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# BAYESIAN ESTIMATION FOR THE STRESS – STRENGTH RELIABILITY EXPONENTIATED Q-EXPONENTIAL DISTRIBUTION BASED ON SINGLY TYPE II CENSORING DATA

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## ABSTRACT

Reliability of the stress$(Y)$ – strength$(X)$ $R = P(Y > X)$ model using Bayesian estimation method is proposed in this paper. We used it to evaluate the component's performance when the two variables $X$ and $Y$ are independent Exponentiated q-Exponential random variables with common parameter $\lambda, \varepsilon$ and $\sigma$. Three different loss functions [weighted, quadratic, and entropy] under two different prior functions [Gamma and Extension of Jeffery], with an empirical Bayes estimator for a type II censored sample are used. A comparison of the three reliability estimators for stress – strength are established with two different types of criteria such as mean squared error (MSE) and mean absolutely percentage error (MAPE). Finally, the results of the simulation study discovered that for small, medium, and large sample sizes, the entropy loss function, weighted and quadratic to the advantage, performed best, respectively.

## KEY WORDS

Bayesian estimation, Reliability, Stress-strength model, Type II censored data, Exponentiated q-Exponential distribution.

## 1. INTRODUCTION

Recent work has produced a number of q-type super statistical distributions, including the q-exponential, q-Weibull, and q-logistic, in the context of statistical mechanics, information theory, and reliability modeling. In tests of reliability and survivability, exponentiated q-Exponential models can be used to simulate a range of component failures. A lot of motivation has been placed on how challenging it is to draw conclusions about the stress-strength reliability (one component or system) concept. Reliability, defined as $R = P(Y > X)$, is a performance metric that may be used to assess the effectiveness of a component if $X$ is its strength and $Y$ is the stress. $R$ is estimated using a number of alternative lifetime distributions.

The calculation of the reliability function has been the subject of much research for several failure models. Widely employed in a range of conventional and Bayesian estimation methodologies as well as in real-world applications. 2015 through the current year. Umesh, Sanjay, and Abhimanyu [9] talk about the traditional and Bayesian estimate of the parameters, reliability function, and hazard function for a novel extension of the exponential distribution using asymmetric loss functions in (2015). In 2016, Li and Hao calculated $R = P(Y < X)$ and utilized Monte Carlo simulation [10] as a comparison. In their analysis, Najarzadegan et al. took into account the Bayes, Maximum Likelihood, and UMVUE estimators. To compare the employed methodologies, real data and simulation were used [11]. Multi-component system dependability was calculated in 2017 by Srinivasa et al. [12]. In 2018 [13], Cheng proved the system's reliability in the strength stress model with parallel components based on the Pareto exponential distribution. To calculate the stress-strength reliability of a multi-component model that followed a Pareto distribution in 2019, Jamal et al. proposed the Maximum Likelihood estimator and the Bayes estimator under two loss functions (linear exponential loss function and squared error loss function) [14]. Jana et al. showed that the Rényi entropy implies artificial biases not warranted by the data and incorrect updating information due to the finite size of the data despite being additive based on $q$ – exponential distribution [15][18]. In 2020, Jha et al. considered the reliability of multi-component stress-strength under progressive Type-II censoring when stress and strength variables obey the unit Gompertz distributions. The reliability was estimated under Bayesian methods. Then, Monte Carlo simulations and three actual data sets were used for comparison of the proposed methods [16]. Also, in the same year, Hassan et al. estimated the multi-component system reliability stress-strength model. The Markov Chain Monte Carlo method was used to assess the performance of estimates [17]. Jana and Bera developed Bayesian estimators for the entropy loss function's parameters in 2020 [5]. The reliability function of a probability distribution on the cone $\Omega$ of positive definite symmetric matrices defines the distribution without any invariance conditions, as established by Hassairi and Roula in their article from 2022 [6]. Additionally, they showed that the exponential probability distribution on $\Omega$ holds up when described by a memory less attribute even without the assumption of an invariance requirement. They looked at the relationship between the uniform distribution on $\Omega$ a limited interval of and the exponential distribution on $\Omega$. We explain the idea of a matrix Pareto distribution and demonstrate how it exhibits the long tail feature.

The concept of competitive hazards, which emphasizes other competing events that preclude viewing the event of interest, is incorporated into the Exponentiated $q$-Exponential distribution. The unknown parameters of this model may be estimated using a variety of techniques, including maximum likelihood estimation, Bayesian estimation, and empirical Bayesian estimation [4]. The latter method, however, runs into a challenging integration that is exceedingly challenging to implement due to its complexity. We questioned if the outcome would have been the same given the same parameters for the competitive risk model and different loss functions (weighted, quadratic, and entropy)? This post was prepared with such attitude in mind.

The following is a description of the paper's structure. The posterior functions of the distributions are shown numerically in Section 2. The Bayesian estimates for model parameters are determined in Section 3. We got the empirical Bayes estimators of EQE reliability based on different loss functions in section 4. In Section 5, a Monte Carlo simulation study is used to validate the correctness of the Bayesian estimates. Finally, in the final part, there are some concluding observations.

**1.1 The Reliability System Methods**

In this paper, the reliability under two prior functions [Gamma and Extension of Jeffery], with three loss distributions [weighted, quadratic, and entropy] are proposed. Bayesian analysis is performed when stress $X$ and strength $Y$ are two independent Exponentiated q-Exponential random variables with parameters $(\lambda, \varepsilon, \alpha)$ and $(\lambda, \varepsilon, \sigma)$, respectively. Let $X \sim EQE(\alpha, \lambda, 1-\varepsilon)$ and $Y \sim EQE(\sigma, \lambda, 1-\varepsilon)$, where EQE means Exponentiated q-Exponential distributions under different shape parameters α and $\sigma$. In particular, the EQE distribution is defined by the probability density function (p.d.f.) and (c.d.f) of $x$ for $\lambda > 0$ as [1]

$$f_\alpha(x, \lambda, 1-\varepsilon) = \alpha(1+\varepsilon)\lambda e_x(-\lambda x)\left(1 - [1 - \lambda\varepsilon x]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha-1} \quad x > 0, \tag{1}$$

$$F_\alpha(x, \lambda, 1-\varepsilon) = 1 - \left(1 - [1 - \lambda\varepsilon x]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha} \quad x > 0$$

The survival function and hazard function when $1 < (1-\varepsilon) < 2$ are given as; respectively:

$$S_\alpha(x, \lambda, 1-\varepsilon) = 1 - \left(1 - [1 - \varepsilon\lambda\xi]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}, \tag{2-10}$$

and

$$h_\alpha(x, \lambda, 1-\varepsilon) = \frac{(1+\varepsilon)\lambda(1-\varepsilon\xi)^{\frac{1}{\varepsilon}}\left(1 - [1 - \varepsilon\lambda\xi]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}}{1 - \left(1 - [1 - \varepsilon\lambda\xi]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}}, \tag{2-11}$$

Similarly, the (p. d. f.) and (c. d. f) of $y$ for $\sigma > 0$ as

$$f_\sigma(y, \lambda, (1-\varepsilon)) = \alpha(1+\varepsilon)\lambda e_y(-\lambda y)\left(1 - [1 - \lambda\varepsilon y]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma-1} \quad y > 0, \tag{2}$$

$$F_\sigma\big(y, \lambda, (1-\varepsilon)\big) = 1 - \left(1 - [1 - \lambda\varepsilon y]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma} \quad y > 0$$

where $e_x = \begin{cases} (1-\varepsilon x)^{\frac{1}{\varepsilon}}; & \text{if } (1-\varepsilon) \neq 1 \\ e^x & ; \text{ if } (1-\varepsilon) = 1 \end{cases}$,

$e_y = \begin{cases} (1-\varepsilon y)^{\frac{1}{\varepsilon}}; & \text{if } (1-\varepsilon) \neq 1 \\ e^y & ; \text{ if } (1-\varepsilon) = 1 \end{cases}$, and $\alpha > 0$

given that $(1-\varepsilon) < 2$.

