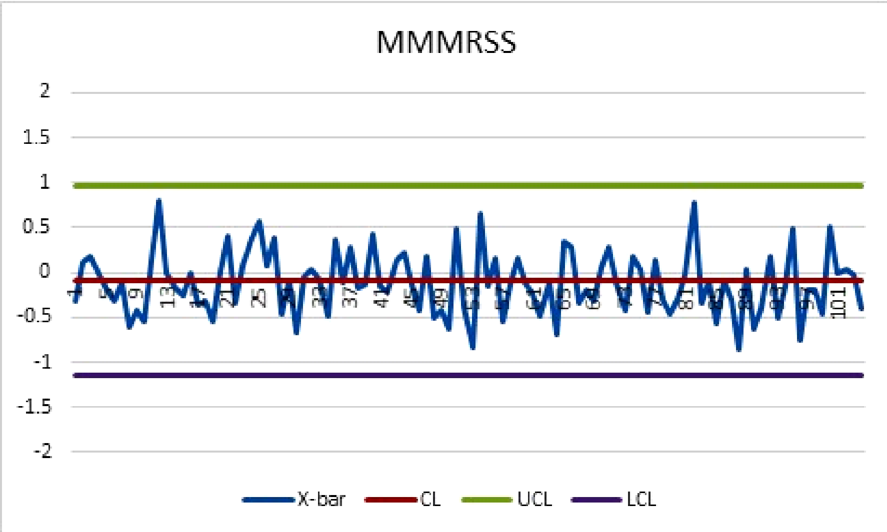
It is of interest to estimate the mean of global temperatures using the suggested technique and compare it with several sampling technique. A summary of the mean and standard deviation based on SRS, RSS, ERSS, MMRSS and MMRSS is presented in the following Table 9.

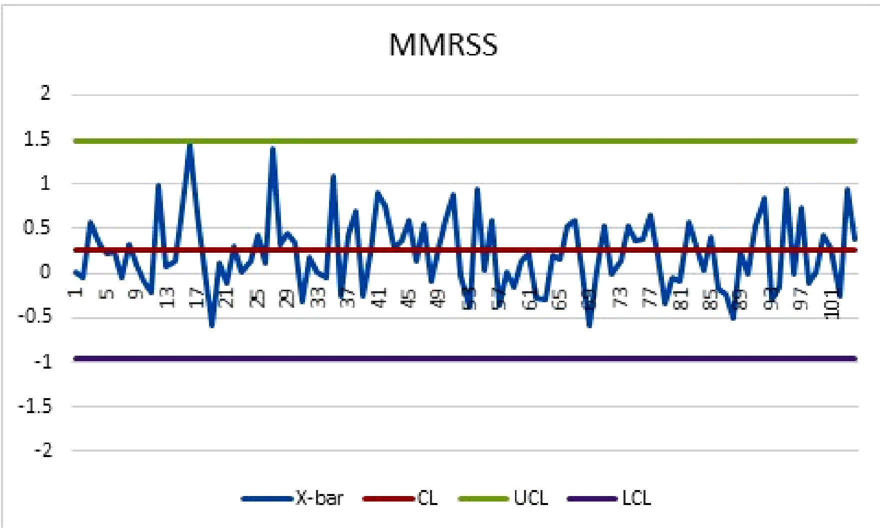
Table 9
A Summary of the Selected Samples using Several Techniques ($m = 4$)

$\bar{X}_{(SRS)}$	0.02848558	$s^2{}_{(SRS)}$	0.1949774
$\bar{X}_{(RSS)}$	0.05387019	$s^2{}_{(RSS)}$	0.1191397
$\bar{X}_{(ERSS)}$	0.07637019	$s^2{}_{(ERSS)}$	0.1135904
$\bar{X}_{(MMRSS)}$	0.1309135	$s^2{}_{(MMRSS)}$	0.1648444
$\bar{X}_{(MMMRSS)}$	0.02425481	$s^2{}_{(MMMRSS)}$	0.1425259

With the purpose of establishing statistical control of the temperature in the process, X-bar SRS, X-bar RSS, X-bar ERSS, X-bar MMRSS and X-bar MMMRSS charts are applied. It is remarkable to mention that our data set is skewed, and since we assume normality of the process in our study, so for that purpose we performed a normalization transformation using the central limit theorem (CLT).

In order to compare the Shewhart X-bar control charts based on the ERSS, RSS, MMRSS, MMMRSS and SRS, we need to collect data under the considered sampling techniques. For this purpose, we draw 104 samples, each of size 4, from the 1656 measurements of the global temperatures under the different sampling techniques. Based on these 104 samples, control limits of the Shewhart- X-bar control charts for the considered sampling techniques are estimated and plotted along with the values of the corresponding plotting-statistics versus sample number in Figure 6.





