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ON ESTIMATING THE RELIABILITY FUNCTION OF AN EXPONENTIAL-PARETO II DISTRIBUTION USING ROBUST MAXIMUM LIKELIHOOD METHOD

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ABSTRACT

This study aimed to estimate the reliability function of the Pareto II distribution, which was contaminated with an exponential distribution. The reliability function of the distribution provides a reasonably good fit for income distributions and property value, and it also explains many experimental phenomena. The simulation method was utilized to estimate the reliability function of the robust maximum likelihood estimators. The criteria MSE and IMSE were used to compare the studied methods to choose the best method. The study showed that the RMLE method is the best in the case of contaminated and no contaminated than the MLE method.

KEYWORDS

Maximum Likelihood Estimator method (MLE), Pareto type II distribution, Reliability function, Robust Maximum Likelihood Estimator method (RMLE).

1. INTRODUCTION

Most statistics studies, particularly in the field of reliability, strive to provide estimates with a high-efficiency level, particularly when the sample data under examination is contaminated.

In this study, we'll use a mixed distribution because weighted distributions are a good tool for modelling statistical data when it doesn't fit into standard distributions due to damage or loss caused by the original observation, which results in a decrease in value or the use of a sampling procedure that gives existing units unequal chances. This concept was proposed by Fisher (1934) and further refined by Rao (1997).

Some authors, including Al-Kadim and Hantoosh (2013), have created double-weighted and double-weighted exponential distributions in recent reports of weighted distributions. Mir et al. (2013) investigated a novel type of length-biased beta distribution, the first kind, and evaluated its parameters. Al-Khadim and Hussein (2014) proposed a new class of length-biased weighted exponential and Rayleigh distributions and used the application to investigate some statistical features. The purpose of this study was to use the simulation approach to develop efficient estimators of the reliability function of the

Pareto II distribution contaminated with an exponential distribution using (RMLE) and compare the results with MLE.

2. THE RELIABILITY FUNCTION

Reliability is abbreviated from "reliable", which means trusting and depending on something. This term was used first after the First World War by 1920 in the productivity improvement process through using statistical control operations; at the end of 1950 and the beginning of 1960, the USA worked to release intercontinental ballistic missiles and competition with the Russians to be the first who will be on the moon. The significant role of reliability appears in study and application, and from then, reliability has become very popular in the applied aspects (Waleed, 2019).

During the last year, many studies dealt with estimating the reliability function of many distributions using different methods; also, there are other techniques in estimating this function through metrics depending on the time average fail and another (Forcina et al., 2020).

Definition 2.1

Let T be a random variable and $P(T \leq t)$ be the probability failure function in the period $(t, t+\Delta t)$, and $F(t)$ be the cumulative distribution function (cdf) at the time of failure t; the Reliability Function, denoted by $R(t)$, can be defined as

$$R(t) = P(T > t) = 1 - P(T \leq t) = 1 - F(t). \quad (1)$$

3. STATISTICAL DISTRIBUTIONS

3.1 Pareto II Distribution

The Pareto distribution is attributed to the Italian economist, Vilfredo Pareto, who used this distribution for the first time and extensively in the subject of economics as a model for studying the income distribution in the applications of dependency theory because it is one of the failure distributions for models of stress, durability, and mechanical engineering (Khaleel, 2012; Al Sarraf et al., 2020). The average time is taken for devices and equipment until failure is essential in determining the reliability of devices and equipment.

Proposition 3.1

If random variable X has a distribution that belongs to the exponential family, and

$$q(\theta) = \ln(1 + \delta_1(\mathrm{t} - \mu)),\ s(\theta) = \alpha\delta_1^{\alpha}, u(\theta) = -(\alpha+1), r(t) = 1$$

Then $X \sim Pareto(\alpha, \delta_1, \mu)$.

Proof:

Since the exponential family form is:

$$f(t;\theta) = \mathrm{r(t)s}(\theta)e^{q(t)u(\theta)} \text{ or } f(t;\theta) = \mathrm{r(t)}e^{\varphi(\theta)+q(t)u(\theta)}$$

And the probability density function of Pareto II is:

$$f(t) = \begin{cases} \dfrac{\alpha\delta_1}{(1+\delta_1(t-\mu))^{(\alpha+1)}} & t > \mu \\ 0 & o.w \end{cases}$$

where $\delta > 0$ is a scale parameter, $\alpha > 0$ is a shape parameter, and $\mu \in R$ is a location parameter then we can rewrite the density function of the Pareto distribution as follows:

$$f(t;\alpha,\delta_1,\mu) = e^{\ln\left[\alpha\delta_1(1+\delta_1(t-\mu))^{-(\alpha+1)}\right]} = e^{\ln\alpha + \ln\delta_1 - (\alpha+1)\ln(1+\delta_1(t-\mu))}$$

where $r(t) = 1$, $\varphi(\theta) = \ln\alpha + \ln\delta_1$, $q(t) = \ln\left(1+\delta_1(t-\mu)\right)$, $u(\theta) = -(\alpha+1)$ we will take the particular case when $\mu = 0$, and then the premise equations become $q(t) = \ln(1+\delta_1 t)$ and the pdf of Pareto II becomes (Chadli and Kermoune, 2021).

$$f(t) = \begin{cases} \dfrac{\alpha\delta_1}{(1+\delta_1 t)^{(\alpha+1)}} & t > 0, \alpha, \delta_1 > 0 \\ 0 & otherwise \end{cases} \quad (2)$$

where δ_1 is a Scale parameter
α is a Shape parameter
μ is location parameter

3.2 Exponential Distribution

The exponential distribution is a continuous probability distribution named after the exponential function. The time intervals between occurrences are estimated using this distribution. The exponential distribution is frequently utilized in time measurement concerns. This includes the post office service, the duration of a telephone call, the period of unloading the freighter, the period of repairing a machine, and the period of waiting for a customer before obtaining the service. In the exact sciences, the exponential distribution represents the lifespan of radioactive atoms before they decay. (Mohammed & Ugwuowo, 2021).

Definition 3.2.1

We say that the random variable T has the exponential distribution if the pdf given by

$$f(t,\delta_2) = \delta_2 e^{-\delta_2 t}, t > 0, \delta_2 > 0$$

and cdf, Reliability

$$F(t) = 1 - e^{-\delta_2 t}, R(t) = e^{-\delta_2 t}.$$

3.3 Exponential-Pareto II

The Pareto distribution has been contaminated with an exponential distribution with a ratio of τ so that we have the exponential-Pareto II distribution

$$f(t,\underline{\theta}) = (1-\tau)\frac{\alpha\delta_1}{\left(1+\delta_1(t-\mu)\right)^{(\alpha+1)}} + \tau\delta_2 e^{-\delta_2 t}, t > 0, \alpha, \delta_1, \delta_2 > 0.$$

3.4 Other Properties

For the random variable t, which has distribution with pdf $f(t)$ and cdf $F(t)$, then the survival function is defined as follows:

$$R(t) = S(t) = 1 - F(t) = P(T > t) \qquad \text{for } t > 0$$

Having the following properties:

1. $S(0) = 1$ and $S(\infty) = 0$.
2. The function $S(t)$ is non-increasing.

and the hazard function is:

$$h(t) = \frac{f(t)}{S(t)}$$

so the survival and hazard function for our distribution is:

$$S_P(t) = \left(\frac{1}{(1+\delta_1 t)^{\alpha}}\right)$$

$$S_E(t) = \mathrm{e}^{-\delta_2 \mathrm{t}}$$

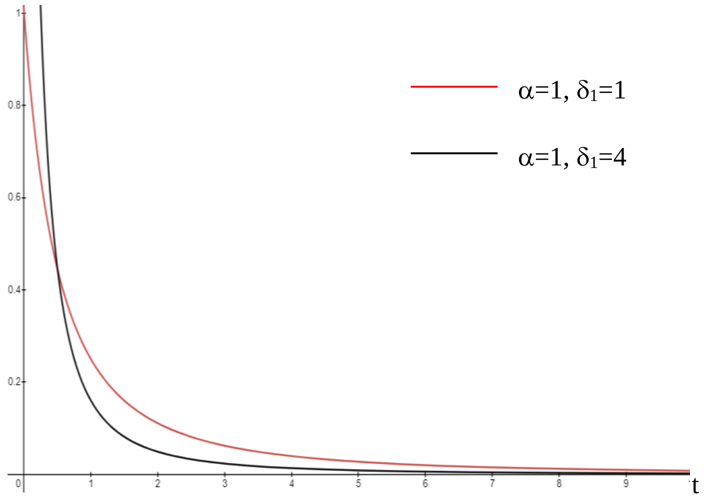
For Pareto and Exponential distribution, respectively.

