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FUZZY EQUIVALENCE FACTORS OF THE RELIABILITY OF A DEPENDENT SERIES SYSTEM

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ABSTRACT

In this work, a series system consisted of n non-identical and dependent units is analyzed. The lifetimes of the system units are assumed to follow an exponential distribution with fuzzy parameters. Hougaard's copula is used to obtain the reliability function of the system. The reliability of the system is improved by using reduction, k cold standby duplication, and mixed standby duplication methods. The reliability equivalence factors for the system are introduced. An algorithm is established to show how to obtain the intervals for the fuzzy reliability equivalence factors for the introduced model. An application based on real data is introduced to show the results and to compare different improvement methods.

KEYWORDS

Copula, reliability, series system, exponential distribution, reduction method, cold standby, mixed standby, equivalence factors, fuzzy number.

1. INTRODUCTION

The reliability of the system can be improved by applying many methods. Reduction method can be used in improving the reliability of the system by multiplying the failure rates of a set of the system units by a factor ρ such that $(0 < \rho < 1)$. There are other methods which can be used in improving systems reliability depending on increasing the redundancy of the units in the system such as cold standby, warm standby and hot standby duplication methods. Usually, cold standby, warm standby and hot standby duplication methods are used in literature. In this work, k standby and mixed standby duplication method are used to improve the reliability of the system.

In k cold standby duplication method, a set of the system units is connected with k units in cold standby configuration where the main unit is working and k units are kept in cold standby mode. When the main unit fails it will be replaced immediately by a cold standby unit. In mixed standby duplication method, a set of the system units is connected with one cold and one warm standby units. When the main unit fails it will be replaced by a warm standby unit and the cold standby unit will works as a warm standby unit. Once the working unit fails it will be replaced immediately by the remaining warm standby unit. These methods are used to improve the efficiency of the system and increase the systems reliability.

Sarhan (2000) introduced the reliability equivalence factors of n independent and non-identical units of a series system by using the survival function and mean time to failure as characteristics to compare different system designs. Sarhan (2002) considered a radar system in an aircraft which consists of three independent and non-identical units with constant failure rates. Sarhan (2004) introduced the reliability equivalence factors of a bridge network system. Sarhan & Mustafa (2006) proposed the reliability equivalence factors of a series system which consists of n independent and non-identical units. Sarhan et al. (2008) introduced the reliability equivalence factors of a parallel-series system assuming that the failure rates of the system units are constant. Sarhan (2009) introduced the reliability equivalence factors of a general series-parallel system and the system units are assumed to be independent and their lifetimes to have exponential distributions.

Xia & Zhang (2007) analyzed the reliability equivalence factors of a parallel system assuming that the failure rates of the system units follow gamma distribution. El-Damcese (2009) introduced the reliability equivalence factors of a series-parallel system where the system units are independent and identical with lifetimes follow Weibull distribution. Reliability equivalence factors for some systems with mixture Weibull failure rates were introduced by Mustafa (2009). Mustafa and El-Faheem (2014) presented the reliability equivalence factors of a system with mixture of n independent and non-identical units subject to delay time. Ezzati and Rasouli (2015) improved the system reliability using linear-exponential function. El-Damcese and Ayoub (2011) obtained the two-dimensional reliability modeling equivalence factors of n independent and identical units for a parallel system by using bivariate Weibull model.

In this paper, analysis of the reliability equivalence factors of a series system consisting of n dependent and non-identical units is introduced. The reliability function of the original system is derived by using the concepts of copula subject to the exponential distribution. The reliability of the original system is improved according to reduction, k cold standby duplication and mixed standby duplication methods. The reliability equivalence factors are introduced to compare different system designs. An algorithm is introduced to show the steps which can be applied to find the intervals for the fuzzy reliability equivalence factors. An application has been carried out to interpret how one can utilize the theoretical results obtained in this study and to compare the different reliability factors of the system.