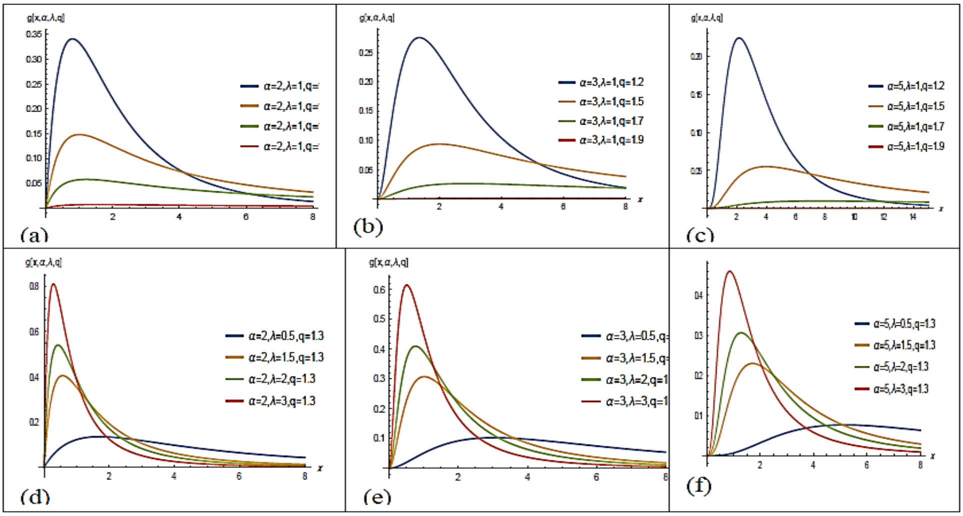


**Figure 1: The Probability Density Function of $\mathbf{f_{\alpha}(x, \lambda, q = 1 - \varepsilon)}$**

In addition to its significance in the field of reliability and life testing, the exponential failure model is one of the popular failure models for its many uses. When the failure rate is constant over time, the exponentiated q-exponential distribution is used. However, there are times when it is necessary to find reliability that begins at a specific time rather than zero, and this particular time represents (the warranty period), necessitating the urgent inclusion of this. Exponentiated q-Exponential distribution parameterized.

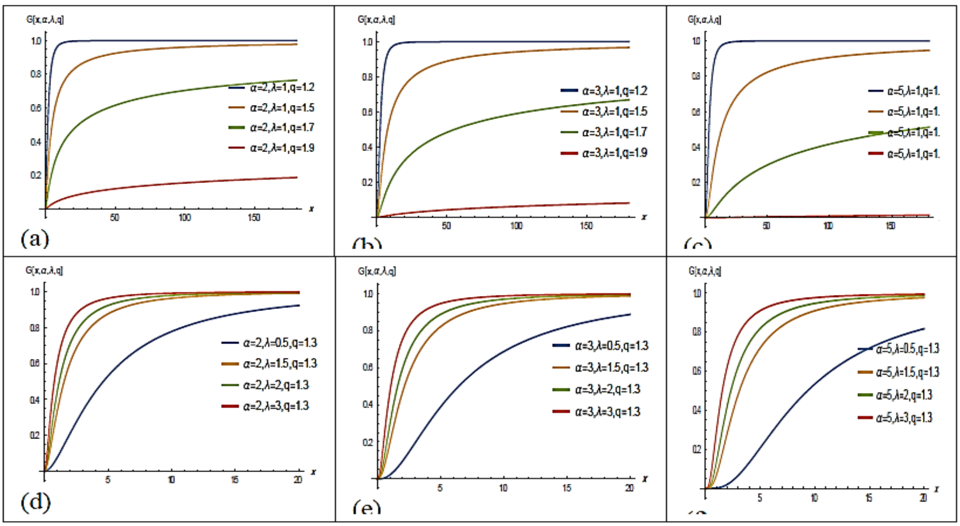


**Figure 2: The Cumulative Function of $\boldsymbol{F_{\alpha}(x, \lambda, q = 1 - \varepsilon)}$**

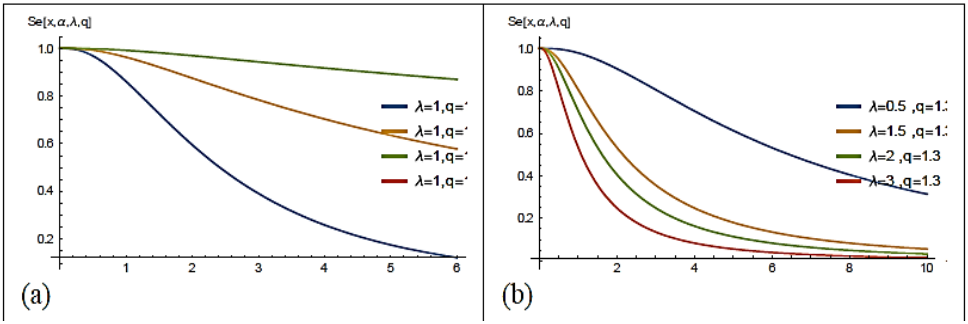


**Figure 3: The Survival Function of $\boldsymbol{S_\alpha(x, \lambda, \mathrm{q} = 1 - \varepsilon)}$**

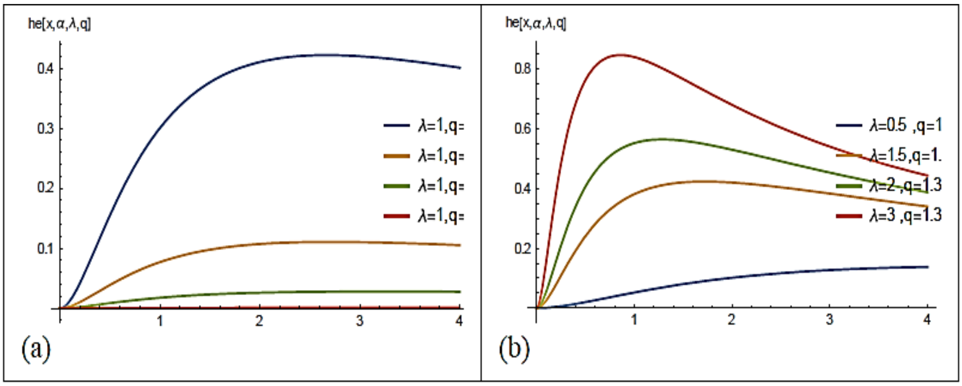


**Figure 4: The Hazard Function of $\boldsymbol{h_\alpha(x, \lambda, \mathrm{q} = 1 - \varepsilon)}$**

The stress-strength reliability is given below:

$$R = P(y > x) = \int_0^\infty \int_0^y f(x) f(y) dx\, dy$$

$$= \int_0^\infty \left( \int_0^y f(x) dx \right) f(y)\, dy$$

$$= \int_0^\infty F_x(y) f(y)\, dy$$

$$= \int_0^\infty \left[ \begin{array}{c} \left(1 - [1 - \Omega\lambda y]^{\frac{1+\varepsilon}{\varepsilon}}\right)^\alpha \ \alpha(1+\varepsilon)\, \lambda(1 - \varepsilon\lambda y)^{\frac{1}{\varepsilon}} \\ \left[1 - (1 - \varepsilon\lambda y)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha - 1} \end{array} \right] dy$$

$$R = P(y > x) = \frac{\sigma}{\alpha + \sigma} \tag{3}$$

This study compares the reliability estimates from the Bayesian estimation method using three different loss functions—weighted, quadratic, and entropy—under two different prior functions—gamma and Jeffrey extension—with the empirical Bayes estimator in the case of data from exponentiated q-exponential distributions. For a controlled type II stress strength sample with two different types of indicators, such as mean squared error (MSE) and mean absolute relative error (MAPE).

## 2. THE POSTERIOR DISTRIBUTIONS

Let $x_1, x_2, \dots, x_n$ and $y_1, y_2, \dots, y_m$ be two random samples. We get the likelihood function as[2]

$$L = \frac{n!}{(n-r)!}[1 - F(x_r)]^{n-r} \prod_{i=1}^{r} f\left(x_i, \underline{\theta}\right),$$

and the posterior function as:

$$P\left(\alpha, \sigma \mid \underline{x}, \underline{y}\right) = \frac{L\left(\underline{x}, \underline{y} \mid \alpha, \sigma\right) g(\alpha, \sigma)}{\int_0^\infty \int_0^\infty L\left(\alpha, \sigma \mid \underline{x}, \underline{y}\right) g(\alpha, \sigma) d\alpha d\sigma}.$$

Suppose that the sample is singly type II censored such that $r < n$ , and $r < m$ then the likelihood functions of $x_r, \dots, x_n$ and $y_r, \dots, y_m$ are given by

$$\mathrm{L}(\alpha, \lambda, (1-\varepsilon) \mid \mathrm{x}) = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!} \left[1 - \left(1 - [1 - \lambda\varepsilon x_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{\mathrm{n}-\mathrm{r}} \alpha^{\mathrm{r}} (1+\varepsilon)^{\mathrm{r}} \lambda^{\mathrm{r}} \prod_{\mathrm{i}=1}^{\mathrm{r}} (1 - \lambda\varepsilon \mathrm{x_i})^{\frac{1}{\varepsilon}}$$

$$. \prod_{\mathrm{i}=1}^{\mathrm{r}} \left[1 - (1 - \lambda\varepsilon \mathrm{x_i})^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1} \tag{1}$$

and

$$\mathrm{L}(\sigma, \lambda, (1-\varepsilon) \mid \mathrm{y}) = \frac{\mathrm{m}!}{(\mathrm{m}-\mathrm{r})!} \left[1 - \left(1 - [1 - \lambda\varepsilon y_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\right]^{\mathrm{m}-\mathrm{r}} \sigma^{\mathrm{r}} (1+\varepsilon)^{\mathrm{r}} \lambda^{\mathrm{r}}$$

$$. \prod_{\mathrm{i}=1}^{\mathrm{r}} (1 - \lambda\varepsilon \mathrm{y_i})^{\frac{1}{\varepsilon}} \tag{2}$$