The Hazard function also can be determined for Pareto and Exponential distribute as follows:

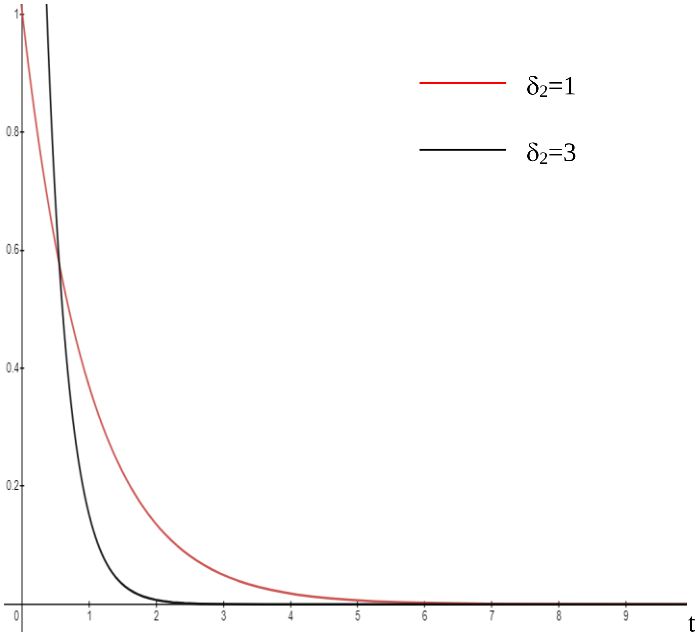
$$h_P(t) = \left(\frac{\alpha\delta_1}{1+\delta_1 t}\right)$$

$$h_E(t) = \delta_2$$

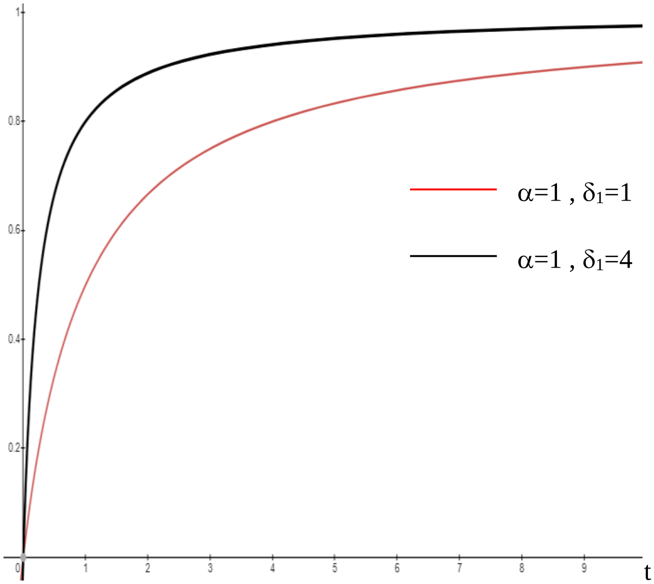
The following are some shapes of some of the distribution functions under study:



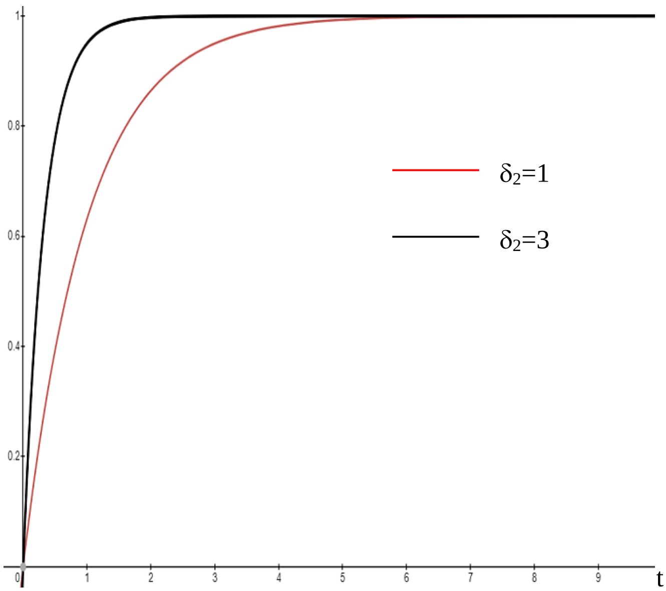
P.D.F of Pareto Type II Lomax Distribution



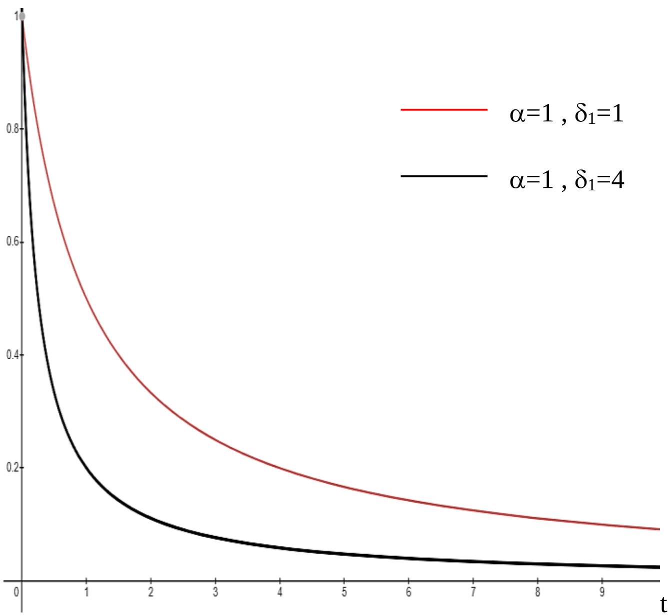
P.D.F of Exponential Distribution



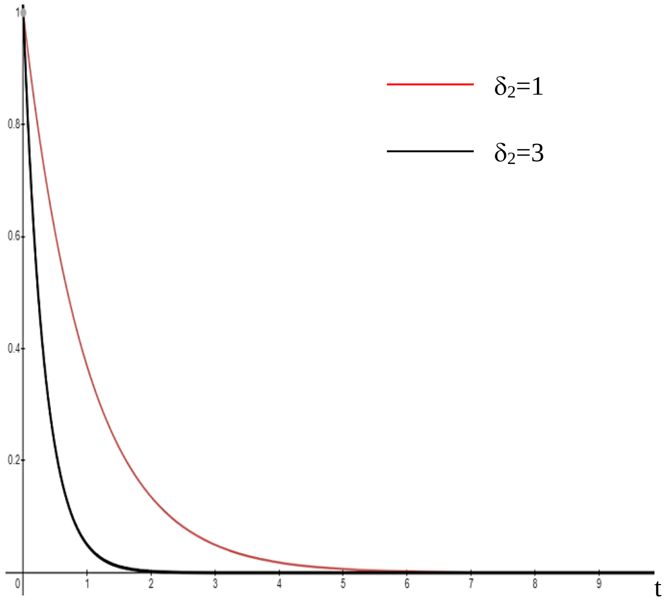
C.D.F of Pareto Type II Lomax Distribution