2. RELIABILITY ANALYSIS

Sklar's theorem states that the joint probability distribution of random variables can be expressed in terms of a copula function and their marginal distributions. That is if $F(x_1, x_2, \dots, x_n)$ is the joint cumulative distribution function with marginal cumulative distribution functions $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ then there exists an n-dimensional copula C such that for all real $x_1, x_2, \dots, x_n$

$$F(x_1, x_2, \dots, x_n) = \mathrm{C}[F_1(x_1), F_2(x_2), \dots, F_n(x_n)]$$

Now, consider a system consisted of n dependent units connected in series. The joint survival function of the original system is derived as follows

$$S(t) = \mathrm{Prob}(\min(T_1, T_2, \dots, T_n) > t) = \mathrm{Prob}(T_1 > t, T_2 > t, \dots, T_n > t)$$

According Hougaard's (1986) copula family, the survival function of the system can be written as

$$S(t) = exp\left(-\left[\{-\ln S_1(t)\}^{1/\gamma} + \{-\ln S_2(t)\}^{1/\gamma} + \cdots + \{-\ln S_n(t)\}^{1/\gamma}\right]^{\gamma}\right), \gamma \geq 1$$

Appling this definition of copula, the reliability of the series system of n dependent units subject to the exponential distribution is obtained as

$$R(t) = exp\left\{-t\left(\lambda_1^{1/\gamma} + \lambda_2^{1/\gamma} + \cdots + \lambda_n^{1/\gamma}\right)^{\gamma}\right\}, \gamma \geq 1$$

$$R(t) = exp\left\{-t\left(\sum_{i=1}^{n} \lambda_i^{1/\gamma}\right)^{\gamma}\right\}, \gamma \geq 1$$

where λ_i is the failure rate of the i-th unit.

3. REDUCTION METHOD

In this method, it is supposed that the failure rates of a set r of the units of the system are decreased by multiplying by a factor $\rho, 0 < \rho < 1$ and hence the system reliability function is obtained as follows.

$$R_r(t) = exp\left\{-t\left(\sum_{i\in r}(\rho\lambda_i)^{1/\gamma} + \sum_{i\notin r}\lambda_i^{1/\gamma}\right)^{\gamma}\right\}, for\ r \subset \{1, 2, \dots, n\} \tag{1}$$

4. K COLD STANDBY DUPLICATION METHOD

In this method, it is supposed that a set c of the system units are duplicated by k cold standby units. The reliability of each unit duplicated with k identical cold standby units will be given by.

$$R_i(t) = e^{-\lambda_i t}\sum_{j=0}^{k}\frac{(\lambda_i t)^j}{j!}$$

The reliability function of the system will be given by

$$R_c(t) = exp\left\{-\left(\sum_{i\in c}\left(\lambda_i t - \ln\left(\sum_{j=0}^{k}\frac{(\lambda_i t)^j}{j!}\right)\right)^{1/\gamma} + \sum_{i\notin c}(\lambda_i t)^{1/\gamma}\right)^{\gamma}\right\}, \quad \text{for } c \subset \{1, 2, \dots, n\} \tag{2}$$

5. MIXED STANDBY DUPLICATION METHOD

In this method, it is supposed that a set m of the units of the system is connected with a warm and a cold standby units. The reliability function of each unit is given by

$$R_i(t) = \frac{-\lambda_i\left(2\beta_i + \lambda_i + \beta_i\lambda_i t + {\beta_i}^2 t\right)e^{-(\lambda_i+\beta_i)t} + (\lambda_i + \beta_i)^2 e^{-\lambda_i t}}{{\beta_i}^2}, \beta_i < \lambda_i$$

where β_i is the failure rate of the warm standby unit. The reliability function of the system will be given by

$$R_m(t) = exp\left\{-\left(\sum_{i\in m}\left(\lambda_i t - \ln\left(\frac{-\lambda_i(2\beta_i+\lambda_i+\beta_i\lambda_i t+\beta_i{}^2 t)e^{-\beta_i t}+(\lambda_i+\beta_i)^2}{\beta_i{}^2}\right)\right)^{1/\gamma} + \sum_{i\notin m}(\lambda_i t)^{1/\gamma}\right)^{\gamma}\right\}, \text{for } m \subset \{1,2,\dots,n\} \quad (3)$$

6. RELIABILITY EQUIVALENCE FACTORS

The system reliability equivalence factors in case of k cold standby duplication method are obtained by equating equation (1) and equation (2) and the result is given as follows.

$$\rho_{r,c} = \left\{\frac{\sum_{i\in c}\left(\lambda_i - \frac{\ln\left(\sum_{j=0}^{k}\frac{(\lambda_i t)^j}{j!}\right)}{t}\right)^{1/\gamma} + \sum_{i\notin c}\lambda_i{}^{1/\gamma} - \sum_{i\notin r}\lambda_i^{1/\gamma}}{\sum_{i\in r}\lambda_i^{1/\gamma}}\right\}^{\gamma} \quad (4)$$