### 2.1 Under Gamma Prior

The Gamma distribution is used as a prior distribution because of its wide importance in Bayesian analysis. Let $\alpha,$ and $\sigma$ be two independent Gamma random variables with common parameter $a > 0$ with different $\mathrm{b}_1$ and $\mathrm{b}_2$ parameters, the p. d. f. is given by [3]

$$g(\alpha) = \frac{b_1^a}{\Gamma(a)} \alpha^{a-1} e^{-b_1 \alpha}, \alpha,\ b_1 > 0. \tag{3}$$

$$g(\sigma) = \frac{b_2^a}{\Gamma(a)} \sigma^{a-1} e^{-b_2 \sigma}, \sigma,\ b_2 > 0. \tag{4}$$

Using the formulae (1), (2), (3) and (4), then the joint likelihood function and prior distributions can be written as

$$L\left(\alpha,\sigma \mid \underline{x},\underline{y}\right) g(\alpha,\sigma)$$
$$=\left[\begin{array}{l}\frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma^2(a)}\alpha^r\sigma^r(1+\varepsilon)^{2r}\lambda^{2r}\left[1-\left(1-[1-\lambda\varepsilon x_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{n-r} \\ \left[1-\left(1-[1-\lambda\varepsilon y_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\right]^{m-r} \\ \prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{i=1}^{r}\left[1-(1-\lambda\varepsilon x_i)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\prod_{i=1}^{r}(1-\lambda\varepsilon y_i)^{\frac{1}{\varepsilon}} \\ \prod_{i=1}^{r}\left[1-(1-\lambda\varepsilon y_i)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\alpha^{a-1}\sigma^{a-1}e^{-b_1\alpha-b_2\sigma}\end{array}\right]$$

and

$$\iint_0^\infty L\left(\alpha,\sigma \mid \underline{x},\underline{y}\right) g(\alpha,\sigma)d\alpha d\sigma = \int_0^\infty\int_0^\infty \frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma^2(a)}$$
$$\alpha^r\sigma^r(1+\varepsilon)^{2r}\lambda^{2r}\alpha^{a-1}\sigma^{a-1}e^{-b_1\alpha-b_2\sigma}$$
$$\left[1-\left(1-[1-\lambda\varepsilon x_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{n-r}$$
$$\left[1-\left(1-[1-\lambda\varepsilon y_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\right]^{m-r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}$$
$$\prod_{i=1}^{r}\left[1-(1-\lambda\varepsilon x_i)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}$$
$$\prod_{j=1}^{r}\left[1-\left(1-\lambda\varepsilon y_j\right)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1} d\alpha d\sigma$$

Let $1-\lambda\varepsilon x_r = \zeta_1$ and $1-\lambda\varepsilon y_r = \zeta_2$, then

$$= \frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma^2(a)}(1+\varepsilon)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}$$
$$\int_0^\infty \alpha^{r+a-1}\,e^{-b_1\alpha}\left[1-\left(1-[\zeta_1]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{n-r}\prod_{i=1}^{r}\left[1-(\zeta_1)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}d\alpha$$
$$\int_0^\infty \sigma^{r+a-1}e^{-b_2\sigma}\left[1-\left(1-[\zeta_2]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\right]^{m-r}\prod_{i=1}^{r}\left[1-(\zeta_2)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}d\sigma$$

We assume that $e^z = 1-[\zeta_1]^{\frac{2-q}{1-q}}$ and $e^w = 1-[\zeta_2]^{\frac{2-q}{1-q}}$, then

$$= \frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma^2(a)}(1+\varepsilon)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}$$

$$.\int_0^\infty \alpha^{r+a-1}\, e^{-b_1\alpha}[1-(e^z)^\alpha]^{n-r}\prod_{i=1}^{r}[e^z]^{\alpha-1}\, d\alpha$$

$$.\int_0^\infty \sigma^{r+a-1}e^{-b_2\sigma}\,[1-(e^w)^\sigma]^{m-r}\prod_{i=1}^{r}[e^w]^{\alpha-1}\, d\sigma$$

$$= \frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma^2(a)}(1+\varepsilon)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}$$

$$.\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r+a-1)!}{(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a}}$$

$$.\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r+a-1)!}{(b_2-\sum_{i=1}^{r} w_i - kw)^{r+a}}$$

The posterior distribution $P_1(\alpha,\sigma \mid \underline{x},\underline{y})$ under gamma prior, is given by

$$P_1(\alpha,\sigma \mid \underline{x},\underline{y}) = \left(\begin{array}{c}\dfrac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}e^{kz\alpha}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a-1)!}{(b_1-\sum_{i=1}^{r} z_i-kz)^{r+a}}}\\ .\dfrac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}e^{kw\sigma}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-1)!}{(b_2-\sum_{i=1}^{r} w_i-kw)^{r+a}}}\\ \alpha^{r+a-1}\,\sigma^{r+a-1}e^{-(b_1-\sum_{i=1}^{r} z_i)\alpha}e^{-(b_2-\sum_{i=1}^{r} w_i)\sigma}\end{array}\right) \tag{5}$$

## 2.2 Under Extension of Jeffery's Prior

Regarding the case of non-informational distributions the Extension of Jeffry as prior distribution have been used for the case of common parameter (c) in order to focus on the rest of the parameters, where the functions for $(\beta,\lambda)$ are given by [4]

$$g(\alpha) = k\frac{n^c}{\alpha^{2c}}\ ,\alpha > 0;\ \mathrm{k,n,c} > 0. \tag{6}$$

$$g(\sigma) = k\frac{m^c}{\sigma^{2c}},\ \sigma > 0;\ k,m,c > 0. \tag{7}$$

Using formulae (1), (2), (6) and (7), it will be:

$$L\left(\alpha,\sigma \mid \underline{x},\underline{y}\right) g(\alpha,\sigma)$$
$$=\left(\begin{array}{c}\frac{n!}{(n-r)!}\frac{m!}{(m-r)!}k^2n^cm^c\alpha^{r-2c}\sigma^{r-2c}(1+\varepsilon)^{2r}\lambda^{2r}\\ \left[1-\left(1-[1-\lambda\varepsilon x_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{n-r}\left[1-\left(1-[1-\lambda\varepsilon y_r]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\right]^{m-r}\\ \prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{i=1}^{r}\left[1-(1-\lambda\varepsilon x_i)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}\\ \prod_{j=1}^{r}\left[1-\left(1-\lambda\varepsilon y_j\right)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\alpha^{a-1}\sigma^{a-1}e^{-b_1\alpha-b_2\sigma}\end{array}\right)$$

We use the same procedure and previous hypotheses to obtain the product of integration:

$$\int_0^\infty\int_0^\infty L(\alpha,\sigma \mid \underline{x},\underline{y})g(\alpha,\sigma)d\alpha d\sigma=\frac{n!}{(n-r)!}\frac{m!}{(m-r)!}k^2n^cm^c$$
$$(1+\varepsilon)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}$$
$$.\int_0^\infty\alpha^{r-2c}\,e^{-b_1\alpha}[1-(e^z)^\alpha]^{n-r}\prod_{i=1}^{r}[e^{z_i}]^{\alpha-1}\,d\alpha$$
$$.\int_0^\infty\sigma^{r-2c}e^{-b_2\sigma}\,[1-(e^w)^\sigma]^{m-r}\prod_{i=1}^{r}[e^{w_i}]^{\alpha-1}\,d\sigma$$
$$=\frac{n!}{(n-r)!}\frac{m!}{(m-r)!}k^2n^cm^c(1+\varepsilon)^{2r}\lambda^{2r}$$
$$\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{j=1}^{r}\left(1-\lambda\varepsilon y_j\right)^{\frac{1}{\varepsilon}}e^{\sum_{i=1}^{r}z_i}\,e^{\sum_{i=1}^{r}w_i}$$
$$.\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r-2c+1}}$$
$$\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r-2c)!}{(b_2-\sum_{i=1}^{r}w_i-kw)^{r-2c+1}}$$

The posterior distribution $P_2(\alpha,\sigma \mid \underline{x},\underline{y})$ under extension of Jeffry prior, is given by

$$P_2(\alpha,\sigma \mid \underline{x},\underline{y})=\left[\begin{array}{c}\dfrac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}e^{kz\alpha}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r-2c+1}}}\\ .\dfrac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}e^{kw\sigma}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r-2c)!}{(b_2-\sum_{i=1}^{r}w_i-kw)^{r-2c+1}}}\\ \alpha^{r-2c}\,\sigma^{r-2c}e^{(\sum_{i=1}^{r}z_i)\alpha}e^{(\sum_{i=1}^{r}w_i)\sigma}\end{array}\right] \tag{8}$$