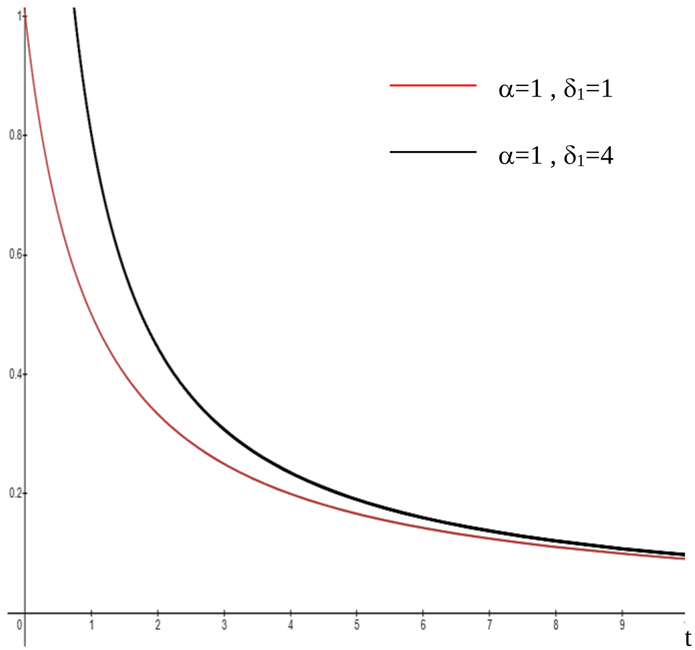
C.D.F of Exponential Distribution



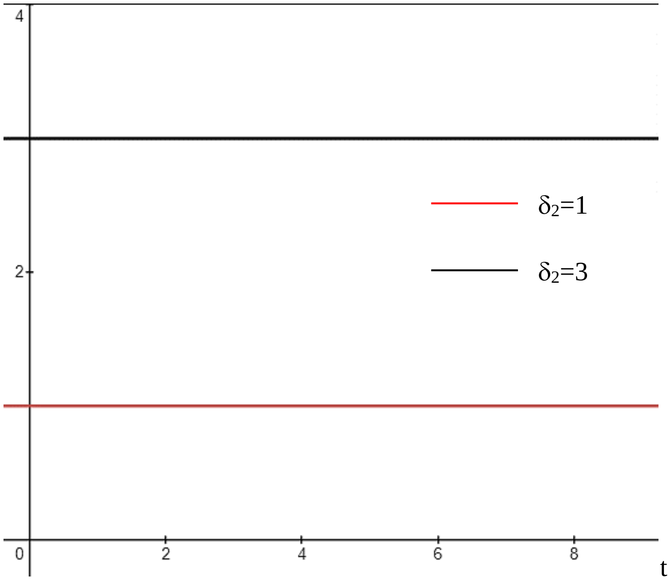
Survival Function of Pareto Type II Lomax Distribution



Survival Function of Exponential Distribution



Hazard Function of Pareto Type II Lomax Distribution



Hazard Function of Exponential Distribution

4. ESTIMATION RELIABILITY FUNCTION

The primary purpose of the estimation methods is to obtain the capabilities of the model parameters to be studied. It must have good specifications this time to make a model able to benefit from it, and the estimation methods differ according to different ideas and methods to achieve two purposes important (Khaleel, 2012).

The first is that the capabilities contain good specifications, the purpose the other is the method appears in an easy-to-implement way; in recent years, intelligent techniques have been used to estimate the function of reliability, and there are several ways to estimate the function of the reliability of the total data, including classical and non-classical methods (Al Sarraf et al., 2020).

4.1 Maximum Likelihood Estimator Method (MLE)

MLE is the most excellent possible method that assumes that the sample represents the whole community (Atta and Abbas, 2014). The value of the estimate will maximize the probability density function (pdf) and can be defined:

Definition 4.1.1

If $T_1, T_2, \dots \dots \dots \dots, T_n$ are random samples of size (n) drawn from a population with a probability density function $f(t, \underline{\theta})$, where $\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_k)$ then the likelihood function, which is symbolized by the symbol (L), is:

$$L(\underline{\theta}; \underline{t}) = f(t_1, \underline{\theta}).f(t_2, \underline{\theta}) \dots \dots f(t_n, \underline{\theta}) = \prod_{i=1}^{n} f(t_i, \underline{\theta})$$

where $\underline{\theta} = (\alpha, \delta_1)$, then the likelihood function for the Pareto II distribution is:

$$L(\alpha, \delta_1; \underline{t}) = \prod_{i=1}^{n} \frac{\alpha\delta_1}{(1+\delta_1 t_i)^{(\alpha+1)}}.$$

For estimating the parameters, we take the natural logarithm to both sides of the equation

$$\text{Ln}\, L = \text{Ln} \prod_{i=1}^{n} \frac{\alpha\delta_1}{(1+\delta_1 t_i)^{(\alpha+1)}}).$$

To find the estimated value of the shape parameter, which makes the likelihood function great as possible, we can find the derivative of the function for the parameter

$$\frac{\partial \text{Ln}\, L}{\partial \alpha} = \frac{\text{n}}{\alpha} - \sum_{i=1}^{n} \text{Ln}(1+\delta_1 t_i) \tag{3}$$

Equating the derivative to zero, equation (3) has one solution.

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} \text{Ln}(1+\hat{\delta}_1 t_i)} \tag{4}$$

In the same way, we find the estimated value of the measurement parameter (scale parameter):

$$\frac{n}{\hat{\delta}_1} - \sum_{i=1}^{n} \frac{t_i}{1+\hat{\delta}_1 t_i} - \hat{\alpha} \sum_{i=1}^{n} \frac{t_i}{1+\hat{\delta}_1 t_i} = 0$$

And by substitute $\hat{\alpha}$ we get

$$\frac{n}{\hat{\delta}_1} - \sum_{i=1}^{n} \frac{1}{1+\hat{\delta}_1 t_i} - \frac{n}{\sum_{i=1}^{n} \text{Ln}\left(1+\hat{\delta}_1 t_i\right)} \sum_{i=1}^{n} \frac{t_i}{1+\hat{\delta}_1 t_i} = 0.$$

The equation is solved numerically using the Newton-Raphson method.

The MLE estimator has the characteristic of steadiness; this feature can obtain the reliability function estimate for the distribution of Pareto II

$$\hat{R}_{MLE}(t) = \left(1+\hat{\delta}_1 t\right)^{-\hat{\alpha}_{MLE}} \tag{5}$$

Here $\hat{\delta}_1$is the MLE of δ_1.