The system reliability equivalence factors in case of mixed standby duplication method are obtained by equating equation (1) and equation (3) and the result is given as follows.

$$\rho_{r,m} = \left\{\frac{\sum_{i\in m}\left(\lambda_i - \frac{\ln\left(\frac{-\lambda_i(2\beta_i+\lambda_i+\beta_i\lambda_i t+\beta_i{}^2 t)e^{-\beta_i t}+(\lambda_i+\beta_i)^2}{\beta_i{}^2}\right)}{t}\right)^{1/\gamma} + \sum_{i\notin m}\lambda_i{}^{1/\gamma} - \sum_{i\notin r}\lambda_i^{1/\gamma}}{\sum_{i\in r}\lambda_i^{1/\gamma}}\right\}^{\gamma} \quad (5)$$

Equations (4) and (5) are in very complicated form and numerical solution can be found by using Maple package program. Now, the parameters of the exponential distribution will be considered as fuzzy number with triangular membership function.

7. ANALYSIS OF FUZZINESS

7.1 Parameter Estimation

The point estimator for the parameter λ is given as

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i}$$

The $(1-\delta)100\%$ confidence interval for the parameter λ is given by

$$\left[\frac{\chi^2\left(2n, {}^{\delta}/_{2}\right)}{2\sum_{i=1}^{n} t_i}, \frac{\chi^2\left(2n, 1-{}^{\delta}/_{2}\right)}{2\sum_{i=1}^{n} t_i}\right]$$

7.2 Parameters as Fuzzy Numbers

Now consider that the parameter λ of the exponential distribution are fuzzy numbers with triangular membership function given as follows

$$\mu(\lambda) = \begin{cases} 0, \lambda < L \\ (\lambda - L)/(M - L), L \le \lambda \le M \\ (U - \lambda)/(U - M), M \le \lambda \le U \\ 0, \lambda > U \end{cases},$$

where *M* is the point estimator of λ and the numbers *L* and *U* are the lower and upper limits of the confidence interval of λ. The intervals for the fuzzy reliability equivalence factors of the system can be obtained by applying the following algorithm.

7.3 Algorithm

Step 1: Generate random samples of different sizes $l_i\ (i = 1, 2, \dots, n)$ from the exponential distribution at fixed values of the parameter λ by using the relation:

$$T = -\left(\frac{\ln(1-u)}{\lambda}\right), 0 < u < 1$$

Step 2: Calculate the point estimates and the confidence intervals for the parameters λ at a level of significance δ.

Step 3: Calculate the intervals for the fuzzy parameters by substituting in the following relation:

$$\left[\tilde{\lambda}^L, \tilde{\lambda}^U\right] = [L + (\alpha - cut)(M - L), U - (\alpha - cut)(U - M)]$$

where $\alpha - cut = \{0, 0.1, 0.2, \dots, 1\}$ and *M* is the point estimator of λ and [*L*,*U*] is the confidence interval limits of λ.

Step 4: Substituting in equations (4) and (5) to obtain the intervals for the fuzzy reliability equivalence factors of the system.

$$\left[\tilde{\rho}_{r,c}^L, \tilde{\rho}_{r,c}^U\right], \left[\tilde{\rho}_{r,m}^L, \tilde{\rho}_{r,m}^U\right].$$

8. REAL DATA APPLICATION

In this application, consider a system consists of three non-identical units connected in series. Real data from Lawless (2003) are used and these data are showed in Table 1.

Table 1
Random Samples with Different Sizes

Sample Size	Random Samples
$l_1 = 15$	$S_1 = \begin{Bmatrix} 0.4, 82.85, 9.88, 89.29, 215.10, 2.75, 0.79, 15.93, \\ 3.91, 0.27, 0.69, 100.58, 27.80, 13.95, 53.24 \end{Bmatrix}$
$l_2 = 19$	$S_2 = \begin{Bmatrix} 0.96, 4.15, 0.19, 0.78, 8.01, 31.75, 7.35, 6.50, 8.27, 33.91, \\ 32.52, 3.16, 4.85, 2.78, 4.67, 1.31, 12.06, 36.71, 72.89 \end{Bmatrix}$
$l_3 = 15$	$S_3 = \begin{Bmatrix} 1.97, 0.59, 2.58, 1.69, 2.71, 25.50, 0.35, 0.99, \\ 3.99, 3.67, 2.07, 0.96, 5.35, 2.90, 13.77 \end{Bmatrix}$

The results for the point estimations and the 95% confidence intervals for the parameters $\lambda_1, \lambda_2, \lambda_3$ are obtained as follows

$$\hat{\lambda}_1 = 0.0242, \hat{\lambda}_2 = 0.0696, \hat{\lambda}_3 = 0.2171$$

Confidence interval for $\lambda_1 = [0.0135, 0.0380]$

Confidence interval for $\lambda_2 = [0.0419, 0.1042]$

Confidence interval for $\lambda_3 = [0.1215, 0.3399]$

The results for the intervals for the fuzzy parameters $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3$ are shown in Table 2.