## 3. THE BAYESIAN ESTIMATORS FOR EQE RELIABILITY

In this section the Bayesian estimators for stress-strength Exponentiated q-Exponential reliability under three loss functions are derived as following:

### 3.1 Under Weighted Loss Function

In this section the Bayesian estimator for R using gamma prior function will be derived under weighted loss function [12] $\hat{R}_{wG}$, where:

$$\hat{R}_{wG} = \left[E\left(R^{-1} \mid \underline{x}, \underline{y}\right)\right]^{-1}$$

$$E\left(R^{-1} \mid \underline{x}, \underline{y}\right) = \int_0^\infty \int_0^\infty R^{-1} P\left(\alpha, \sigma \mid \underline{x}, \underline{y}\right) d\alpha d\sigma$$

$$= \int_0^\infty \int_0^\infty \sigma^{-1}\alpha P(\alpha, \sigma \mid \underline{x}, \underline{y}) d\alpha d\sigma + \int_0^\infty \int_0^\infty P(\alpha, \sigma \mid \underline{x}, \underline{y}) d\alpha d\sigma$$

$$= \mathrm{A}_1 + \mathrm{A}_2. \tag{9}$$

where

$$A_1 = \int_0^\infty \int_0^\infty \sigma^{-1}\alpha \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} e^{kz\alpha}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^r z_i - kz)^{r+a}}}$$
$$. \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} e^{kw\sigma}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{(b_2 - \sum_{i=1}^r w_i - kw)^{r+a}}} \alpha^{r+a-1}\, \sigma^{r+a-1}$$
$$. e^{-(b_1 - \sum_{i=1}^r z_i)\alpha} e^{-(b_2 - \sum_{i=1}^r w_i)\sigma}\, d\alpha d\sigma$$

$$= \left[\frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a)!}{(b_1 - \sum_{i=1}^r z_i - kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^r z_i - kz)^{r+a}}} . \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-2)!}{[b_2 - \sum_{i=1}^r w_i - kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^r w_i - kw]^{r+a}}}\right],$$

and

$$A_2 = \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^r z_i - kz)^{r+a}}} \int_0^\infty \alpha^{r+a-1}\, e^{-(b_1 - \sum_{i=1}^r z_i - kz)\alpha} d\alpha$$
$$\frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{(b_2 - \sum_{i=1}^r w_i - kw)^{r+a}}}$$
$$. \int_0^\infty \sigma^{r+a-1} e^{-(b_2 - \sum_{i=1}^r w_i - kw)\sigma} d\sigma = 1$$

Then the Bayesian estimators for EQE reliability based on weighted loss function $\widehat{R}_{WG}$ is given as

$$\hat{R}_{WG} = \left[ 1 + \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-2)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \right]^{-1}. \tag{10}$$

### 3.2 Under Quadratic Loss Function

The derivation of Bayesian estimator for EQE reliability using gamma prior under quadratic loss[8] function $\hat{R}_{QG}$ is obtained as follows

$$\hat{R}_{QG} = \frac{E\left(R^{-1} \mid \underline{x}, \underline{y}\right)}{E\left(R^{-2} \mid \underline{x}, \underline{y}\right)}$$

Such that

$$E\left(R^{-2} \mid \underline{x}, \underline{y}\right) = \int_0^{\infty}\int_0^{\infty} (\alpha^2\sigma^{-2} + 2\alpha\sigma^{-1} + 1) P(\alpha, \sigma \mid \underline{x}, \underline{y}) d\alpha d\sigma = B_1 + 2B_2 + 1. \tag{11}$$

where

$$B_1 = \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \int_0^{\infty} \alpha^{r+a+1} e^{-(b_1 - \sum_{i=1}^{r} z_i - kz)\alpha} d\alpha$$

$$\frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{(b_2 - \sum_{i=1}^{r} w_i - kw)^{r+a}}} . \int_0^{\infty} \sigma^{r+a-3} e^{-(b_2 - \sum_{i=1}^{r} w_i - kw)\sigma} d\sigma$$

$$= \left[ \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a+1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} . \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-3)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-2}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \right],$$

and

$$B_2 = \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \int_0^\infty \alpha^{r+a}\, e^{-(b_1 - \sum_{i=1}^{r} z_i - kz)\alpha} d\alpha$$

$$\cdot \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{(b_2 - \sum_{i=1}^{r} w_i - kw)^{r+a}}} \int_0^\infty \sigma^{r+a-2} e^{-(b_2 - \sum_{i=1}^{r} w_i - kw)\sigma} d\sigma$$

$$= \left[ \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-2)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \right]$$

Then

$$E\left(R^{-2} \mid \underline{x}, \underline{y}\right)$$

$$= \left[ \begin{gathered} 1 + \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a+1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \\ * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-3)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-2}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \\ +2 \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \\ * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-2)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \end{gathered} \right]$$

Now, the Bayesian estimators for EQE reliability based on quadratic loss function $\hat{R}_{QG}$ is given as

$$\hat{R}_{QG}=\frac{\left[1+\dfrac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a-1)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a}}}\cdot\dfrac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-2)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-1)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a}}}\right]}{\left[1+\dfrac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a+1)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a+2}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a-1)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a}}}\cdot\dfrac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-3)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a-2}}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-1)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a}}}+2\dfrac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\dfrac{(r+a-1)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a}}}\cdot\dfrac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-2)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\dfrac{(r+a-1)!}{[b_2-\sum_{i=1}^{r}w_i-kw]^{r+a}}}\right]} \qquad (12)$$

### 3.3 Under Entropy Loss Function

The derivation of Bayesian estimator for EQE reliability using gamma prior under entropy loss $\hat{R}_{tEG}$ function [7] as:

$$\hat{R}_{tEG}=\left[E\left(R^{-t}\mid \underline{x},\underline{y}\right)\right]^{-\frac{1}{t}},$$

where $t\neq 0$. If $\mathrm{t}=1$, then like formulae (10) the $\hat{R}_{wG}$ is

$$\hat{R}_{1EG}=\left[E\left(R^{-1}\mid \underline{x},\underline{y}\right)\right]^{-1}=\hat{R}_{wG}.$$

If $t=2$, then like formulae (11), we get

$$\hat{R}_{2EG} = \left[ 1 + \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a+1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \cdot \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-3)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-2}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} + 2 \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \cdot \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-2)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \right]^{-\frac{1}{2}}, \quad (13)$$

Now, the Bayesian estimators for EQE reliability based on entropy loss $\hat{R}_{tEG}$ function is given as

$$\hat{R}_{tEG} = \left[ E\left( R^{-t} \mid \underline{x}, \underline{y} \right) \right]^{-\frac{1}{t}},$$

$$\hat{R}_{tEG} = \left[ \sum_{j=0}^{t} C_j^t \left( \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a+j-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a+j}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-j-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a-j}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r+a-1)!}{[b_2 - \sum_{i=1}^{r} w_i - kw]^{r+a}}} \right) \right]^{\frac{-1}{t}}, \quad (14)$$

where

$$E\left[ R^{-t} \mid \underline{x}, \underline{y} \right] = \int_0^\infty \int_0^\infty \sigma^{-t} (\sigma + \alpha)^t P(\alpha, \sigma \mid \underline{x}, \underline{y}) d\alpha d\sigma$$

$$= \int_0^\infty \int_0^\infty \sigma^{-t} \sum_{j=0}^{t} \binom{t}{j} \sigma^{t-j} \alpha^j P\left( \alpha, \sigma \,\middle|\, \underline{x}, \underline{y} \right) d\alpha d\sigma$$

$$\sum_{j=0}^{t} \binom{t}{j} \left( \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r+a-1)!}{(b_1 - \sum_{i=1}^{r} z_i - kz)^{r+a}}} \int_0^\infty \alpha^{r+a-1+j} \, e^{-(b_1 - \sum_{i=1}^{r} z_i - kz)\alpha} d\alpha \right)$$

$$
.\left(\begin{array}{c}\dfrac{\sum_{k=0}^{m-r}(-1)^{k}\binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^{k}\binom{m-r}{k}\dfrac{(r+a-1)!}{(b_2-\sum_{i=1}^{r}w_i-kw)^{r+a}}}\\ \int_0^{\infty}\sigma^{r+a-1-j}e^{-(b_2-\sum_{i=1}^{r}w_i-kw)\sigma}d\sigma\end{array}\right).
$$