4.2 Robust Maximum Likelihood Estimator Method (RMLE)

This method is based on scoring fisher encoding and is considered part of the maximum likelihood method. This method is the derivation of the reliability function estimate for the distribution of Pareto II contaminated with the distribution of ace using the robust (ML) method (Chadli and Kermoune, 2021).

$$T_1, T_2, \dots \dots, T_{n_1} \sim f_1 \Rightarrow 1 - F_1(t) = (1+\delta_1 t)^{-\alpha_1}$$

$$T_{n_1+1}, \dots \dots, T_n \sim f_2 \Rightarrow 1 - F_2(t) = e^{-\delta_2 t}$$

Such that

$$\begin{bmatrix} f_1(t;\delta_1,\alpha_1) = \dfrac{\alpha_1\delta_1}{(1+\delta_1 t)^{(\alpha_1+1)}} \\ f_2(t;\delta_2) = \delta_2 e^{-\delta_2 t} \end{bmatrix}$$

Then

$$\begin{aligned} f(t;\underline{\delta},\alpha) &= (1-\tau) f_1(t;\delta_1,\alpha_1) + \tau f_2(t;\delta_2) \\ &= (1-\tau)\frac{\alpha_1\delta_1}{(1+\delta_1 t)^{(\alpha_1+1)}} + \tau\ \delta_2 e^{-\delta_2 t_n} \end{aligned} \tag{6}$$

So that τ represents contaminate levels in the observations understudy and from the equation (5)

$$\begin{aligned} L &= \prod_{i=1}^{n} f(t_i;\underline{\theta}) \\ &= \prod_{i=1}^{n}\left[(1-\tau)\frac{\alpha_1\delta_1}{(1+\delta_1 t_i)^{(\alpha_1+1)}} + \tau\ \delta_2 e^{-\delta_2 t_i}\right] \end{aligned}$$

$$lnL = \sum_{i=1}^{n} ln\left[(1-\tau)\frac{\alpha_1\delta_1}{(1+\delta_1 t_i)^{(\alpha_1+1)}} + \tau\ \delta_2 e^{-\delta_2 t_i}\right] \tag{7}$$

Taking the partial derivative for $\underline{\theta}$ to (7), we get

$$\frac{\partial}{\partial\underline{\theta}} lnL = \begin{bmatrix} \dfrac{\partial}{\partial\alpha} lnL \\ \dfrac{\partial}{\partial\delta_1} lnL \\ \dfrac{\partial}{\partial\delta_2} lnL \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = G \text{ such that } \underline{\theta} = \begin{bmatrix} \alpha \\ \delta_1 \\ \delta_2 \end{bmatrix} \tag{8}$$

The second derivative is as follows:

$$\frac{\partial^2}{\partial\underline{\theta}\partial\underline{\theta}} lnL = \begin{bmatrix} \dfrac{\partial}{\partial\alpha^2} lnL & \dfrac{\partial}{\partial\alpha\partial\delta_1} lnL & \dfrac{\partial}{\partial\alpha\partial\delta_2} lnL \\ \dfrac{\partial}{\partial\alpha\partial\delta_1} lnL & \dfrac{\partial}{\partial\delta_1^2} lnL & \dfrac{\partial}{\partial\delta_1\partial\delta_2} lnL \\ \dfrac{\partial}{\partial\delta_2\partial\alpha} lnL & \dfrac{\partial}{\partial\delta_2\partial\delta_1} lnL & \dfrac{\partial}{\partial\delta_2^2} lnL \end{bmatrix}$$

$$\frac{\partial^2}{\partial\underline{\theta}\partial\underline{\theta}} lnL = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = H \tag{9}$$

It is also $H = \frac{\partial}{\partial\underline{\theta}}(G)$.

To find the estimate of parameters (α, δ_1, δ_2) in formula (6), Multivariable Newton-Raphson numerical method of solving this equation is adopted and based on formulas (8), (9) as follows:

$$\underline{\theta}_{n+1} = \theta_n - H^{-1}G \tag{10}$$

Formula (10) is the only robust maximum likelihood estimator method parameter of (α, δ_1, δ_2) as follows:

$$\underline{\hat{\theta}}_{RMLE} = \underline{\theta}_{n+1} \tag{11}$$

Using stability, the reliability function is estimated to be as follows:

$$R(t) = (1-\tau)R_1(t) + \tau R_2(t)$$
$$= (1-\tau)(1+\delta_1 t)^{-\alpha_1} + \tau(\mathrm{e}^{-\delta_2 \mathrm{t}})$$

$$\boxed{\hat{R}_{RMLE}(t) = (1-\hat{\tau})(1+\hat{\delta}_{1_{RMLE}}t)^{-\hat{\alpha}_{RMLE}} + \hat{\tau}(\mathrm{e}^{-\hat{\delta}_{2RMLE}\mathrm{t}})} \tag{12}$$

5. SIMULATION

In our study, the formulation of the simulation model includes four basic and important stages for estimating the reliability function of Pareto II distribution, respectively:

Stage 1:

This is one of the most critical stages on which the rest of the subsequent stages depend, where the default values are selected as follows: Choosing arbitrary values for α, δ_1, δ_2 with $\tau = 10\%, 20\%, 30\%$

Table 1
The Default Value of the Scale Parameter

Model	**1**	**2**	**3**
α	1.2	2.4	3.1
δ_1	1.3	1.4	2.3
δ_2	2.1	3.5	1.6

For simulation n = (50, 150, 250) virtual samples size
The repetition of each experiment is r =1000.

Stage 2:

Pareto II distribution and exponential distribution data are generated using the cdf

$$F_1(t) = 1 - (1+\delta_1 t)^{-\alpha}$$
$$F_2(t) = 1 - e^{-\delta_2 t}$$

Suppose the U represents the cdf of the Pareto II distribution

$$U = 1 - (1+\delta_1 t)^{-\alpha}$$

$$\Rightarrow t = \delta_1^{-1}\left((1-U)^{-1/\alpha} - 1\right) \tag{13}$$

As for generating exponential distribution and letting U ~ uniform (0,1) of the distribution

$$U = 1 - e^{-\delta_2 t}$$

Then

$$t = -\delta_2^{-1} ln(1-u) \qquad (14)$$

The distribution of the contaminated Pareto II under study is obtained according to the following formula and based on the equation (13), (14) as follows

$$t = \begin{cases} \delta_1^{-1}\left((1-U)^{-1/\alpha} - 1\right) \\ \quad -\delta_2^{-1}\ln(1-u) \end{cases}$$

Stage 3:

At this stage, the function of the reliability of the Pareto II model was estimated according to the MLE method of formula (5) and RMLE method of formula (12)

Stage 4:

This is the last stage where the two methods of estimation of the reliability function are compared on the adoption of MSE and IMSE according to the following formulas:

$$\text{MSE}(\hat{R}(t)) = \frac{1}{r}\sum_{i=1}^{r}\left[\hat{R}_i(t_j) - R(t_j)\right]^2$$

$$\text{IMSE}(\hat{R}(t)) = \frac{1}{n_t}\sum_{j=1}^{n_t}\left\{\frac{1}{r}\sum_{i=1}^{r}\left[R_i(t_j) - R(t_j)\right]^2\right\} = \frac{1}{n_t}\sum_{j=1}^{n_t}\left(\text{MSE}(\hat{R}(t_j))\right).$$

The following tables show reliability function estimate, MSE, and IMSE on all sample sizes.

Table 2
Times of estimating the reliability function

t_i	Model 1	Model 2	Model 3
t_1	0.1	0.1	0.1
t_2	0.18	0.15	0.2
t_3	0.26	0.2	0.3
t_4	0.34	0.25	0.4
t_5	0.42	0.3	0.5
t_6	0.5	0.35	0.6
t_7	0.58	0.4	0.7
t_8	0.66	0.45	0.8
t_9	0.74	0.5	0.9
t_{10}	0.82	0.55	1

a. Generating random numbers for the Pareto II distribution with two parameters (α, δ_1), an exponential distribution with a single-parameter (δ_2). According to the available generation function in (VB.NET) language:

   $t\sim$ Pareto (α, δ_1) or $t\sim exp(\delta_2)$.

b. Find estimators:
   At this stage, the estimation of the reliability function of the Pareto II distribution and the contaminated Pareto II distribution through the exponential distribution is dealt with through estimation methods in the theoretical part of this thesis, according to the methods: MLE and RMLE.

c. Comparison:
   After finding estimators of reliability function two criteria were used to evaluate the accuracy of the estimation method: Mean squares error (MSE) and Integral Mean square Error (IMSE). The method that yields the smallest value of MSE and IMSE is considered the best-fitted.