Table 2
The Results for the Intervals for the Fuzzy Parameters $\tilde{\boldsymbol{\lambda}}_1, \tilde{\boldsymbol{\lambda}}_2, \tilde{\boldsymbol{\lambda}}_3$

$\alpha - cut$	$\left[\tilde{\lambda}_1^{\ L}, \tilde{\lambda}_1^{\ U}\right]$	$\left[\tilde{\lambda}_2^{\ L}, \tilde{\lambda}_2^{\ U}\right]$	$\left[\tilde{\lambda}_3^{\ L}, \tilde{\lambda}_3^{\ U}\right]$
0.1	[0.0145, 0.0366]	[0.0446, 0.1007]	[0.1310, 0.3276]
0.2	[0.0156, 0.0352]	[0.0474, 0.0972]	[0.1406, 0.3153]
0.3	[0.0167, 0.0338]	[0.0502, 0.0938]	[0.1501, 0.3030]
0.4	[0.0177, 0.0324]	[0.0529, 0.0903]	[0.1597, 0.2907]
0.5	[0.0188, 0.0311]	[0.0557, 0.0869]	[0.1693, 0.2785]
0.6	[0.0199, 0.0297]	[0.0585, 0.0834]	[0.1788, 0.2662]
0.7	[0.0209, 0.0283]	[0.0612, 0.0799]	[0.1884, 0.2539]
0.8	[0.0220, 0.0269]	[0.0640, 0.0765]	[0.1979, 0.2416]
0.9	[0.0231, 0.0255]	[0.0668, 0.0730]	[0.2075, 0.2293]

Now, let us suppose that: $\beta_1 = \beta_2 = \beta_3 = 0.1, \gamma = 1.1, t = 10, k = 2$.

The intervals for the fuzzy reliability equivalence factors of the system are obtained and the results are shown in Tables 3, 4.

Table 3
The Results for the Intervals of the Fuzzy Reliability Equivalence Factors $\left[\tilde{\rho}^{L}_{r,c}, \tilde{\rho}^{U}_{r,c}\right]$

α-cut = 0.1							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.003, 0.017]	-	-	-	-	-	-
{2}	[0.582, 0.613]	[0.023, 0.084]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.609, 0.669]	[0.119, 0.308]	[0.469, 0.529]	[0.006, 0.179]	-	-
{1, 2}	[0.698, 0.714]	[0.255, 0.325]	-	[0.018, 0.064]	-	-	-
{1, 3}	[0.870, 0.872]	[0.654, 0.708]	[0.214, 0.387]	[0.530, 0.584]	[0.104, 0.270]	-	-
{2, 3}	[0.891, 0.892]	[0.713, 0.752]	[0.343, 0.477]	[0.610, 0.646]	[0.247, 0.376]	[0.092, 0.249]	[0.011, 0.154]
{1, 2, 3}	[0.901, 0.902]	[0.739, 0.774]	[0.399, 0.523]	[0.644, 0.678]	[0.310, 0.431]	[0.164, 0.313]	[0.083, 0.226]
α-cut = 0.2							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.003, 0.015]	-	-	-	-	-	-
{2}	[0.582, 0.610]	[0.026, 0.080]	-	-	-	-	-
{3}	[0.852, 0.854]	[0.613, 0.666]	[0.130, 0.298]	[0.473, 0.526]	[0.015, 0.170]	-	-
{1, 2}	[0.699, 0.712]	[0.260, 0.321]	-	[0.020, 0.061]	-	-	-
{1, 3}	[0.870, 0.871]	[0.658, 0.705]	[0.225, 0.378]	[0.534, 0.582]	[0.114, 0.262]	-	-
{2, 3}	[0.891, 0.892]	[0.716, 0.750]	[0.350, 0.470]	[0.612, 0.645]	[0.254, 0.369]	[0.101, 0.240]	[0.018, 0.147]
{1, 2, 3}	[0.901, 0.902]	[0.741, 0.773]	[0.406, 0.517]	[0.646, 0.677]	[0.316, 0.425]	[0.173, 0.306]	[0.091, 0.218]