## 4. THE EMPIRICAL BAYES ESTIMATOR FOR EQE RELIABILITY

### 4.1 Using Gamma Prior

The empirical Bayes estimators of EQE reliability corresponding to Gamma prior distribution are obtained based on different loss functions. However, if the prior Gamma parameters ( $b_1$ and $b_2$ ) are unknown, it may use the empirical Bayes approach to get its estimation from likelihood function and probability density function of prior distribution as [8]

$$
\begin{aligned}
f(\underline{x},y \mid b_1 b_2) &= \int_0^{\infty}\int_0^{\infty} L(\alpha,\sigma \mid \underline{x},\underline{y})g(\alpha,\sigma)d\alpha d\sigma\\
&= \int_0^{\infty}\int_0^{\infty}\frac{n!}{(n-r)!}\frac{m!}{(m-r)!}\frac{b_1^a b_2^a}{\Gamma a\Gamma a}\alpha^r\sigma^r(1+\varepsilon)^{2r}\lambda^{2r}\,\alpha^{a-1}\sigma^{a-1}e^{-b_1\alpha-b_2\sigma}\\
&\qquad \prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{i=1}^{r}\left[1-(1-\lambda\varepsilon x_i)^{\frac{1+\varepsilon}{\varepsilon}}\right]^{\alpha-1}\\
&\quad .\left[1-\left(1-[1-\lambda\varepsilon x]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha}\right]^{n-r}\Bigg[1\\
&\qquad -\left(1-[1-\lambda\varepsilon y]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\sigma}\Bigg]^{m-r}\prod_{i=1}^{r}(1-\lambda\varepsilon y_i)^{\frac{1}{\varepsilon}}.\prod_{i=1}^{r}\Bigg[1\\
&\qquad -(1-\lambda\varepsilon y_i)^{\frac{1+\varepsilon}{\varepsilon}}\Bigg]^{\alpha-1} d\alpha d\sigma
\end{aligned}
$$

Indeed the joint likelihood is

$$
f(\underline{x},y \mid b_1 b_2) = \left(\begin{array}{c}
\dfrac{n!}{(n-r)!}\dfrac{m!}{(m-r)!}\dfrac{b_1^a b_2^a}{\Gamma a\Gamma a}(1+\varepsilon)^{2r}\lambda^{2r}\prod_{i=1}^{r}(1-\lambda\varepsilon x_i)^{\frac{1}{\varepsilon}}\prod_{i=1}^{r}(1-\lambda\varepsilon y_i)^{\frac{1}{\varepsilon}}\\
\sum_{k=0}^{n-r}(-1)^{k}\binom{n-r}{k}\dfrac{(r+a-1)!}{(b_1-\sum_{i=1}^{r}z_i-kz)^{r+a}}\\
\sum_{k=0}^{m-r}(-1)^{k}\binom{m-r}{k}\dfrac{(r+a-1)!}{(b_2-\sum_{i=1}^{r}w_i-kw)^{r+a}}
\end{array}\right)
$$

Taking the natural log , we get

$$\ln f\left(\underline{x}, \mathrm{y} \mid b_1 b_2\right) = \ln\frac{n!}{(n-r)!} + \ln\frac{m!}{(m-r)!} + 2r\ln\lambda + 2r\ln(1+\varepsilon) + \sum_{i=1}^{r}\ln(1-\lambda\varepsilon \mathrm{x}_i)^{\frac{1}{\varepsilon}} + \sum_{i=1}^{r}\ln(1-\lambda\varepsilon \mathrm{y}_i)^{\frac{1}{\varepsilon}} + \ln\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r+a-1)!}{(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a}} + \ln\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r+a-1)!}{(b_2-\sum_{i=1}^{r} w_i - kw)^{r+a}} + a\ln b_1 + a\ln b_2 - \ln\Gamma a\Gamma a$$

Now, we derivative with respect $b_1$ to obtain the MLE estimators of $b_1$ as follows

$$\frac{\partial L}{\partial b_1} = \frac{a}{b_1} + \frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}(r+a-1)!\,\frac{-(r+a)(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a-1}}{[(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a}]^2}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r+a-1)!}{(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a}}}$$

$$\Rightarrow\ b_1 = \frac{a\ \sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\ \frac{1}{(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r+a)}{(b_1-\sum_{i=1}^{r} z_i - kz)^{r+a+1}}} \tag{15}$$

In the same way we derive relative to $b_2$, we get;

$$b_2 = \frac{a\ \sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\ \frac{1}{(b_2-\sum_{i=1}^{r} w_i - kw)^{r+a}}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r+a)}{(b_2-\sum_{i=1}^{r} w_i - kw)^{r+a+1}}} \tag{16}$$

### 4.2 For Extension of Jeffery Prior

The Bayesian estimators for EQE reliability is derived in this section for extension of Jeffery prior as in formulae (6) and (7) under the three loss functions as follows:

#### 4.2.1 Weighted Loss Function

The Bayesian estimators of EQE reliability for extension of Jeffery prior under the weighted loss functions $\hat{\mathrm{R}}_{\mathrm{wJ}}$ is obtained from formulae (9) as follows:

$$\hat{R}_{wJ} = [D_1 + D_2]^{-1}$$

where

$$D_1 = \frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}}\int_0^\infty \alpha^{r-2c+1}\ e^{-(-\sum_{i=1}^{r} z_i - kz)\alpha}d\alpha \cdot \frac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c}}}\int_0^\infty\ \sigma^{r-2\mathrm{c}-1}e^{-(-\sum_{i=1}^{r} w_i - kw)\sigma}d\sigma$$

$$D_1 = \left( \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c+1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+2}}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} * \frac{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c-1}}}{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c}}} \right),$$

and

$$D_2 = \int_0^\infty \int_0^\infty \left( \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} e^{kz\alpha}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+1}}} * \frac{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} e^{kw\sigma}}{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c+1}}} \alpha^{r-2c}\, \sigma^{r-2c} e^{(\sum_{i=1}^{r} z_i)\alpha} e^{(\sum_{i=1}^{r} w_i)\sigma} \right) = 1$$

$$\hat{R}_{wJ} = \left[ 1 + \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c+1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+2}}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} * \frac{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c-1}}}{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c}}} \right]^{-1} \tag{17}$$

**4.2.2 Under Quadratic Loss Function**

From formulae (8) and (11), assuming that $E(R^{-2} \mid x, y) = D_1 + 2D_2 + 1$, then:

$$D_1 = \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \int_0^\infty \alpha^{r-2c+2}\, e^{-(-\sum_{i=1}^{r} z_i - kz)\alpha} d\alpha \cdot \frac{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k}}{\sum_{k=0}^{m-r} (-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c}}} \int_0^\infty \sigma^{r-2c-2} e^{-(-\sum_{i=1}^{r} w_i - kw)\sigma} d\sigma$$

$$= \left( \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c+2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+3}}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \cdot \frac{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c-1}}}{\sum_{k=0}^{n-r} (-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \right),$$

and

$$D_2 = \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \int_0^\infty \alpha^{r-2c+1}\, e^{-(-\sum_{i=1}^{r} z_i - kz)\alpha} d\alpha$$

$$\cdot \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} w_i - kw)^{r-2c}}} \int_0^\infty \sigma^{r-2c-1} e^{-(-\sum_{i=1}^{r} w_i - kw)\sigma} d\sigma$$

$$= \left( \begin{array}{c} \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c+1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ \cdot \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \end{array} \right).$$

Now

$$E(R^{-2} \mid x, y) = \left[ \begin{array}{c} \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c+2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+3}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ \cdot \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c-1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ +2 \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c+1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ \cdot \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \frac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} + 1 \end{array} \right]$$

Here the Bayesian estimators of EQE reliability for extension of Jeffery prior under the Quadratic loss functions

$$\hat{R}_{QJ}=\frac{\left[1+\frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c+1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c+2}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}\cdot\frac{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c-1}}}{\sum_{k=0}^{m-r}(-1)^k\binom{m-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}w_i-kw)^{r-2c}}}\right]}{\left[\frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c+2)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c+3}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}\cdot\frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-2)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c-1}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}+2\frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c+1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c+2}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}\cdot\frac{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}{\sum_{k=0}^{n-r}(-1)^k\binom{n-r}{k}\frac{(r-2c-1)!}{(-\sum_{i=1}^{r}z_i-kz)^{r-2c}}}+1\right]} \tag{18}$$

**4.2.3 Under Entropy Loss Function**

Likewise, from formulae (10) and (11) the Bayesian estimators of EQE reliability $\hat{R}_{tEJ}$ for extension of Jeffery prior under the entropy loss functions are obtained as

$$\hat{R}_{tEJ}=\left[E\left(R^{-t}\mid \underline{x},\underline{y}\right)\right]^{-\frac{1}{t}}\ t\neq 0,$$