6. RESULTS

From tables (3), (4), and (5), we observe that the estimators of the reliability function by using all studied estimation methods revealed that the estimator values of parameters are close to the real values of the reliability function in all models and samples sizes, in addition, we notice the followings:

1. For all models, we observed that the RMLE method for reliability function estimates was close to the real value of the reliability function when the sample size increased.
2. It shows that the estimated and real value of the reliability function decreases by increasing time t_i.
3. The increase in the value of scale and shape parameters leads to an increased estimated reliability function for all estimation methods.
4. By increasing the sample size, the (MSE) values for estimating the reliability function decrease for all models in the RMLE method. In contrast, in the MLE method, increases in some models.

Table 3
Estimators of the Parameters, the Reliability Function of the Estimation Methods when Contamination Ratio (10%) and for all Selected Sample Sizes and all Model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}
50	0.1	0.858286	0.910695	0.785232	0.1	0.727628	0.835957	0.72754	0.1	0.558951	0.539503	0.549086
	0.18	0.767824	0.746286	0.696576	0.15	0.628739	0.769255	0.629594	0.2	0.351066	0.32372	0.336023
	0.26	0.692519	0.787433	0.629304	0.2	0.547325	0.710548	0.549251	0.3	0.238805	0.209376	0.221206
	0.34	0.629043	0.733574	0.573895	0.25	0.479653	0.658584	0.482594	0.4	0.171856	0.143189	0.153277
	0.42	0.574965	0.584211	0.526967	0.3	0.422921	0.61235	0.426725	0.5	0.128818	0.102238	0.110248
	0.5	0.528465	0.638903	0.486647	0.35	0.374993	0.571017	0.379468	0.6	0.099504	0.075547	0.081562
	0.58	0.488154	0.597259	0.451647	0.4	0.334219	0.533903	0.339165	0.7	0.078617	0.057407	0.06168
	0.66	0.452952	0.55893	0.420998	0.45	0.299308	0.500443	0.304536	0.8	0.0632	0.044648	0.04748
	0.74	0.42201	0.523606	0.393956	0.5	0.269241	0.470163	0.274578	0.9	0.051497	0.035413	0.037093
	0.82	0.394649	0.49101	0.369936	0.55	0.243204	0.442664	0.248503	1	0.042415	0.028562	0.029349
150	0.1	0.858286	0.801632	0.803727	0.1	0.727628	0.805461	0.72926	0.1	0.558951	0.625121	0.519409
	0.18	0.767824	0.68369	0.700539	0.15	0.628739	0.729094	0.631479	0.2	0.351066	0.422196	0.302859
	0.26	0.692519	0.590716	0.623246	0.2	0.547325	0.66324	0.551145	0.3	0.238805	0.301399	0.191186
	0.34	0.629043	0.51604	0.562131	0.25	0.479653	0.606042	0.48445	0.4	0.171856	0.224325	0.127879
	0.42	0.574965	0.455095	0.511962	0.3	0.422921	0.556037	0.428556	0.5	0.128818	0.172476	0.089398
	0.5	0.528465	0.404666	0.469714	0.35	0.374993	0.512061	0.381316	0.6	0.099504	0.136113	0.064725
	0.58	0.488154	0.362431	0.4335	0.4	0.334219	0.473174	0.341082	0.7	0.078617	0.109734	0.048224
	0.66	0.452952	0.326683	0.402055	0.45	0.299308	0.438616	0.306575	0.8	0.0632	0.090058	0.036804
	0.74	0.42201	0.296139	0.374478	0.5	0.269241	0.407764	0.276789	0.9	0.051497	0.075033	0.028672
	0.82	0.394649	0.269823	0.3501	0.55	0.243204	0.380102	0.250926	1	0.042415	0.063331	0.022738
250	0.1	0.858286	0.849461	0.845349	0.1	0.727628	0.655649	0.72961	0.1	0.558951	0.581314	0.558048
	0.18	0.767824	0.735498	0.749456	0.15	0.628739	0.540313	0.632701	0.2	0.351066	0.371987	0.347739
	0.26	0.692519	0.668477	0.671568	0.2	0.547325	0.44978	0.553259	0.3	0.238805	0.254556	0.23344
	0.34	0.629043	0.604333	0.607382	0.25	0.479653	0.377835	0.487345	0.4	0.171856	0.183042	0.165112
	0.42	0.574965	0.560188	0.553781	0.3	0.422921	0.32002	0.432067	0.5	0.128818	0.136728	0.121281
	0.5	0.528465	0.503947	0.508471	0.35	0.374993	0.273089	0.385263	0.6	0.099504	0.105262	0.091628
	0.58	0.488154	0.464054	0.46974	0.4	0.334219	0.23464	0.345295	0.7	0.078617	0.08305	0.070732
	0.66	0.452952	0.42933	0.436291	0.45	0.299308	0.202873	0.310902	0.8	0.0632	0.06687	0.055535
	0.74	0.42201	0.404864	0.407134	0.5	0.269241	0.176423	0.2811	0.9	0.051497	0.054771	0.044204
	0.82	0.394649	0.380946	0.381502	0.55	0.243204	0.154243	0.255114	1	0.042415	0.045521	0.035587

