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POISSON-PRANAV DISTRIBUTION AND ITS APPLICATIONS

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ABSTRACT

This research paper deals with formulation of Poisson-Pranav probability distribution by combining Poisson distribution and Pranav distribution for count data.Some important structural and statistical properties of this model are derived and discussed like coefficient of variation, skewness, kurtosis, reliability analysis and order statistics are beening obtained. Also for obtaining estimate of unknown parameter of this distribution maximum likelihood estimation method is used. At last we fitted our proposed model to real life data set to check its suitability and applicability compared to other related models.

KEYWORDS

Poisson distribution, Pranav distribution, compound mechanism, Poisson-Pranav distribution, Count Data, Maximum likelihood estimation.

1. INTRODUCTION

In our day to day life we many times deal with count data and decision making for dealing with count data becomes important. For improving decision making while dealing with count data we fit a valid probability model to count data. Probability model fitted to count data can be obtained by various techniques. One such technique is Compounding. In compounding mechanism we can combine any two distributions irrespective of their nature (discrete or continuous) to form a new model as we have done in this paper by combining Poisson distribution and Pranav distribution and obtaining Poisson-Pranav distribution. Compounding technique is mostly used while dealing with over dispersed data and in situations where parameter of one distribution is a random variable following some other distribution as the case is for count data sometimes. Flexibility in applying probability models to count data is an important feature which is ensured by using compounding technique. There are many practical situations where need for compounding technique arises. The resulting distribution obtained by combining two distributions will be discrete or continuous depending on the nature of the parent distribution. It is well known that Greenwood and Yule (1920) first developed a new compounded model as combination of Poisson model and negative binomial model by considering parameter of Poisson model as gamma variate. Sankaran (1970) formulated the Discrete Poisson-Lindley Distribution and studied its vital properties. Gerstenkorn (1993 & 1996) obtained some compounded models by compounding gamma distribution & exponential distribution and also combined

Polya with beta distribution. Mahmoudi et al. (2010) generalized the Poisson-Lindley distribution of Sankaran (1970) and proved that his generalized distribution is more flexible for analyzing count data. Gupta and Ong (2004) formulated a new generalized negative binomial distribution by considering parameter of Poisson distribution as generalized gamma variate and the resulting distribution was fitted to various data sets and proved as better alternative to negative binomial distribution. El-Monseff and Sohsah (2014) obtained Poisson Weighted Lindley Distribution and studied its vital properties and applied to some real life situations. Shankar (2016) introduced Aradhana distribution and its applications in real life and also studied some of its important properties. Ahmad et al. (2017) obtained a new discrete compound distribution with Applications in various fields of real life and obtained its various crucial properties. Razika and Halim (2017) introduced Poisson Quasi-Lindley distribution and its applications by combining Poisson and Quasi-Lindley distribution. The research paper which we have formulated deals with formulation of Poisson-Pranav distribution by combining Poisson distribution with Pranav distribution and then we studied its vital properties and fitted our model to count data.

2. DEFINITION OF PROPOSED MODEL (POISSON-PRANAV DISTRIBUTION)

If Z is a poison variate i.e., $Z|\lambda \sim \mathrm{P}(\lambda), \lambda$ being itself a Pranav variate with parameter θ, then the resulting distribution determined by marginalizing over λ is Poison-Pranav distribution obtained by compounding Poisson distribution and Pranav distribution, which is denoted by $PPD(Z;\theta)$. Our proposed model i.e., Poison-Pranav model is discrete model as parent distribution (Poisson) is a discrete distribution. A random variable Z follows PPD (θ) with probability mass function given in the theorem below.

Theorem 2.1

The probability mass function of a Poisson-Pranav Distribution i.e., $PPD(Z;\theta)$ is given by

$$P(Z=z)=\frac{\theta^4}{(\theta^4+6)}\left[\frac{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}{(1+\theta)^{z+4}}\right]; z=0,1,2,3,...,;\theta>0$$

Proof:

The p.m.f of a Poisson-Pranav Distribution i.e., $PPD(Z;\theta)$ can be obtained as

If $Z \sim \mathrm{P}(\lambda)$ then p.m.f of Poisson distribution is given as

$$g(z\,|\,\lambda)=\frac{e^{-\lambda}\lambda^z}{(z)!}\ ;\ z=0,1,2,3,...,;\lambda>0$$