If $t=1\rightarrow\hat{R}_t=\hat{R}_w$ same the formulae (17), and if $t=2\rightarrow\hat{R}_{2EJ}=\left[E\left(R^{-2}\mid \underline{x},y\right)\right]^{-\frac{1}{2}}$ then

$$\left[E\left(R^{-2} \mid \underline{x}, \underline{y}\right)\right]^{-\frac{1}{2}} = \left[\begin{array}{l} 1 + \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c+2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+3}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ * \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-2)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c-1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ +2 \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c+1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+2}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \\ * \dfrac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} \end{array}\right]^{-\frac{1}{2}} \tag{19}$$

and $E\left[R^{-t} \mid \underline{x}, \underline{y}\right] = \int_0^\infty \int_0^\infty \sigma^{-t} (\sigma+\alpha)^t P(\alpha, \sigma \mid \underline{x}, \underline{y}) d\alpha d\sigma$. Now, the Bayesian estimators of EQE reliability $\hat{R}_{tEJ}$ is given by

$$\hat{R}_{tEJ} = \left[\sum_{j=0}^{t} \binom{t}{j} \left( \frac{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c+j)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c+j+1}}}{\sum_{k=0}^{n-r}(-1)^k \binom{n-r}{k} \dfrac{(r-2c-1)!}{(-\sum_{i=1}^{r} z_i - kz)^{r-2c}}} * \frac{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \dfrac{(r-2c-j)!}{[-\sum_{i=1}^{r} w_i - kw]^{r-2c-j+1}}}{\sum_{k=0}^{m-r}(-1)^k \binom{m-r}{k} \dfrac{(r-2c-1)!}{[-\sum_{i=1}^{r} w_i - kw]^{r-2c}}} \right)\right]^{\frac{-1}{t}}. \tag{20}$$

## 5. THE SIMULATION STUDY

Simulation is a strategy for employing computer models to duplicate or simulate real-world phenomena. It is often difficult to understand and analyze processes in real life, hence it is desirable to depict these processes using specialized models in a way that is comparable to real-life images. Various phases of the use of techniques for evaluating the system reliability of single systems were used in simulation testing. On six different samples, the suggested estimation techniques were tested (15, 30, 50). Statistics based on mean absolute error (MAPE) and mean squared error for each sample, repeated 1000 times (MSE). For this, the phases of the Monte Carlo simulation are as follows:

**Phase 1:**

- First: calculate the sample size using the formulas n = m (15, 30, 50) and r = 5, 8, 20. The sample size affects how accurate and effective the findings of estimate techniques are. The linked tables were used to determine the sample size.

- Second: Calculating the values of the three parameters (α = 0.9, **σ** = 0.3, **λ** = 0.4, and ε = 4). The Exponentiated Q-Exponential Distribution's two parameters have default values.
- Third: Choosing the sample frequency is the third step (L). The studies were repeated (L = 1,000) times for each event in order to achieve great uniformity.
- Finding the values of the constants (a=4, **b1**=0.4, **b2**=0.8, t=1, c=1) is the fourth step.

**Phase 2**: Create random samples as $u_1, \dots, u_n$ that follow a continuous uniform distribution that is well-defined on the interval (0, 1).

**Phase 3**: Create random samples that follow a uniform continuous distribution in the range (0, 1) as $\mathrm{w}_1, \dots, \mathrm{w}_\mathrm{m}$.

**Phase 4**: Using the (CDF), F(x) and F(y), where U is a uniform random variable on the interval (0,1), are used to assemble discrete values for the two random variables, respectively, by F(x) and F(y), and then by the inverse of distribution function method obtained from

$$F(X) = \left(1 - [1 - \lambda\varepsilon x]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha} = U_i,$$

and

$$F(Y) = 1 - \left(1 - [1 - \lambda\varepsilon y]^{\frac{1+\varepsilon}{\varepsilon}}\right)^{\alpha} = W_j, \forall i = 1, \dots, n, and\ j = 1, \dots, m.$$

Convert the previously mentioned random uniform samples to samples that are random having followed EQED.

$$x_i = \frac{\left(1 - U_i^{\frac{1}{\alpha}}\right)^{\frac{1+\varepsilon}{\varepsilon}} - 1}{\lambda\varepsilon}, \text{ and } y_j = \frac{\left(1 - w_j^{\frac{1}{\sigma}}\right)^{\frac{1+\varepsilon}{\varepsilon}} - 1}{\lambda\varepsilon}.$$

**Phase 5**: Using the formula (1) to recall R.

**Phase 6**: Using formulae (5), and (8), determine the posterior distributions under (Gamma and extension of Jeffery) prior.

**Phase 7**: Using formulae (10), (12), (14), (17), (18), and (20), determine R of the Bayes, respectively.

**Phase 8**: Compute MSE, and MAPE criteria based on replication of (L=1000) and (n=15, m = 15) represented the smallest sample size, (n = 30, m = 30) for moderate, and (n = 50, m = 50) for large sample sizes in three experiments with varied parameter values as follows:

$$MSE = \frac{1}{L}\sum_{i=1}^{n}\left(\hat{R}_i - R\right)^2 \text{ and } MAPE = \frac{1}{L}\sum_{i=1}^{n}\frac{|\hat{R}_i - R|}{|R|}.$$

## 6. CONCLUSIONS

Tables (1–8) provide the findings of the simulation study, which show how the predicted dependability of this system varies as sample sizes change using loss functions. Finally, it was discovered that the entropy loss function of the Bayesian estimator, weighted and Quadratic to the advantage, performed best for small, medium, and large sample sizes.

**Table 1**
***α=0.9, σ=0.3, R= 0.2500***
***λ=0.4, ε =4, a=4, b1=0.4, b2=0.8, t=1, c=1***

| n, r | | | Criteria | Weighted | Quadratic | Entropy | Best |
|---|---|---|---|---|---|---|---|
| 15,5 | $\hat{R}_{Gamma}$ | | | 0.4164 | 2.1620e-04 | 0.3346 | |
| | | Gamma | MSE<br>MAPE | 0.0070<br>0.1673 | 0.2498<br>0.9996 | 0.0072<br>0.3383 | W |
| | $\hat{R}_{E\ Gamma}$ | | | 0.9322 | 5.2211e-04 | 0.6045 | |
| | | E Gamma | MSE<br>MAPE | 0.4654<br>2.7289 | 0.0622<br>0.9979 | 0.1257<br>1.4179 | Q |
| | $\hat{R}_{Jeffery}$ | | | 0.1134 | 0.0013 | 0.0014 | |
| | | Jeffery | MSE<br>MAPE | 0.1495<br>0.7733 | 0.2487<br>0.9975 | 0.2486<br>0.9972 | W |
| 30,8 | $\hat{R}_{Gamma}$ | | | 0.4233 | 2.0882e-04 | 0.6170 | |
| | | Gamma | MSE<br>MAPE | 0.0059<br>0.1534 | 0.2498<br>0.9996 | 0.1347<br>1.4680 | W |
| | $\hat{R}_{E\ Gamma}$ | | | 0.9766 | 6.7767e-04 | 0.7765 | |
| | | E Gamma | MSE<br>MAPE | 0.5279<br>2.9063 | 0.0622<br>0.9973 | 0.2772<br>2.1059 | Q |
| | $\hat{R}_{Jeffery}$ | | | 0.1037 | 0.0224 | 0.0023 | |
| | | Jeffery | MSE<br>MAPE | 0.1570<br>0.7925 | 0.2281<br>0.9551 | 0.2477<br>0.9954 | W |
| 50,20 | $\hat{R}_{Gamma}$ | | | 0.2782 | 1.5336e-06 | 0.4482 | |
| | | Gamma | MSE<br>MAPE | 0.0492<br>0.4435 | 0.2500<br>1.0000 | 0.0393<br>0.7927 | E |
| | $\hat{R}_{E\ Gamma}$ | | | 0.9632 | 1.3084e-04 | 0.5214 | |
| | | E Gamma | MSE<br>MAPE | 0.5086<br>2.8526 | 0.0624<br>0.9995 | 0.0737<br>1.0856 | Q |
| | $\hat{R}_{Jeffery}$ | | | 2.7233e-05 | 0.9995 | 2.3414e-04 | |
| | | Jeffery | MSE<br>MAPE | 0.2500<br>0.9999 | 0.2495<br>0.9989 | 0.2498<br>0.9995 | Q |