Table 4
Estimators of the Parameters, the Reliability Function of the Estimation Methods when Contamination Ratio (20%) and for all Selected Sample Sizes and all Model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}
50	0.1	0.852986	0.703913	0.748643	0.1	0.725079	0.86812	0.666145	0.1	0.591528	0.807989	0.586915
	0.18	0.758647	0.63511	0.641284	0.15	0.624608	0.812509	0.556021	0.2	0.392742	0.670124	0.384671
	0.26	0.679935	0.872632	0.56893	0.2	0.541687	0.762501	0.469978	0.3	0.281025	0.567246	0.270298
	0.34	0.613559	0.715801	0.514583	0.25	0.472676	0.717343	0.401672	0.4	0.211349	0.488098	0.198818
	0.42	0.557075	0.664022	0.470837	0.3	0.414812	0.676404	0.346655	0.5	0.16443	0.425684	0.150871
	0.5	0.508629	0.61677	0.434163	0.35	0.365967	0.639156	0.301749	0.6	0.130991	0.375448	0.117038
	0.58	0.466783	0.573583	0.402674	0.4	0.324483	0.605152	0.264651	0.7	0.106135	0.334313	0.092268
	0.66	0.43041	0.534054	0.375227	0.45	0.289052	0.574013	0.233662	0.8	0.087071	0.300133	0.073638
	0.74	0.398609	0.49782	0.351052	0.5	0.258633	0.545414	0.207515	0.9	0.072101	0.271369	0.059338
	0.82	0.370656	0.46456	0.32959	0.55	0.232389	0.519077	0.185248	1	0.060135	0.246895	0.048193
150	0.1	0.852986	0.882801	0.846651	0.1	0.725079	0.615899	0.628809	0.1	0.591528	0.640236	0.589007
	0.18	0.758647	0.806832	0.74939	0.15	0.624608	0.502259	0.510823	0.2	0.392742	0.441418	0.389104
	0.26	0.679935	0.742699	0.668881	0.2	0.541687	0.412314	0.420885	0.3	0.281025	0.320885	0.276813
	0.34	0.613559	0.687851	0.601425	0.25	0.472676	0.350305	0.351298	0.4	0.211349	0.242717	0.206855
	0.42	0.557075	0.640417	0.544326	0.3	0.414812	0.232056	0.296715	0.5	0.16443	0.189355	0.159841
	0.5	0.508629	0.598997	0.495571	0.35	0.365967	0.24449	0.253349	0.6	0.130991	0.151426	0.126437
	0.58	0.466783	0.562522	0.453621	0.4	0.324483	0.205312	0.218477	0.7	0.106135	0.123573	0.101707
	0.66	0.43041	0.530159	0.41728	0.45	0.289052	0.190086	0.190115	0.8	0.087071	0.10256	0.082831
	0.74	0.398609	0.501256	0.385604	0.5	0.258633	0.165584	0.166798	0.9	0.072101	0.086347	0.068093
	0.82	0.370656	0.475288	0.357838	0.55	0.232389	0.14268	0.147433	1	0.060135	0.073593	0.056384
250	0.1	0.852986	0.882801	0.846651	0.1	0.725079	0.625899	0.628809	0.1	0.591528	0.610186	0.591087
	0.18	0.758647	0.806832	0.74939	0.15	0.624608	0.502259	0.510823	0.2	0.392742	0.405457	0.389227
	0.26	0.679935	0.742699	0.668881	0.2	0.541687	0.412314	0.420885	0.3	0.281025	0.28611	0.274491
	0.34	0.613559	0.687851	0.601425	0.25	0.472676	0.350305	0.351298	0.4	0.211349	0.211117	0.202538
	0.42	0.557075	0.640417	0.544326	0.3	0.414812	0.252056	0.296715	0.5	0.16443	0.161251	0.154146
	0.5	0.508629	0.598997	0.495571	0.35	0.365967	0.24449	0.253349	0.6	0.130991	0.126594	0.119917
	0.58	0.466783	0.562522	0.453621	0.4	0.324483	0.205312	0.218477	0.7	0.106135	0.101635	0.094801
	0.66	0.43041	0.530159	0.41728	0.45	0.289052	0.212786	0.220115	0.8	0.087071	0.083128	0.075866
	0.74	0.398609	0.501256	0.385604	0.5	0.258633	0.155584	0.166798	0.9	0.072101	0.069066	0.061298
	0.82	0.370656	0.475288	0.357838	0.55	0.232389	0.13268	0.147433	1	0.060135	0.058158	0.049915

Table 5
Estimators of the Parameters, the Reliability Function of the Estimation Methods when Contamination Ratio (30%) and for all Selected Sample Sizes and all Model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}	t_i	R_{real}	R_{MLE}	R_{RMLE}
50	0.1	0.847686	0.779013	0.789011	0.1	0.722531	0.944669	0.628808	0.1	0.624105	0.8339	0.608466
	0.18	0.74947	0.610848	0.668551	0.15	0.620476	0.919823	0.516509	0.2	0.434418	0.710289	0.413046
	0.26	0.667351	0.54087	0.577403	0.2	0.53605	0.896595	0.432566	0.3	0.323245	0.61523	0.299331
	0.34	0.598074	0.498088	0.507153	0.25	0.4657	0.87482	0.36832	0.4	0.250842	0.540183	0.225917
	0.42	0.539185	0.441647	0.45198	0.3	0.406703	0.854357	0.318056	0.5	0.200042	0.479652	0.175049
	0.5	0.488792	0.570808	0.40782	0.35	0.356941	0.835084	0.277922	0.6	0.162479	0.429948	0.138056
	0.58	0.445413	0.524929	0.371819	0.4	0.314747	0.816893	0.245275	0.7	0.133653	0.388513	0.110239
	0.66	0.407868	0.483448	0.341948	0.45	0.278797	0.799689	0.218264	0.8	0.110942	0.35352	0.088824
	0.74	0.375208	0.44588	0.316757	0.5	0.248026	0.783388	0.195575	0.9	0.092704	0.323634	0.072059
	0.82	0.346662	0.411797	0.295196	0.55	0.221575	0.767918	0.176257	1	0.077855	0.297857	0.058772
150	0.1	0.847686	0.812514	0.815466	0.1	0.722531	0.344405	0.636272	0.1	0.624105	0.722068	0.615756
	0.18	0.74947	0.700425	0.703659	0.15	0.620476	0.210213	0.53024	0.2	0.434418	0.546441	0.423131
	0.26	0.667351	0.60488	0.615137	0.2	0.53605	0.131282	0.451692	0.3	0.323245	0.428304	0.310091
	0.34	0.598074	0.541897	0.544244	0.25	0.4657	0.083719	0.391457	0.4	0.250842	0.344977	0.236436
	0.42	0.539185	0.468664	0.486808	0.3	0.406703	0.054418	0.343807	0.5	0.200042	0.283973	0.184944
	0.5	0.488792	0.423138	0.439727	0.35	0.356941	0.035999	0.305093	0.6	0.162479	0.237949	0.147181
	0.58	0.445413	0.393802	0.40068	0.4	0.314747	0.024202	0.27294	0.7	0.133653	0.202356	0.118557
	0.66	0.407868	0.449509	0.367919	0.45	0.278797	0.016517	0.245759	0.8	0.110942	0.174255	0.096352
	0.74	0.375208	0.419373	0.340122	0.5	0.248026	0.011429	0.222455	0.9	0.092704	0.151673	0.078837
	0.82	0.346662	0.392703	0.31628	0.55	0.221575	0.008012	0.202253	1	0.077855	0.133248	0.064854
250	0.1	0.847686	0.711424	0.819505	0.1	0.722531	1.219201	0.642331	0.1	0.624105	0.607466	0.623546
	0.18	0.74947	0.557141	0.709176	0.15	0.620476	1.335869	0.536938	0.2	0.434418	0.398734	0.428151
	0.26	0.667351	0.445285	0.621218	0.2	0.53605	1.457176	0.458451	0.3	0.323245	0.276964	0.31176
	0.34	0.598074	0.362066	0.550346	0.25	0.4657	1.583073	0.398005	0.4	0.250842	0.200867	0.235635
	0.42	0.539185	0.298777	0.492622	0.3	0.406703	1.713513	0.350024	0.5	0.200042	0.150719	0.182549
	0.5	0.488792	0.249725	0.445092	0.35	0.356941	1.84845	0.310932	0.6	0.162479	0.116243	0.14385
	0.58	0.445413	0.211076	0.405526	0.4	0.314747	1.987842	0.278389	0.7	0.133653	0.09171	0.114746
	0.66	0.407868	0.18018	0.372232	0.45	0.278797	2.13165	0.250822	0.8	0.110942	0.073746	0.092367
	0.74	0.375208	0.155164	0.343918	0.5	0.248026	2.279834	0.227147	0.9	0.092704	0.06027	0.07488
	0.82	0.346662	0.134674	0.319593	0.55	0.221575	2.432359	0.20659	1	0.077855	0.049949	0.061051

To reach the best estimator through preference between different estimated methods, this study has generally depended on the following statistical measures for comparison: Mean Square Error (MSE) and Integral Mean Square Error (IMSE).

