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# A MULTI-SERVER QUEUE WITH REVERSE BALKING AND IMPATIENT CUSTOMERS

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## ABSTRACT

In this paper, we study a Markovian multiple-server queuing system with reverse balking and reneging. Reverse balking is a recently added concept in queuing theory, introduced by Jain et al. (2014). Reverse balking states that probabilistic decision of joining or not joining a queue by an arrival depends on the system size. Higher is the system size, more the probability of joining the system. On the other hand, the customers may get impatient due to excess wait for their service. The impatient customers may decide to abandon the queue without receiving the service (called as reneging). Keeping the key concepts of reverse balking and reneging in view, we develop a multiple-server queuing system with reverse balking and reneging. The stationary probabilities of system size are obtained using iterative method. The performance measures like average system size, average rate of reneging, and average rate of reverse balking are obtained. Finally, the sensitivity analysis of the model is carried out.

## KEYWORDS

Reverse Balking, Reneging, Sensitivity Analysis, Multiple-server Queuing System.

## 1. INTRODUCTION

Business environment is highly challenging these days due to uncertainty. Uncertainty appears in all dimensions of an operating firm for example uncertain economic environment, uncertain natural calamities, uncertain customer behavior and else. Hence margin for error is very low for business firms. Every firm is looking for risk management and precise prediction of future. Consumer behavior is one of the most uncertain characteristics of business environment. Due to higher level of expectations customers' impatience increases in a particular firm. Thus customer impatience is a vital issue for corporate world.

Customers' impatience is harmful to any business. It leads to loss of potential customers. Impatience is of three types: balking, reneging and jockeying. In reneging, a customer joins the queue, waits for some time and leaves the queue without getting service due to impatience, Gross and Harris (1985). Wang et al. (2010) present a nice review on queuing systems with impatient customers. They survey the queuing

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systems according to customer impatience behavior, solution methods, and associated optimization aspects.

Queuing models with balking and reneging find their applications in a variety of areas. For instance, in call centers usually a calling customer hangs up before service agent, that is, he is reneged. In packet-switched communication networks with time critical traffic a packet loses its value if it is not transmitted within a given time interval. Such lost packets can be considered as reneged customers. The patients who abandon the OPD rooms in hospitals without being attended can also be considered as reneged customers.

In the case of balking as described above, the chances of an arriving customer for not joining the queue are more if the system size is large and vice-versa. But when we talk about the businesses like investment, the chances for customers to invest are more with the firms having large number of customers already with them. If we consider an investment firm as a queuing system, the probability of balking (not joining the firm) will be less when the system size is large and vice-versa. This kind of balking is called Reverse Balking, Jain et al. (2014).

For example, in case of a Mutual Fund where the purchase of a mutual fund plan refers to an arrival of a customer into the queuing system (Mutual Fund), the processed claims can be considered as the departures from the system, the processing department is considered as a service facility, and the capacity of the system (the number of mutual fund plans it can accommodate) is taken as finite. The claims are processed on first-come, first- served basis. The probability of customers joining a Mutual Fund is more when there are large number of mutual fund customers already with it and vice-versa (reverse balking).

Recently, Jain et al. (2014) incorporated the concept of reverse balking in queuing theory. Reverse balking is a phenomenon in which the probability of not joining the queuing system is more when there is less number of customers in the system and vice-versa. Queues with reverse balking find their applications in investment business, restaurants, hospitals, schools, business of quality products etc. Kumar et al. (2014) study an M/M/1/N queuing system with reverse balking wherein they include the concept of reverse reneging. Kumar et al. (2015) incorporate the customers with feedback in an M/M/1/N queue with reverse balking. They derive the stationary probabilities of system size.

In this paper, we generalize the work of Jain et al. (2014) by considering the multiserver case and reneging. A multiple-server finite capacity Markovian queuing model with reverse balking and reneging is developed. Steady-state solution of the model is derived iteratively.