α-cut = 0.3							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.004, 0.014]	-	-	-	-	-	-
{2}	[0.583, 0.606]	[0.029, 0.076]	-	-	-	-	-
{3}	[0.852, 0.854]	[0.617, 0.663]	[0.141, 0.289]	[0.477, 0.523]	[0.024, 0.161]	-	-
{1, 2}	[0.699, 0.710]	[0.264, 0.317]	-	[0.022, 0.058]	-	-	-
{1, 3}	[0.870, 0.871]	[0.662, 0.703]	[0.235, 0.369]	[0.537, 0.579]	[0.123, 0.253]	-	-
{2, 3}	[0.891, 0.892]	[0.719, 0.748]	[0.358, 0.463]	[0.614, 0.643]	[0.260, 0.362]	[0.109, 0.232]	[0.025, 0.139]
{1, 2, 3}	[0.901, 0.902]	[0.744, 0.771]	[0.413, 0.510]	[0.648, 0.675]	[0.323, 0.418]	[0.181, 0.298]	[0.099, 0.210]
α-cut = 0.4							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.004, 0.013]	-	-	-	-	-	-
{2}	[0.584, 0.604]	[0.031, 0.072]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.622, 0.660]	[0.152, 0.279]	[0.482, 0.521]	[0.034, 0.151]	-	-
{1, 2}	[0.700, 0.709]	[0.267, 0.313]	-	[0.024, 0.055]	-	-	-
{1, 3}	[0.870, 0.871]	[0.666, 0.700]	[0.244, 0.360]	[0.542, 0.576]	[0.133, 0.244]	-	-
{2, 3}	[0.891, 0.892]	[0.721, 0.746]	[0.365, 0.455]	[0.617, 0.641]	[0.267, 0.355]	[0.118, 0.223]	[0.033, 0.130]
{1, 2, 3}	[0.901, 0.902]	[0.746, 0.769]	[0.419, 0.504]	[0.651, 0.673]	[0.329, 0.411]	[0.190, 0.290]	[0.107, 0.203]

α-cut = 0.5							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.005, 0.012]	-	-	-	-	-	-
{2}	[0.585, 0.602]	[0.034, 0.068]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.625, 0.657]	[0.162, 0.268]	[0.485, 0.517]	[0.043, 0.141]	-	-
{1, 2}	[0.700, 0.708]	[0.271, 0.310]	-	[0.026, 0.052]	-	-	-
{1, 3}	[0.870, 0.871]	[0.669, 0.698]	[0.254, 0.351]	[0.545, 0.574]	[0.142, 0.235]	-	-
{2, 3}	[0.891, 0.892]	[0.724, 0.744]	[0.372, 0.448]	[0.619, 0.639]	[0.274, 0.348]	[0.127, 0.215]	[0.040, 0.122]
{1, 2, 3}	[0.901, 0.902]	[0.748, 0.767]	[0.426, 0.497]	[0.653, 0.671]	[0.336, 0.405]	[0.198, 0.282]	[0.115, 0.195]
α-cut = 0.6							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.005, 0.011]	-	-	-	-	-	-
{2}	[0.586, 0.599]	[0.037, 0.064]	-	-	-	-	-
{3}	[0.852, 0.853]	[0.629, 0.654]	[0.173, 0.258]	[0.488, 0.514]	[0.052, 0.131]	-	-
{1, 2}	[0.700, 0.707]	[0.275, 0.305]	-	[0.028, 0.049]	-	-	-
{1, 3}	[0.870, 0.871]	[0.669, 0.695]	[0.264, 0.341]	[0.547, 0.571]	[0.151, 0.226]	-	-
{2, 3}	[0.891, 0.892]	[0.726, 0.742]	[0.379, 0.440]	[0.621, 0.637]	[0.281, 0.340]	[0.135, 0.206]	[0.048, 0.113]
{1, 2, 3}	[0.901, 0.902]	[0.750, 0.766]	[0.433, 0.490]	[0.654, 0.669]	[0.343, 0.398]	[0.206, 0.273]	[0.122, 0.186]