From tables (6), (7), and (8), we noticed the following MSE and IMSE:

Table 6
MSE and IMSE of Different Reliability Functions of the Estimation Methods when Contamination Ratio (10%) and for all Selected Sample Sizes and all Model

n	Model 1			Model 2			Model 3		
	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}
50	0.1	2.250309E-03	4.858889E-04	0.1	6.998072E-04	3.861636E-05	0.1	1.201458E-03	5.320549E-05
	0.18	5.392157E-03	5.386792E-04	0.15	1.12377E-03	6.673375E-05	0.2	1.91287E-03	8.180721E-05
	0.26	8.469187E-03	5.29987E-04	0.2	1.492125E-03	9.250076E-05	0.3	2.175066E-03	9.251582E-05
	0.34	1.107072E-02	4.963619E-04	0.25	1.80433E-03	1.137146E-04	0.4	2.297216E-03	9.521197E-05
	0.42	0.0130877	4.549982E-04	0.3	2.068049E-03	1.295006E-04	0.5	2.384221E-03	9.348809E-05
	0.5	1.454195E-02	4.135913E-04	0.35	2.291329E-03	1.39783E-04	0.6	2.465094E-03	8.902382E-05
	0.58	1.550602E-02	3.752789E-04	0.4	2.48095E-03	1.449756E-04	0.7	2.545381E-03	8.29215E-05
	0.66	1.606589E-02	3.410841E-04	0.45	2.642374E-03	1.457682E-04	0.8	2.624172E-03	7.600965E-05
	0.74	1.630435E-02	3.110906E-04	0.5	0.00278	1.429705E-04	0.9	2.699375E-03	6.89056E-05
	0.82	1.629439E-02	2.849986E-04	0.55	2.897422E-03	1.374086E-04	1	2.769244E-03	6.203743E-05
IMSE		1.60537E-05	5.70997E-07		0.000292	1.65985E-05		0.001216	4.19068E-05
150	0.1	2.981022E-04	4.523206E-05	0.1	7.72153E-04	3.841052E-05	0.1	1.308426E-03	3.744739E-05
	0.18	7.07444E-04	7.05091E-05	0.15	1.29281E-03	9.170143E-05	0.2	2.204282E-03	8.089691E-06
	0.26	1.128089E-03	8.662079E-05	0.2	1.759265E-03	1.554974E-04	0.3	2.62958E-03	3.061269E-06
	0.34	1.520791E-03	9.676995E-05	0.25	2.154567E-03	2.233289E-04	0.4	2.884459E-03	4.224473E-06
	0.42	1.872821E-03	1.028052E-04	0.3	2.481518E-03	2.916865E-04	0.5	3.079908E-03	6.390037E-06
	0.5	2.181579E-03	1.059021E-04	0.35	2.749274E-03	3.587222E-04	0.6	3.251333E-03	8.293859E-06
	0.58	2.448706E-03	1.068624E-04	0.4	2.968092E-03	4.235213E-04	0.7	3.408774E-03	9.718928E-06
	0.66	2.677587E-03	1.062525E-04	0.45	3.147376E-03	4.857058E-04	0.8	3.554185E-03	1.071043E-05
	0.74	2.872291E-03	1.04491E-04	0.5	3.295068E-03	5.452033E-04	0.9	3.687301E-03	1.136143E-05
	0.82	3.036965E-03	1.018884E-04	0.55	3.417624E-03	6.021151E-04	1	3.807614E-03	1.17575E-05
IMSE		1.82549E-06	9.03116E-08		0.000296	3.96564E-05		0.001652	6.15194E-06
250	0.1	1.715984E-04	1.825705E-04	0.1	5.748741E-04	1.394744E-06	0.1	1.334821E-03	4.496356E-05
	0.18	4.312875E-04	1.959391E-04	0.15	9.895966E-04	3.923531E-06	0.2	2.346737E-03	7.416502E-05
	0.26	7.252114E-04	1.850087E-04	0.2	1.385006E-03	9.445255E-06	0.3	2.863621E-03	8.848575E-05
	0.34	1.025787E-03	1.699588E-04	0.25	1.743425E-03	1.738917E-05	0.4	3.176932E-03	9.637596E-05
	0.42	1.318658E-03	1.555495E-04	0.3	2.061191E-03	2.640132E-05	0.5	3.409398E-03	9.964743E-05
	0.5	1.59574E-03	1.425652E-04	0.35	2.340016E-03	3.512229E-05	0.6	3.605329E-03	9.910718E-05
	0.58	1.852593E-03	1.308589E-04	0.4	2.583486E-03	4.255025E-05	0.7	3.779877E-03	9.567277E-05
	0.66	2.087143E-03	1.201729E-04	0.45	2.795692E-03	4.81202E-05	0.8	3.937797E-03	9.028887E-05
	0.74	2.298905E-03	1.103189E-04	0.5	2.980559E-03	5.16412E-05	0.9	4.080372E-03	8.37755E-05
	0.82	2.488454E-03	1.011892E-04	0.55	3.141693E-03	5.319325E-05	1	4.207989E-03	7.676574E-05
IMSE		5.1886E-07	5.5393E-08		0.000162	2.27813E-06		0.00128	3.31952E-05

Table 7
MSE and IMSE of Different Reliability Functions of the Estimation Methods when Contamination Ratio (20%) and for all Selected Sample Sizes and all Model

n	Model 1			Model 2			Model 3		
	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}
50	0.1	2.918598E-03	1.749769E-03	0.1	2.134556E-03	1.140441E-03	0.1	2.656809E-03	1.235834E-04
	0.18	7.220213E-03	1.851344E-03	0.15	2.776232E-03	1.595681E-03	0.2	4.971905E-03	2.550952E-04
	0.26	1.169052E-02	1.703395E-03	0.2	3.483747E-03	1.824924E-03	0.3	6.332754E-03	3.519896E-04
	0.34	1.573347E-02	1.495535E-03	0.25	4.506754E-03	1.891618E-03	0.4	7.253055E-03	4.169551E-04
	0.42	1.912971E-02	1.290872E-03	0.3	5.419181E-03	1.85745E-03	0.5	7.976979E-03	4.549906E-04
	0.5	2.184142E-02	1.109735E-03	0.35	6.294027E-03	1.766471E-03	0.6	8.598001E-03	4.719176E-04
	0.58	2.391416E-02	9.5632E-04	0.4	7.126278E-03	1.646787E-03	0.7	9.149814E-03	4.734008E-04
	0.66	0.0254271	8.288749E-04	0.45	0.0078689	1.515341E-03	0.8	9.644251E-03	4.643271E-04
	0.74	2.646811E-02	7.238112E-04	0.5	8.519783E-03	1.381988E-03	0.9	1.008586E-02	4.485762E-04
	0.82	2.712174E-02	6.37334E-04	0.55	9.105021E-03	1.252295E-03	1	1.047733E-02	4.290373E-04
IMSE		7.31765E-05	4.97897E-06		0.000964	0.00026744		0.001258	6.34364E-05
150	0.1	5.294003E-04	3.103319E-05	0.1	1.444049E-03	2.231462E-04	0.1	3.69571E-03	5.181576E-06
	0.18	1.423348E-03	6.644172E-05	0.15	2.68786E-03	2.137516E-04	0.2	6.970534E-03	8.585028E-06
	0.26	2.509571E-03	9.714346E-05	0.2	4.01313E-03	1.971583E-04	0.3	8.945039E-03	1.02105E-05
	0.34	3.669295E-03	1.216307E-04	0.25	5.326665E-03	1.780797E-04	0.4	1.031233E-02	1.07486E-05
	0.42	4.827618E-03	1.405697E-04	0.3	6.575722E-03	1.592238E-04	0.5	1.140306E-02	1.050374E-05
	0.5	5.93812E-03	1.549608E-04	0.35	7.732392E-03	1.419458E-04	0.6	1.234438E-02	9.739941E-06
	0.58	6.973763E-03	1.657035E-04	0.4	8.784357E-03	1.267726E-04	0.7	1.318259E-02	8.692116E-06
	0.66	7.920817E-03	1.735268E-04	0.45	9.729044E-03	1.137743E-04	0.8	1.393434E-02	7.539642E-06
	0.74	8.774449E-03	1.790082E-04	0.5	1.056975E-02	1.027984E-04	0.9	1.460634E-02	6.401823E-06
	0.82	9.535606E-03	1.826044E-04	0.55	1.131314E-02	9.35994E-05	1	1.520274E-02	5.347851E-06
IMSE		2.01312E-06	5.07173E-08		0.000785	1.78456E-05		0.004403	3.30259E-06
250	0.1	1.77805E-04	5.617738E-04	0.1	1.64929E-03	1.283722E-03	0.1	3.85894E-03	1.515624E-04
	0.18	3.821272E-04	7.55458E-04	0.15	3.055009E-03	1.139096E-03	0.2	7.287358E-03	2.772871E-04
	0.26	5.510316E-04	7.386644E-04	0.2	4.530662E-03	9.179534E-04	0.3	9.366387E-03	3.448568E-04
	0.34	6.736662E-04	6.477544E-04	0.25	5.970214E-03	7.445766E-04	0.4	1.081478E-02	3.789823E-04
	0.42	7.561485E-04	5.456197E-04	0.3	7.318316E-03	6.338605E-04	0.5	1.197488E-02	3.910767E-04
	0.5	8.077051E-04	4.553825E-04	0.35	8.549176E-03	5.707293E-04	0.6	1.297817E-02	3.874856E-04
	0.58	8.366562E-04	3.828289E-04	0.4	9.654392E-03	5.370765E-04	0.7	1.387245E-02	3.729959E-04
	0.66	8.495392E-04	3.270017E-04	0.45	0.0106356	5.189179E-04	0.8	1.467493E-02	3.515323E-04
	0.74	8.511946E-04	2.84832E-04	0.5	1.149997E-02	5.071045E-04	0.9	1.539255E-02	3.262075E-04
	0.82	8.450873E-04	2.530348E-04	0.55	1.225744E-02	4.963116E-04	1	1.602969E-02	2.993423E-04
IMSE		7.80789E-05	5.74471E-05		0.000926	9.06277E-05		0.008217	0.000231944