Rest of the paper is structured as follows: in section 2, the literature review is provided; in section 3, assumptions under which the model is developed are presented; section 4 deals with the mathematical formulation; in section 5 steady-state solution is derived; section 6 deals with measures of performance; numerical illustrations and sensitivity analysis of the model is performed in section 7. Finally, the paper is concluded in section 8.

## 2. LITERATURE REVIEW

Haight (1957) introduces the notion of customer impatience in queuing theory. He performs the steady- state analysis of an M/M/1 queue with balking. Barrer (1957a) discusses a queuing problem which is characterized by the impatience of the customers and the indifference of the clerks. Barrer (1957b) studies a queuing problem with impatient customers where the customers are served ordered basis. Haight (1959) further studies a single server queue with reneging. Ancker and Gafarian (1963a) perform the stationary analysis of a finite capacity Markovian queue with balking and reneging. Ancker and Gafarian (1963b) also derive the results for a single server queuing system with balking. Rao (1965) study a non-Markovian single server queuing model with balking, reneging and interruptions. They solve this model by using supplementary variable technique. Rao (1967) obtains the busy period of an M/G/1 queue with balking. Robert (1979) provides a detailed account of the reneging in single channel queues.

Bae and Kim (2010) study a general input, exponential service times, single server queue with constant customer patience times. Manoharan and Jose (2011) introduce the concept of random balking in M/M/1 queuing model. Liau (2011) study a queuing system with balking index and reneging rate. Kapodistria (2011) studies queues with synchronized abandonments. She studies single server as well as multiserver queues. Choudhury and Medhi (2011) study balking in a single server finite buffer Markovian queuing system with position dependent reneging. They derive stationary results of the model. Kumar (2012) studies a single server queuing problem having correlated input, catastrophes, restoration and impatient customers. The model finds its application in communication networks. Kumar and Sharma (2012) study single as well as multi-channel queues with balking, and retention of reneged customers. Kumar (2013) considers a finite capacity Markovian multiserver queuing model with balking, reneging and retention of reneged customers. He derives the transient solution of the model using matrix method. The economic analysis of the model is also carried out. Kumar et al. (2014) develop the cost model for a finite capacity, single server Markovian feedback queue with retention of reneged customers and perform the optimization for service rate, system capacity. Kumar and Sharma (2014) consider a multi-channel Markovian feedback queue with balking and retention of reneged customers.

Burak (2015) proposes a non-stationary multiserver queuing model with abandonment and balking for inbound call centers. He shows that the uniformization with steady-state detection can be used in a very effective way to evaluate transient behavior of multiserver queues. Kumar and Som (2015) consider an M/M/1/N queue with reverse balking, reverse reneging and customer retention. They derive the stationary probabilities of system size and obtain expressions for important performance measures. Som (2019) uses iterative technique to solve a queuing system with heterogeneous service rate and reverse balking.

The concept of reverse balking is the most recent one and is suited to many practical situations as described in the introduction section. We incorporate the concept of reverse balking into a finite capacity Markovian multiserver queuing model with reneging. We derive the steady-state solution and perform the sensitivity analysis of the model.

## 3. MODEL ASSUMPTIONS

1. The arrival process is Poisson with parameter $\lambda$.
2. There are multiple-servers, say c. The service times follow exponential distribution with parameter $\mu$ such as $\mu_n = n\,\mu$ when $n < c$ and $\mu_n = c\,\mu$ when $n \geq c$.
3. The system capacity is taken as finite, say $N$.
4. The queue discipline is First-Come, First-Served.
5. (a) When the system is empty, the customers balk with probability $q'$ and may not balk with probability $p'$ (= $1 - q'$).

   (b) When the system is not empty, customers balk with a probability $1 - \frac{n}{N-1}$ and do not balk with probability $\frac{n}{N-1}$.

   The balking described in (a) and (b) is called reverse balking.
6. Each customer upon joining the queue waits for some time for his service to begin. If he does not receive service by then, he leaves the queue without getting service (i.e. reneged). The reneging times follow the exponentially distribution with parameter $\xi$.