α-cut = 0.7

{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.006, 0.010]	-	-	-	-	-	-
{2}	[0.588, 0.598]	[0.040, 0.060]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.632, 0.651]	[0.183, 0.247]	[0.492, 0.511]	[0.061, 0.121]	-	-
{1, 2}	[0.701, 0.706]	[0.278, 0.301]	-	[0.030, 0.046]	-	-	-
{1, 3}	[0.870, 0.871]	[0.675, 0.692]	[0.273, 0.331]	[0.551, 0.568]	[0.160, 0.216]	[0.004, 0.064]	-
{2, 3}	[0.891, 0.892]	[0.728, 0.740]	[0.386, 0.432]	[0.623, 0.635]	[0.288, 0.332]	[0.143, 0.196]	[0.056, 0.105]
{1, 2, 3}	[0.901, 0.902]	[0.752, 0.764]	[0.440, 0.482]	[0.657, 0.668]	[0.349, 0.390]	[0.214, 0.264]	[0.130, 0.178]

α-cut = 0.8

{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.006, 0.009]	-	-	-	-	-	-
{2}	[0.590, 0.596]	[0.043, 0.056]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.635, 0.648]	[0.193, 0.235]	[0.495, 0.508]	[0.070, 0.110]	-	-
{1, 2}	[0.702, 0.705]	[0.281, 0.297]	-	[0.032, 0.043]	-	-	-
{1, 3}	[0.870, 0.871]	[0.678, 0.689]	[0.282, 0.321]	[0.553, 0.565]	[0.169, 0.206]	[0.013, 0.053]	-
{2, 3}	[0.891, 0.892]	[0.730, 0.738]	[0.394, 0.424]	[0.625, 0.633]	[0.295, 0.325]	[0.151, 0.187]	[0.063, 0.096]
{1, 2, 3}	[0.901, 0.902]	[0.754, 0.762]	[0.446, 0.475]	[0.658, 0.666]	[0.355, 0.383]	[0.222, 0.255]	[0.137, 0.169]

α-cut = 0.9							
{r} \ {c}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.007, 0.008]	-	-	-	-	-	-
{2}	[0.592, 0.594]	[0.046, 0.052]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.638, 0.644]	[0.202, 0.224]	[0.498, 0.504]	[0.079, 0.099]	-	-
{1, 2}	[0.703, 0.704]	[0.285, 0.292]	-	[0.034, 0.040]	-	-	-
{1, 3}	[0.870, 0.871]	[0.680, 0.686]	[0.291, 0310]	[0.556, 0.562]	[0.177, 0.196]	[0.022, 0.042]	-
{2, 3}	[0.891, 0.892]	[0.731, 0.736]	[0.401, 0.416]	[0.626, 0.631]	[0.302, 0.317]	[0.159, 0.177]	[0.070, 0.087]
{1, 2, 3}	[0.901, 0.902]	[0.756, 0.759]	[0.453, 0.467]	[0.660, 0.664]	[0.362, 0.376]	[0.229, 0.246]	[0.144, 0.161]

Table 4

The Results for the Intervals of the Fuzzy Reliability Equivalence Factors $\left[\tilde{\rho}^{L}_{r,m}, \tilde{\rho}^{U}_{r,m}\right]$

α-cut = 0.1							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.005, 0.021]	-	-	-	-	-	-
{2}	[0.584, 0.615]	[0.028, 0.091]	-	-	-	-	-
{3}	[0.853, 0.855]	[0.611, 0.671]	[0.126, 0.313]	[0.472, 0.533]	[0.012, 0.185]	-	-
{1, 2}	[0.700, 0.715]	[0.260, 0.331]	-	[0.022, 0.070]	-	-	-
{1, 3}	[0.870, 0.873]	[0.221, 0.392]	[0.221, 0.392]	[0.533, 0.587]	[0.111, 0.276]	-	-
{2, 3}	[0.892, 0.893]	[0.715, 0.754]	[0.349, 0.481]	[0.612, 0.649]	[0.253, 0.381]	[0.098, 0.255]	[0.016, 0.161]
{1, 2, 3}	[0.902, 0.903]	[0.740, 0.776]	[0.404, 0.527]	[0.646, 0.681]	[0.315, 0.435]	[0.170, 0.319]	[0.089, 0.232]