**Table 2**
***α=3.7, σ=2, R= 0.3509***
**λ=0.4, ε =4, a=4, b1=0.4, b2=0.8, t=1, c=1**

| ***n, r*** | | | ***Criteria*** | ***Weighted*** | ***Quadratic*** | ***Entropy*** | ***Best*** |
|---|---|---|---|---|---|---|---|
| ***15,5*** | $\hat{R}_{Gamma}$ | | | ***0.0361*** | ***1.3000e-05*** | ***0.3206*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1108<br>0.9028*** | ***0.1232<br>0.1862*** | ***0.16808<br>0.0863*** | ***W<br>E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.8986*** | ***0.0061*** | ***0.4961*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3000<br>1.5608*** | ***0.1189<br>0.9828*** | ***0.0211<br>0.4138*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***1.7924e-05*** | ***0.4477*** | ***0.0012*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.4213<br>1.0000*** | ***0.0406<br>0.3103*** | ***0.4197<br>0.9981*** | ***Q*** |
| ***30,8*** | $\hat{R}_{Gamma}$ | | | ***0.0411*** | ***6.3725e-06*** | ***0.4569*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1112<br>0.9172*** | ***0.1231<br>1.0000*** | ***0.0112<br>0.3020*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9627*** | ***0.0036*** | ***0.6827*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3743<br>1.7436*** | ***0.1206<br>0.9898*** | ***0.1101<br>0.9456*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1081*** | ***0.0356*** | ***0.0016*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.2927<br>0.8335*** | ***0.3764<br>0.9452*** | ***0.4192<br>0.9975*** | ***W*** |
| ***50,20*** | $\hat{R}_{Gamma}$ | | | ***0.0453*** | ***3.7424e-07*** | ***0.3819*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1119<br>0.9290*** | ***0.1231<br>0.9452*** | ***0.17614<br>0.0884*** | ***W<br>E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9422*** | ***1.5685e-06*** | ***0.4085*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3497<br>1.6852*** | ***0.1231<br>1.0000*** | ***0.0033<br>0.1640*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.0910*** | ***0.0508*** | ***0.0052*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.3115<br>0.8598*** | ***0.3580<br>0.9217*** | ***0.4146<br>0.9920*** | ***W*** |

**Table 3**
***α=0.9, σ=0.3, R= 0.2500***
**λ=0.4, ε =4, C=2, t=2, a=4, b1=2.1, b2=1.5**

| *n, r* | | | *Criteria* | *Weighted* | *Quadratic* | *Entropy* | *Best* |
|---|---|---|---|---|---|---|---|
| *15,5* | $\hat{R}_{Gamma}$ | | | *0.0857* | *1.3177e-05* | *0.4465* | |
| | | *Gamma* | *MSE*<br>*MAPE* | *0.0931*<br>*1.1427* | *0.0625*<br>*0.9999* | *0.0386*<br>*0.7858* | *E* |
| | $\hat{R}_{E\ Gamma}$ | | | *1.3023e-06* | *2.3068e-11* | *1.4670e-07* | |
| | | *E Gamma* | *MSE*<br>*MAPE* | *0.0635*<br>*1.0001* | *0.0625*<br>*1.0000* | *0.0625*<br>*1.0000* | *Q, E* |
| | $\hat{R}_{Jeffery}$ | | | *0.1001* | *6.8076e-05* | *9.8447e-05* | |
| | | *Jeffery* | *MSE*<br>*MAPE* | *0.1231*<br>*0.5406* | *0.4199*<br>*0.9983* | *0.4212*<br>*0.9998* | *W* |
| *30,8* | $\hat{R}_{Gamma}$ | | | *0.0657* | *2.4992e-05* | *0.3747* | |
| | | *Gamma* | *MSE*<br>*MAPE* | *0.0728*<br>*1.0629* | *0.0625*<br>*0.9999* | *0.0156*<br>*0.4988* | *E* |
| | $\hat{R}_{E\ Gamma}$ | | | *6.8833e-06* | *3.1538e-10* | *5.1551e-07* | |
| | | *E Gamma* | *MSE*<br>*MAPE* | *0.0635*<br>*1.0001* | *0.0625*<br>*1.0000* | *0.0625*<br>*1.0000* | *Q, E* |
| | $\hat{R}_{Jeffery}$ | | | *0.1006* | *1.1240e-07* | *2.0421e-07* | |
| | | *Jeffery* | *MSE*<br>*MAPE* | *0.3919*<br>*0.9549* | *0.4213*<br>*0.9994* | *0.4213*<br>*0.9998* | *W* |
| *50,20* | $\hat{R}_{Gamma}$ | | | *0.0393* | *3.8300e-07* | *0.6884* | |
| | | *Gamma* | *MSE*<br>*MAPE* | *0.0583*<br>*0.9571* | *0.0625*<br>*1.0000* | *0.1922*<br>*1.7534* | *W* |
| | $\hat{R}_{E\ Gamma}$ | | | *6.6539e-05* | *5.2622e-13* | *2.6994e-06* | |
| | | *E Gamma* | *MSE*<br>*MAPE* | *0.0625*<br>*0.9997* | *0.0635*<br>*1.0000* | *0.0635*<br>*1.0000* | *W* |
| | $\hat{R}_{Jeffery}$ | | | *0.1000* | *1.2477e-06* | *2.8737e-04* | |
| | | *Jeffery* | *MSE*<br>*MAPE* | *0.3915*<br>*0.9541* | *0.4213*<br>*1.0000* | *0.4210*<br>*0.9996* | *W* |

**Table 4**
***α=3.7, σ=2, R= 0.3509***
**λ=0.4, ε =3.5, a=4, b1=0.9, b2=1.8**

| ***n, r*** | | | ***Criteria*** | ***Weighted*** | ***Quadratic*** | ***Entropy*** | ***Best*** |
|---|---|---|---|---|---|---|---|
| ***15,5*** | $\hat{R}_{Gamma}$ | | | ***0.0554*** | ***3.9004e-05*** | ***0.5416*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1149<br>0.9578*** | ***0.1231<br>0.9999*** | ***0.0364<br>0.5433*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.5439*** | ***4.9746e-05*** | ***0.1170*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.0372<br>0.5499*** | ***0.1231<br>0.9999*** | ***0.0547<br>0.6666*** | ***W*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***1.1801e-04*** | ***5.6138e-07*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.5125<br>0.9333*** | ***0.5623<br>0.9998*** | ***0.5645<br>1.0018*** | ***W*** |
| ***30,8*** | $\hat{R}_{Gamma}$ | | | ***0.0377*** | ***6.8088e-06*** | ***0.6952*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1109<br>0.9075*** | ***0.1231<br>1.0000*** | ***0.1185<br>0.9811*** | ***W*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9414*** | ***5.9047e-06*** | ***0.4580*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3487<br>1.6827*** | ***0.1231<br>1.0000*** | ***0.0115<br>0.3053*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1003*** | ***2.2004e-06*** | ***2.7600e-06*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.5126<br>0.9337*** | ***0.5625<br>1.0000*** | ***0.5625<br>1.0000*** | ***W*** |
| ***50,20*** | $\hat{R}_{Gamma}$ | | | ***0.0490*** | ***9.1795e-07*** | ***0.6147*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.1128<br>0.9397*** | ***0.1231<br>1.0000*** | ***0.4897<br>1.9942*** | ***W*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9821*** | ***1.4739e-06*** | ***0.5681*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3984<br>1.7989*** | ***0.1231<br>1.0000*** | ***0.0472<br>0.6191*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***8.1082e-06*** | ***1.1104e-07*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.5125<br>0.9333*** | ***0.5625<br>1.0000*** | ***0.5625<br>1.0000*** | ***W*** |

**Table 5**
***α=0.9, σ=0.3, R= 0.2500***
**λ=0.4, ε =4, a=2, b1=0.4, b2=0.8**

| n, r | | | Criteria | Weighted | Quadratic | Entropy | Best |
|---|---|---|---|---|---|---|---|
| 15,5 | $\hat{R}_{Gamma}$ | | | 0.0304 | 1.1073e-05 | 0.2512 | |
| | | Gamma | MSE<br>MAPE | 0.0565<br>0.9215 | 0.0625<br>1.0000 | 0.003826<br>0.0050 | E |
| | $\hat{R}_{E\ Gamma}$ | | | 0.8190 | 7.0260e-05 | 0.3346 | |
| | | E Gamma | MSE<br>MAPE | 0.3238<br>2.2760 | 0.0625<br>0.9997 | 0.0072<br>0.3383 | E |
| | $\hat{R}_{Jeffery}$ | | | 0.0471 | 2.6842e-05 | 3.7829e-07 | |
| | | Jeffery | MSE<br>MAPE | 0.5140<br>0.9372 | 0.5625<br>1.0000 | 0.5625<br>1.0000 | W |
| 30,8 | $\hat{R}_{Gamma}$ | | | 0.0341 | 4.1475e-06 | 0.2996 | |
| | | Gamma | MSE<br>MAPE | 0.0571<br>0.9362 | 0.0625<br>1.0000 | 0.0025<br>0.1985 | E |
| | $\hat{R}_{E\ Gamma}$ | | | 0.9508 | 1.5047e-04 | 0.6170 | |
| | | E Gamma | MSE<br>MAPE | 0.4911<br>2.8033 | 0.0624<br>0.9994 | 0.1347<br>1.4680 | Q |
| | $\hat{R}_{Jeffery}$ | | | 1.9664e-08 | 0.0125 | 3.7503e-06 | |
| | | Jeffery | MSE<br>MAPE | 0.5625<br>1.000 | 0.5454<br>0.9834 | 0.5625<br>1.0000 | Q |
| 50,20 | $\hat{R}_{Gamma}$ | | | 0.0484 | 7.6745e-07 | 0.4659 | |
| | | Gamma | MSE<br>MAPE | 0.0617<br>0.9935 | 0.0625<br>1.0000 | 0.0466<br>0.8636 | E |
| | $\hat{R}_{E\ Gamma}$ | | | 0.9512 | 9.5245e-05 | 0.4482 | |
| | | E Gamma | MSE<br>MAPE | 0.4917<br>2.8048 | 0.0625<br>0.9996 | 0.0393<br>0.7927 | E |
| | $\hat{R}_{Jeffery}$ | | | 0.1000 | 7.4073e-06 | 7.7531e-06 | |
| | | Jeffery | MSE<br>MAPE | 0.5125<br>0.9333 | 0.5625<br>1.0000 | 0.5625<br>1.0000 | W |