## 4. STOCHASTIC MODEL FORMULATION

Let $P_n(t)$ be the probability that there are n customers in the system at time t.

The Chapman-Kolmogorov equations of the model are:

$$\frac{dP_0(t)}{dt} = -\lambda\, p' P_0(t) + \mu\, P_1(t); \qquad n = 0 \qquad (1)$$

$$\frac{dP_1(t)}{dt} = \lambda\, p' P_0(t) - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu\right\} P_1(t) + (2\mu)\, P_2(t); \qquad n = 1 \qquad (2)$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right) \lambda\, P_{n-1}(t) - \left\{\left(\frac{n}{N-1}\right)\lambda + n\mu\right\} P_n(t) + \{(n+1)\mu\}\, P_{n+1}(t) \qquad 2 \leq n < c \qquad (3)$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right) \lambda\, P_{n-1}(t) - \left\{\left(\frac{n}{N-1}\right)\lambda + c\mu + (n-c)\,\xi\right\} P_n(t) + [c\mu + \{(n+1) - c\}\xi]\, P_{n+1}(t) \qquad c \leq n \leq N-1 \qquad (4)$$

$$\frac{dP_N(t)}{dt} = \lambda\, P_{N-1}(t) - [c\mu + (N-c)\xi]\, P_N(t); \qquad n = N \qquad (5)$$

## 5. STEADY- STATE SOLUTION

In steady state $\lim_{t\to\infty} P_n(t) = P_n$ , $\lim_{t\to\infty} P_n{}'(t) = 0$. Therefore the equations (1) to (5) become:

$$0 = -\lambda\, p' P_0 + \mu\, P_1; \qquad n = 0 \qquad (6)$$

$$0 = \lambda\, p' P_0 - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu\right\} P_1 + (2\mu)\, P_2; \qquad n = 1 \qquad (7)$$

$$0 = \left(\frac{n-1}{N-1}\right) \lambda\, P_{n-1} - \left\{\left(\frac{n}{N-1}\right)\lambda + n\mu\right\} P_n + \{(n+1)\mu\}\, P_{n+1} \qquad 2 \le n < c \qquad (8)$$

$$0 = \left(\frac{n-1}{N-1}\right) \lambda\, P_{n-1} - \left\{\left(\frac{n}{N-1}\right)\lambda + c\mu + (n-c)\,\xi\right\} P_n + [c\mu + \{(n+1) - c\}\,\xi]\, P_{n+1} \qquad c \le n \le N-1 \qquad (9)$$

$$\frac{dP_N(t)}{dt} = \lambda\, P_{N-1}(t) - [c\mu + (N-c)\xi]\, P_N(t) \qquad n = N \qquad (10)$$

Solving (6) – (10) we obtain:

$$P_n = \begin{cases} \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \displaystyle\prod_{r=1}^{n} \dfrac{\lambda}{r\mu}\right] p'^{P_0}, 1 \le n < c \\ \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \displaystyle\prod_{s=c}^{n} \dfrac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \dfrac{\lambda}{r\mu}\right] p' P_0, c \le n \le N-1 \\ \left[\dfrac{(N-2)!}{(N-1)^{N-1}} \displaystyle\prod_{s=c}^{n} \dfrac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \dfrac{\lambda}{r\mu}\right] p' P_0, n = N \end{cases}$$

Using $\sum_{n=1}^{N} P_n = 1$, we get

$$P_0 + \sum_{n=1}^{c-1} P_n + \sum_{n=c}^{N-1} P_n + P_N = 1$$

$$P_0 = \left\{1 + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{r\mu}\right] p' + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{s=c}^{n} \frac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \frac{\lambda}{r\mu}\right] p' + \left[\frac{(N-2)!}{(N-1)^{N-1}} \prod_{s=c}^{n} \frac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \frac{\lambda}{r\mu}\right] p'\right\}^{-1}$$