α-cut = 0.2							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.006, 0.020]	-	-	-	-	-	-
{2}	[0.585, 0.611]	[0.031, 0.087]	-	-	-	-	-
{3}	[0.853, 0.855]	[0.616, 0.669]	[0.137, 0.304]	[0.476, 0.530]	[0.021, 0.176]	-	-
{1, 2}	[0.700, 0.713]	[0.264, 0.327]	-	[0.024, 0.067]	-	-	-
{1, 3}	[0.870, 0.872]	[0.231, 0.383]	[0.231, 0.383]	[0.537, 0.585]	[0.120, 0.268]	-	-
{2, 3}	[0.892, 0.893]	[0.718, 0.752]	[0.356, 0.474]	[0.614, 0.647]	[0.259, 0.374]	[0.107, 0.246]	[0.024, 0.153]
{1, 2, 3}	[0.902, 0.903]	[0.743, 0.775]	[0.411, 0.521]	[0.648, 0.679]	[0.322, 0.429]	[0.179, 0.311]	[0.097, 0.224]
α-cut = 0.3							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.006, 0.019]	-	-	-	-	-	-
{2}	[0.586, 0.608]	[0.034, 0.083]	-	-	-	-	-
{3}	[0.853, 0.855]	[0.620, 0.666]	[0.148, 0.294]	[0.480, 0.527]	[0.030, 0.167]	-	-
{1, 2}	[0.701, 0.711]	[0.268, 0.323]	-	[0.026, 0.064]	-	-	-
{1, 3}	[0.870, 0.872]	[0.241, 0.374]	[0.241, 0.374]	[0.540, 0.582]	[0.130, 0.259]	-	-
{2, 3}	[0.892, 0.893]	[0.721, 0.750]	[0.363, 0.467]	[0.617, 0.645]	[0.266, 0.367]	[0.116, 0.238]	[0.031, 0.145]
{1, 2, 3}	[0.902, 0.903]	[0.745, 0.773]	[0.418, 0.515]	[0.650, 0.677]	[0.328, 0.423]	[0.188, 0.304]	[0.105, 0.217]

α-cut = 0.4							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.007, 0.017]	-	-	-	-	-	-
{2}	[0.587, 0.606]	[0.037, 0.079]	-	-	-	-	-
{3}	[0.853, 0.855]	[0.624, 0.663]	[0.159, 0.284]	[0.485, 0.524]	[0.040, 0.158]	-	-
{1, 2}	[0.701, 0.710]	[0.272, 0.319]	-	[0.028, 0.061]	-	-	-
{1, 3}	[0.871, 0.872]	[0.251, 0.365]	[0.251, 0.365]	[0.544, 0.580]	[0.139, 0.250]	-	-
{2, 3}	[0.892, 0.893]	[0.723, 0.748]	[0.370, 0.460]	[0.619, 0.644]	[0.273, 0.360]	[0.125, 0.230]	[0.039, 0.137]
{1, 2, 3}	[0.902, 0.903]	[0.748, 0.771]	[0.424, 0.508]	[0.653, 0.676]	[0.335, 0.416]	[0.196, 0.296]	[0.113, 0.209]
α-cut = 0.5							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.008, 0.016]	-	-	-	-	-	-
{2}	[0.587, 0.603]	[0.040, 0.075]	-	-	-	-	-
{3}	[0.853, 0.855]	[0.628, 0.660]	[0.169, 0.274]	[0.488, 0.521]	[0.049, 0.148]	-	-
{1, 2}	[0.701, 0.709]	[0.276, 0.315]	-	[0.031, 0.057]	-	-	-
{1, 3}	[0.871, 0.872]	[0.261, 0.356]	[0.261, 0.356]	[0.547, 0.577]	[0.148, 0.241]	-	-
{2, 3}	[0.891, 0.892]	[0.726, 0.746]	[0.378, 0.453]	[0.621, 0.641]	[0.280, 0.353]	[0.133, 0.221]	[0.047, 0.128]
{1, 2, 3}	[0.902, 0.903]	[0.750, 0.769]	[0.431, 0.501]	[0.655, 0.674]	[0.341, 0.409]	[0.204, 0.288]	[0.121, 0.201]