Table 8
MSE and IMSE of Different Reliability Functions of the Estimation Methods when Contamination Ratio (30%) and for all Selected Sample Sizes and all Model

n	Model 1			Model 2			Model 3		
	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}	t_i	R_{MLE}	R_{RMLE}
50	0.1	3.516891E-03	2.356396E-03	0.1	2.187147E-03	3.043205E-03	0.1	4.148772E-03	1.943965E-04
	0.18	8.910536E-03	2.737544E-03	0.15	4.178239E-03	3.952539E-03	0.2	8.386572E-03	4.07918E-04
	0.26	1.475501E-02	2.607931E-03	0.2	6.364717E-03	4.264153E-03	0.3	1.133277E-02	6.160593E-04
	0.34	2.028407E-02	2.314541E-03	0.25	8.578406E-03	4.234987E-03	0.4	1.356875E-02	7.829885E-04
	0.42	2.516633E-02	1.991617E-03	0.3	1.071434E-02	4.042524E-03	0.5	1.542291E-02	8.986982E-04
	0.5	0.0292955	1.690635E-03	0.35	1.271136E-02	3.785005E-03	0.6	1.703519E-02	9.668134E-04
	0.58	3.267928E-02	1.428384E-03	0.4	1.453823E-02	3.51043E-03	0.7	1.846305E-02	9.962064E-04
	0.66	3.537948E-02	1.207034E-03	0.45	1.618362E-02	3.239755E-03	0.8	1.973179E-02	9.966237E-04
	0.74	3.747977E-02	1.02304E-03	0.5	1.764882E-02	2.980874E-03	0.9	2.085527E-02	9.767521E-04
	0.82	3.906856E-02	8.711134E-04	0.55	1.894277E-02	2.736044E-03	1	2.184405E-02	9.435779E-04
IMSE		0.000127912	9.4575E-06		0.003183	0.001016826		0.011556	0.000596229
150	0.1	8.714378E-04	1.080823E-03	0.1	3.139311E-03	2.924822E-03	0.1	5.914771E-03	1.775172E-04
	0.18	2.527529E-03	1.603763E-03	0.15	5.958273E-03	2.614565E-03	0.2	1.176647E-02	4.283657E-04
	0.26	4.707276E-03	1.63467E-03	0.2	9.007598E-03	2.010539E-03	0.3	1.573328E-02	6.385748E-04
	0.34	7.169872E-03	1.441204E-03	0.25	1.205243E-02	1.480512E-03	0.4	1.871193E-02	7.841238E-04
	0.42	9.735074E-03	1.188955E-03	0.3	1.495582E-02	1.10881E-03	0.5	2.118076E-02	8.677421E-04
	0.5	1.227667E-02	9.525649E-04	0.35	1.764419E-02	8.793201E-04	0.6	2.333588E-02	9.007953E-04
	0.58	1.471207E-02	7.575866E-04	0.4	2.008418E-02	7.521788E-04	0.7	2.525418E-02	8.961672E-04
	0.66	1.699172E-02	6.071095E-04	0.45	2.226786E-02	6.901214E-04	0.8	2.696721E-02	8.654895E-04
	0.74	1.908986E-02	4.954563E-04	0.5	2.420256E-02	6.65134E-04	0.9	0.0284911	8.182222E-04
	0.82	2.099723E-02	4.145771E-04	0.55	0.0259042	6.583156E-04	1	2.983795E-02	7.61563E-04
IMSE		1.17879E-05	1.09977E-06		0.002273	0.000201887		0.016392	0.00056478
250	0.1	8.826911E-04	7.63044E-04	0.1	3.067521E-03	2.610367E-03	0.1	6.653884E-03	2.858496E-04
	0.18	2.559202E-03	1.203124E-03	0.15	5.878946E-03	2.424384E-03	0.2	1.305231E-02	5.484273E-04
	0.26	4.76051E-03	1.27503E-03	0.2	8.960622E-03	1.887974E-03	0.3	1.727716E-02	7.12577E-04
	0.34	7.241561E-03	1.150824E-03	0.25	1.207347E-02	1.380438E-03	0.4	2.040429E-02	8.067251E-04
	0.42	9.820903E-03	9.618609E-04	0.3	0.0150726	1.013018E-03	0.5	2.298607E-02	8.469869E-04
	0.5	0.0123725	7.759497E-04	0.35	1.787577E-02	7.84356E-04	0.6	2.524302E-02	8.463765E-04
	0.58	1.481442E-02	6.199823E-04	0.4	2.044214E-02	6.59781E-04	0.7	2.725854E-02	8.169189E-04
	0.66	1.709781E-02	4.997757E-04	0.45	2.275763E-02	6.027145E-04	0.8	2.906481E-02	7.690445E-04
	0.74	1.919754E-02	4.119507E-04	0.5	2.482486E-02	5.84319E-04	0.9	3.067699E-02	7.110522E-04
	0.82	2.110492E-02	3.500826E-04	0.55	2.665642E-02	5.846734E-04	1	0.0321061	6.490514E-04
IMSE		1.00457E-06	7.32646E-08		0.001472	0.000117018		0.011565	0.000359874

7. CONCLUSIONS

During conducting the simulation experiments and according to the analyses of the results from the practical part, the following conclusions have been drawn:

1. It was shown that the real value of the reliability function and estimated reliability function decrease with the increase of time ti; and it is always between (0-1), which coincides with the theoretical aspect of characteristics of the reliability function.
2. The values of the two statistical measures (MSE) in estimating the reliability function were decreased by increasing the sample sizes and all estimation methods, which aligns with statistical theory.
3. The results of (IMSE) in estimating the reliability function in the robust Maximum Likelihood estimator (RMLE) methods were better than the maximum likelihood estimator (MLE) method in the presence or absence of contamination.

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