## 6. MEASURES OF PERFORMANCE

### 6.1 Expected System Size

$$L_s = \sum_{n=1}^{N} nP_n$$

$$L_s = \sum_{n=1}^{c-1} nP_n + \sum_{n=c}^{N-1} nP_n + NP_N$$

$$L_s = n\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{r=1}^{n}\frac{\lambda}{r\mu}\right]p'P_0$$

$$+ n\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$

$$+ N\left[\frac{(N-2)!}{(N-1)^{N-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$

### 6.2 Average Rate of Reneging

$$R_r = \sum_{n=c}^{N}(n-c)\xi P_n$$

$$R_r = \sum_{n=c}^{N-1}(n-c)\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]\xi p'P_0$$

$$+\left[\frac{(N-2)!}{(N-1)^{N-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]\xi p'P_0$$

### 6.3 Average Rate of Reverse Balking

$$R_b{}' = q'\lambda P_0 + \sum_{n=1}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda P_n$$

$$R_b{}' = q'\lambda P_0 + \sum_{n=1}^{c-1}\left(1-\frac{n}{N-1}\right)\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{r=1}^{n}\frac{\lambda}{r\mu}\right]p'P_0$$

$$+\sum_{n=c}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$

## 7. SENSITIVITY ANALYSIS OF THE MODEL

This section deals with the numerical illustration. The sensitivity analysis is discussed. The results are computed using MS-Excel.

**Table 1**
**Variation in $L_s$, $R_r$ and $R_b'$ with respect to μ**
**$\lambda$ =10, $\xi$ =0.1, $q'$ =0.8, c=3, N=10**

| (μ) | Expected System Size ($L_s$) | Average Rate of Reneging ($R_r$) | Average Rate of Reverse Balking ($R_b'$) |
|---|---|---|---|
| 3.0 | 0.58730 | 0.00074 | 8.25796 |
| 3.1 | 0.56672 | 0.00060 | 8.25999 |
| 3.2 | 0.54789 | 0.00049 | 8.26106 |
| 3.3 | 0.53057 | 0.00040 | 8.26137 |
| 3.4 | 0.51455 | 0.00033 | 8.26110 |
| 3.5 | 0.49966 | 0.00028 | 8.26035 |
| 3.6 | 0.48576 | 0.00024 | 8.25923 |
| 3.7 | 0.47274 | 0.00020 | 8.25781 |
| 3.8 | 0.46051 | 0.00017 | 8.25616 |
| 3.9 | 0.44899 | 0.00015 | 8.25431 |
| 4.0 | 0.43811 | 0.00013 | 8.25232 |
| 4.1 | 0.42781 | 0.00011 | 8.25021 |
| 4.2 | 0.41804 | 0.00009 | 8.24801 |
| 4.3 | 0.40875 | 0.00008 | 8.24575 |
| 4.4 | 0.39991 | 0.00007 | 8.24343 |
| 4.5 | 0.39148 | 0.00006 | 8.24108 |
| 4.6 | 0.38343 | 0.00006 | 8.23870 |
| 4.7 | 0.37573 | 0.00005 | 8.23631 |
| 4.8 | 0.36836 | 0.00004 | 8.23392 |
| 4.9 | 0.36129 | 0.00004 | 8.23152 |
| 5.0 | 0.35451 | 0.00003 | 8.22914 |

From Table 1, we can see that the increase in service rate leads to the decrease in average system size and average reneging rate. The average reverse balking rate increases as the service rate increases. That is, the results are consistent with the functioning of the model.