α-cut = 0.6							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.008, 0.015]	-	-	-	-	-	-
{2}	[0.588, 0.601]	[0.043, 0.071]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.631, 0.657]	[0.179, 0.264]	[0.491, 0.518]	[0.058, 0.138]	-	-
{1, 2}	[0.702, 0.708]	[0.280, 0.311]	-	[0.033, 0.054]	-	-	-
{1, 3}	[0.870, 0.872]	[0.270, 0.347]	[0.270, 0.347]	[0.550, 0.574]	[0.157, 0.232]	[0.002, 0.082]	-
{2, 3}	[0.891, 0.892]	[0.728, 0.744]	[0.385, 0.445]	[0.623, 0.639]	[0.287, 0.345]	[0.142, 0.212]	[0.054, 0.120]
{1, 2, 3}	[0.901, 0.902]	[0.752, 0.767]	[0.438, 0.494]	[0.657, 0.672]	[0.348, 0.402]	[0.212, 0.279]	[0.129, 0.193]
α-cut = 0.7							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.009, 0.014]	-	-	-	-	-	-
{2}	[0.590, 0.600]	[0.046, 0.066]	-	-	-	-	-
{3}	[0.854, 0.855]	[0.635, 0.654]	[0.190, 0.253]	[0.495, 0.515]	[0.068, 0.127]	-	-
{1, 2}	[0.702, 0.707]	[0.283, 0.307]	-	[0.035, 0.051]	-	-	-
{1, 3}	[0.871, 0.872]	[0.279, 0.337]	[0.279, 0.337]	[0.554, 0.571]	[0.166, 0.222]	[0.010, 0.071]	-
{2, 3}	[0.891, 0.892]	[0.730, 0.742]	[0.392, 0.437]	[0.626, 0.637]	[0.294, 0.338]	[0.150, 0.203]	[0.062, 0.111]
{1, 2, 3}	[0.901, 0.902]	[0.754, 0.766]	[0.445, 0.487]	[0.659, 0.670]	[0.354, 0.395]	[0.220, 0.270]	[0.136, 0.184]

α-cut = 0.8							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.010, 0.013]	-	-	-	-	-	-
{2}	[0.592, 0.598]	[0.049, 0.063]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.638, 0.651]	[0.199, 0.242]	[0.498, 0.511]	[0.077, 0.117]	-	-
{1, 2}	[0.703, 0.706]	[0.287, 0.302]	-	[0.037, 0.048]	-	-	-
{1, 3}	[0.871, 0.872]	[0.288, 0.327]	[0.288, 0.327]	[0.556, 0.568]	[0.175, 0.212]	[0.019, 0.060]	-
{2, 3}	[0.891, 0.892]	[0.732, 0.740]	[0.399, 0.429]	[0.627, 0.635]	[0.300, 0.330]	[0.158, 0.193]	[0.069, 0.102]
{1, 2, 3}	[0.901, 0.902]	[0.756, 0.763]	[0.451, 0.479]	[0.660, 0.668]	[0.361, 0.388]	[0.228, 0.261]	[0.143, 0.176]
α-cut = 0.9							
{r} \ {m}	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
{1}	[0.010, 0.012]	-	-	-	-	-	-
{2}	[0.594, 0.596]	[0.052, 0.059]	-	-	-	-	-
{3}	[0.853, 0.854]	[0.641, 0.647]	[0.209, 0.230]	[0.501, 0.508]	[0.086, 0.106]	-	-
{1, 2}	[0.704, 0.705]	[0.283, 0.307]	-	[0.040, 0.045]	-	-	-
{1, 3}	[0.871, 0.872]	[0.290, 0.298]	[0.297, 0.316]	[0.559, 0.565]	[0.183, 0.202]	[0.028, 0.048]	-
{2, 3}	[0.891, 0.892]	[0.734, 0.738]	[0.406, 0.421]	[0.629, 0.633]	[0.307, 0.322]	[0.166, 0.184]	[0.077, 0.093]
{1, 2, 3}	[0.901, 0.902]	[0.757, 0.761]	[0.458, 0.421]	[0.662, 0.666]	[0.367, 0.381]	[0.236, 0.252]	[0.151, 0.167]

Notice that the values of the equivalence factors ρ must be between 0 and 1. So that any value does not meet this criteria is ignored in Tables 3, 4.

9. CONCLUSION

Most papers in literature deal with the equivalence factors for systems consisted of independent units. In this paper, analysis of the reliability of a dependent system using the concept of copula was introduced. The reliability of the system was improved by using the reduction method and k cold standby duplication method. New method of mixed standby duplication method was established to improve the reliability function of the system. All papers in literature considered that the equivalence factors are deterministic values which give no flexible analysis of the systems. This analysis considered that the equivalence factors are fuzzy and provided an algorithm to find the intervals for the reliability equivalence factors of the system. An application based on real data was introduced to show the implement of the theoretical analysis.

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