***Table 6***
***α=3.7, σ=2, R= 0.3509***
**λ=0.4, ε =4, a=4, b1=0.4, b2=0.8**

| *n, r* | | | *Criteria* | *Weighted* | *Quadratic* | *Entropy* | *Best* |
|---|---|---|---|---|---|---|---|
| ***15,5*** | $\hat{R}_{Gamma}$ | | | ***0.0434*** | ***0.0074*** | ***0.2912*** | |
| | | ***Gamma*** | ***MSE***<br>***MAPE*** | ***0.1115***<br>***0.9237*** | ***0.1231***<br>***0.9999*** | ***0.0036***<br>***0.1701*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.5640*** | ***9.2849e-04*** | ***0.1257*** | |
| | | ***E Gamma*** | ***MSE***<br>***MAPE*** | ***0.0454***<br>***0.6073*** | ***0.1225***<br>***0.9974*** | ***0.0507***<br>***0.6419*** | ***W*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***1.6599e-04*** | ***1.3375e-04*** | |
| | | ***Jeffery*** | ***MSE***<br>***MAPE*** | ***0.3915***<br>***0.9541*** | ***0.4211***<br>***0.9997*** | ***0.4212***<br>***0.9998*** | ***W*** |
| ***30,8*** | $\hat{R}_{Gamma}$ | | | ***0.9274*** | ***0.0064*** | ***0.5155*** | |
| | | ***Gamma*** | ***MSE***<br>***MAPE*** | ***0.3323***<br>***1.6429*** | ***0.1187***<br>***0.9819*** | ***0.0271***<br>***0.4692*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.8716*** | ***1.4545e-04*** | ***0.3613*** | |
| | | ***E Gamma*** | ***MSE***<br>***MAPE*** | ***0.2711***<br>***1.4839*** | ***0.1230***<br>***0.9996*** | ***1.0751e-04***<br>***0.0295*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***1.3322e-14*** | ***5.6179e-05*** | ***1.5199e-05*** | |
| | | ***Jeffery*** | ***MSE***<br>***MAPE*** | ***0.4213***<br>***1.0000*** | ***0.4209***<br>***0.9999*** | ***0.4213***<br>***1.0000*** | ***Q*** |
| ***50,20*** | $\hat{R}_{Gamma}$ | | | ***0.9219*** | ***1.3092e-06*** | ***0.4137*** | |
| | | ***Gamma*** | ***MSE***<br>***MAPE*** | ***0.3260***<br>***1.6271*** | ***0.1231***<br>***1.0000*** | ***0.0039***<br>***0.1790*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9412*** | ***2.3536e-04*** | ***0.3830*** | |
| | | ***E Gamma*** | ***MSE***<br>***MAPE*** | ***0.3484***<br>***1.6822*** | ***0.1230***<br>***0.9993*** | ***0.0010***<br>***0.0916*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***2.3391e-07*** | ***1.7596e-05*** | ***4.1195e-05*** | |
| | | ***Jeffery*** | ***MSE***<br>***MAPE*** | ***0.1341***<br>***1.0000*** | ***0.1231***<br>***0.9999*** | ***0.1231***<br>***0.9999*** | ***Q, E*** |

**Table 7**
***α=0.9, σ=0.3, R= 0.2500***
**λ=0.4, ε =4, a=4, b1=2.1, b2=1.5**

| ***n, r*** | | | ***Criteria*** | ***Weighted*** | ***Quadratic*** | ***Entropy*** | ***Best*** |
|---|---|---|---|---|---|---|---|
| ***15,5*** | $\hat{R}_{Gamma}$ | | | ***0.9361*** | ***9.1981e-04*** | ***0.6193*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.4707<br>2.7443*** | ***0.0620<br>0.9963*** | ***0.1364<br>1.4773*** | ***Q*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.8781*** | ***1.8448e-04*** | ***0.4445*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.3945<br>2.5123*** | ***0.0624<br>0.9993*** | ***0.0378<br>0.7780*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1196*** | ***6.5235e-07*** | ***7.4369e-09*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.1458<br>1.2784*** | ***0.0625<br>0.9999*** | ***0.0625<br>1.0000*** | ***Q, E*** |
| ***30,8*** | $\hat{R}_{Gamma}$ | | | ***0.9654*** | ***5.4700e-05*** | ***0.7260*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.5118<br>2.8617*** | ***0.0625<br>1.0002*** | ***0.2266<br>1.9039*** | ***Q*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9702*** | ***4.1529e-04*** | ***0.7305*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.5186<br>2.8807*** | ***0.0623<br>0.9983*** | ***0.2309<br>1.9220*** | ***Q*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***1.4532e-05*** | ***6.5758e-05*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.0630<br>1.0037*** | ***0.0625<br>0.9999*** | ***0.0625<br>0.9997*** | ***Q, E*** |
| ***50,20*** | $\hat{R}_{Gamma}$ | | | ***0.9513*** | ***3.2438e-06*** | ***0.5085*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.4918<br>2.8050*** | ***0.0625<br>1.0000*** | ***0.0668<br>1.0341*** | ***Q*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.9567*** | ***1.8873e-04*** | ***0.4795*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.4995<br>2.8269*** | ***0.0624<br>0.9992*** | ***0.0527<br>0.9180*** | ***E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***6.3174e-06*** | ***1.7129e-08*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.1125<br>1.2000*** | ***0.0625<br>1.0000*** | ***0.0625<br>1.0000*** | ***Q, E*** |

**Table 8**
***α=3.7, σ=2, R= 0.3509***
**λ=0.4, ε =4, a=4, b1=2.1, b2=1.5**

| ***n, r*** | | | ***Criteria*** | ***Weighted*** | ***Quadratic*** | ***Entropy*** | ***Best*** |
|---|---|---|---|---|---|---|---|
| ***15,5*** | $\hat{R}_{Gamma}$ | | | ***0.9389*** | ***4.6379e-04*** | ***0.6365*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.3458<br>1.6757*** | ***0.1228<br>0.9987*** | ***0.0816<br>0.8139*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.0181*** | ***8.0348e-06*** | ***0.0019*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.1107<br>0.9484*** | ***0.1231<br>1.0000*** | ***0.1218<br>0.9945*** | ***Q, E*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***1.3569e-06*** | ***6.2729e-07*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.1231<br>1.0000*** | ***0.1363<br>1.0438*** | ***0.1231<br>1.0000*** | ***W, E*** |
| ***30,8*** | $\hat{R}_{Gamma}$ | | | ***0.9445*** | ***8.8062e-05*** | ***0.5543*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.3524<br>1.6917*** | ***0.1231<br>0.9997*** | ***0.0414<br>0.5795*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.1927*** | ***5.2524e-07*** | ***0.0227*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.0250<br>0.4508*** | ***0.1231<br>1.0000*** | ***0.1077<br>0.9353*** | ***W*** |
| | $\hat{R}_{Jeffery}$ | | | ***9.1969e-04*** | ***0.0491*** | ***1.6927e-07*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.1225<br>0.9974*** | ***0.1128<br>0.9400*** | ***0.1231<br>1.0000*** | ***Q*** |
| ***50,20*** | $\hat{R}_{Gamma}$ | | | ***0.9247*** | ***1.5201e-06*** | ***0.3946*** | |
| | | ***Gamma*** | ***MSE<br>MAPE*** | ***0.3292<br>1.6352*** | ***0.1231<br>1.0000*** | ***0.0019<br>0.1247*** | ***E*** |
| | $\hat{R}_{E\ Gamma}$ | | | ***0.5052*** | ***4.9047e-09*** | ***0.0391*** | |
| | | ***E Gamma*** | ***MSE<br>MAPE*** | ***0.0238<br>0.4397*** | ***0.1231<br>1.0000*** | ***0.0972<br>0.8886*** | ***W*** |
| | $\hat{R}_{Jeffery}$ | | | ***0.1000*** | ***3.6460e-06*** | ***1.5228e-08*** | |
| | | ***Jeffery*** | ***MSE<br>MAPE*** | ***0.1530<br>1.0850*** | ***0.1231<br>1.0000*** | ***0.1231<br>1.0000*** | ***Q, E*** |

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