**Table 2**
**Variation in $L_s$, $R_r$ and $R_b'$ with respect to λ when μ = 3, ξ = 0.1, q′ = 0.8, c = 3, N = 10**

| Mean Arrival Rate (λ) | Expected System Size ($L_s$) | Average Rate of Reneging ($R_r$) | Average Rate of Reverse Balking ($R_b'$) |
|---|---|---|---|
| 5 | 0.29905 | 0.00001 | 4.10322 |
| 6 | 0.35450 | 0.00003 | 4.93749 |
| 7 | 0.41003 | 0.00008 | 5.77227 |
| 8 | 0.46650 | 0.00018 | 6.60563 |
| 9 | 0.52505 | 0.00038 | 7.43524 |
| 10 | 0.58730 | 0.00074 | 8.25796 |
| 11 | 0.65557 | 0.00138 | 9.06939 |
| 12 | 0.73319 | 0.00250 | 9.86309 |
| 13 | 0.82493 | 0.00436 | 10.62960 |
| 14 | 0.93730 | 0.00738 | 11.35519 |
| 15 | 1.07892 | 0.01209 | 12.02055 |
| 16 | 1.26051 | 0.01922 | 12.59966 |
| 17 | 1.49435 | 0.02960 | 13.05966 |
| 18 | 1.79298 | 0.04416 | 13.36238 |
| 19 | 2.16673 | 0.06371 | 13.46872 |
| 20 | 2.62057 | 0.08878 | 13.34601 |
| 21 | 3.15088 | 0.11936 | 12.97729 |
| 22 | 3.74367 | 0.15476 | 12.36948 |
| 23 | 4.37553 | 0.19362 | 11.55642 |
| 24 | 5.01751 | 0.23411 | 10.59444 |
| 25 | 5.64072 | 0.27431 | 9.55130 |

From Table 2 it is clearly visible that the increase in average arrival rate increases the expected system size. An increase in expected system size leads to high confidence of customers with a particular firm and as a result the average rate of reverse balking decreases therefore. On the other hand, rate of reneging increases gradually due to increasing system size that leads to high level of impatience.

**Table 3**
**Variation in $L_s$ and $R_r$ with respect to $\xi$**
**When $\mu = 3$, $\lambda = 2$, $q' = 0.2$, $c = 3$, $N = 10$**

| Rate of Reneging ($\xi$) | Expected System Size ($L_s$) | Rate of Reneging ($R_r$) |
|---|---|---|
| 0.05 | 0.354518 | 0.000018 |
| 0.06 | 0.354516 | 0.000021 |
| 0.07 | 0.354515 | 0.000024 |
| 0.08 | 0.354513 | 0.000028 |
| 0.09 | 0.354512 | 0.000031 |
| 0.1 | 0.354510 | 0.000035 |
| 0.11 | 0.354509 | 0.000038 |
| 0.12 | 0.354507 | 0.000041 |
| 0.13 | 0.354505 | 0.000045 |
| 0.14 | 0.354504 | 0.000048 |
| 0.15 | 0.354502 | 0.000052 |

From Table 3, it can be observe that increasing rate of reneging causes decrease in expected system size and increase in average rate of reneging. This is because increasing rate of reneging states that more and more customers are moving out of the system without completing their service.

**Table 4**
**Variation in $L_s$ with respect to $q'$**
**When $\lambda = 10$, $\mu = 3$, $\xi = 0.1$, $c = 3$, $N = 10$**

| Probability of Reverse Balking when System is Empty ($q'$) | Expected System Size ($L_s$) |
|---|---|
| 0.1 | 1.01918 |
| 0.2 | 0.99310 |
| 0.3 | 0.96146 |
| 0.4 | 0.92229 |
| 0.5 | 0.87253 |
| 0.6 | 0.80719 |
| 0.7 | 0.71763 |
| 0.8 | 0.58730 |
| 0.9 | 0.38018 |
| 1.0 | 0.00000 |

We can observe from Table 4 that with the increase in probability of reverse balking when there is no customer in the system, the expected system size decreases. When q'=1 (probability that an arriving customer never joins the system) the expected system size drops to zero. This establishes the functioning of our model.

## 8. CONCLUSION AND FUTURE WORK

In this paper a multiple-server finite capacity Markovian queuing system with reverse balking and reneging is developed. Steady-state solution of the model is derived. Necessary measures of performance are obtained. Sensitivity analysis of the model is also performed.

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