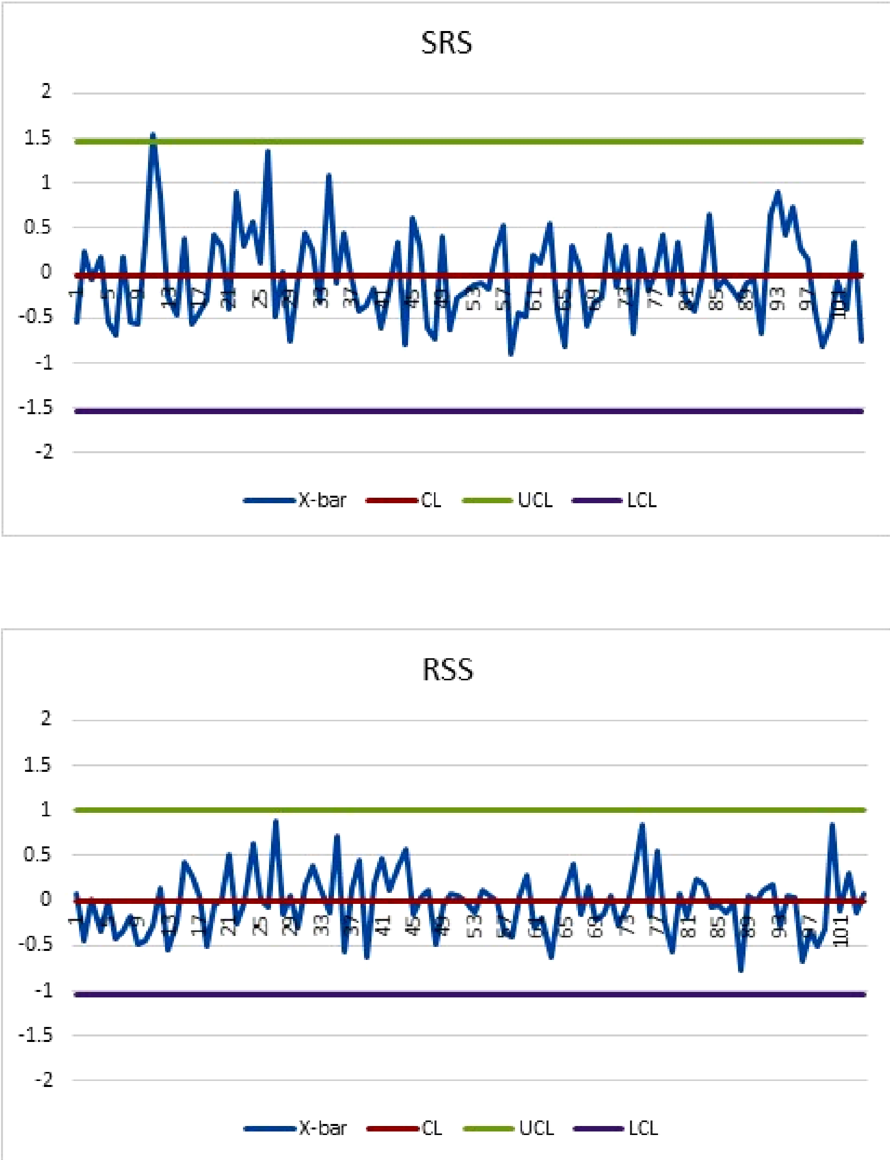


Figure 6: Shewhart-Type Mean Control Chart using Different Sampling Techniques

The distance between the estimated control limits based on MMMRSS is less as compared with the distance between the estimated control limits based on SRS and MMRSS and comparable with the based on under RSS and ERSS. The variation among the sample means estimated based on MMMRSS have also less variability as compared with those estimated based on SRS and MMRSS. These interesting features make the MMMRSS control chart more efficient in detection random shifts in the process mean as compared with the control charts based on SRS and MMRSS.

7. CONCLUSION

In order to overcome the problem of using MMRSS when the distribution is asymmetric, a cost-effective and efficient sampling technique (MMMRSS) for precise estimation of the population parameters is suggested. The MMMRSS is a feasible and informative sampling strategy that utilizes as few “actual" measurements of units as possible. This technique demonstrates its superiority in estimating the population mean better than the SRS and MMRSS techniques for all symmetric and asymmetric distributions considered in this study. The ARL results of the suggested chart are computed using simulation study and compared with that of considered charts. We found that the MMMRSS technique produces an effective control chart for the mean, which is not only better than the SRS and MMRSS techniques but also comparable with RSS and ERSS techniques. The present study can be extended by constructing other types of control charts based on MMMRSS.

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