And its parameter λ follows Pranav distribution (PD) with p.d.f

$$h(\lambda;\theta)=\frac{\theta^4}{\theta^4+6}(\theta+\lambda^3)e^{-\theta\lambda}\,;\lambda>0,\theta>0$$

We have

$$P(Z=z)=\int_0^\infty g(z\,|\,\lambda).h(\lambda;\theta)d\lambda$$

$$P(Z=z)=\int_0^\infty \frac{e^{-\lambda}\lambda^z}{(z)!}\left(\frac{\theta^4}{\theta^4+6}\right)(\theta+\lambda^3)e^{-\theta\lambda}d\lambda$$

$$P(Z=z)=\frac{\theta^4}{z!(\theta^4+6)}\left(\int_0^\infty \theta e^{-(1+\theta)\lambda}\lambda^z\,d\lambda+\int_0^\infty e^{-(1+\theta)\lambda}\lambda^{z+3}d\lambda\right)$$

$$P(Z=z)=\frac{\theta^4}{(\theta^4+6)}\left[\frac{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}{(1+\theta)^{z+4}}\right];z=0,1,2,3,...,;\theta>0... \quad (2.1)$$

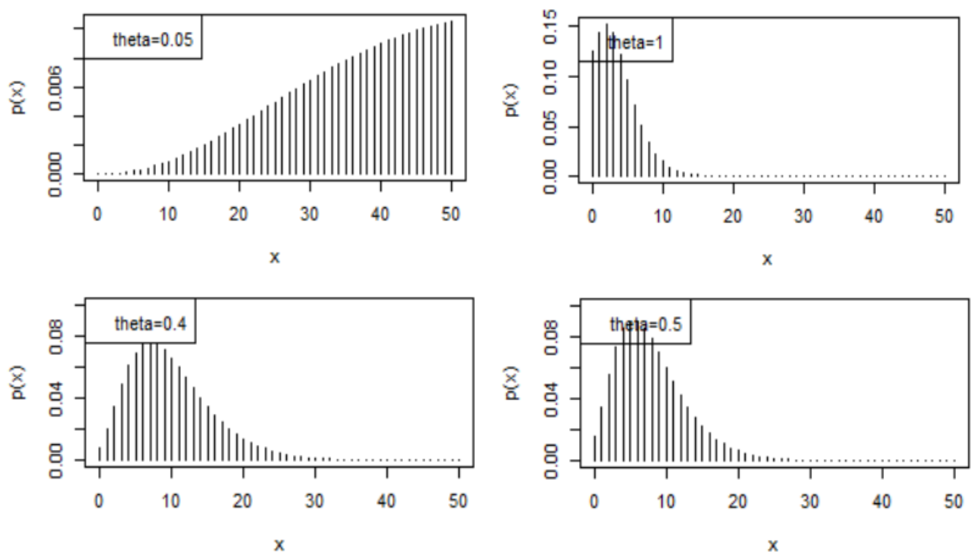
which is the p.m.f. of PPD

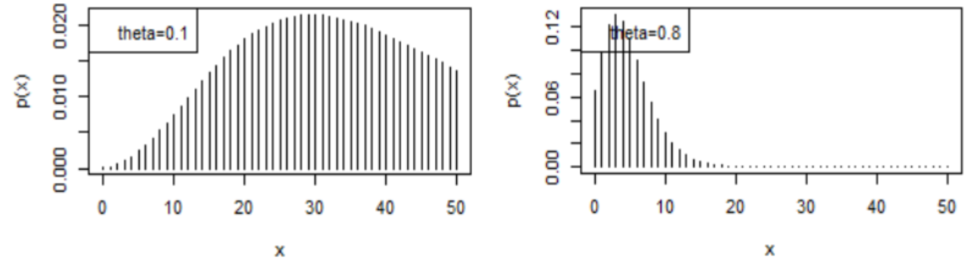
The corresponding c.d.f of Poisson-Pranav distribution is obtained as:

$$F_Z(z)=\sum_{n=0}^{z}\frac{\theta^4}{(\theta^4+6)}\left[\frac{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}{(1+\theta)^{z+4}}\right]\ldots \quad (2.2)$$

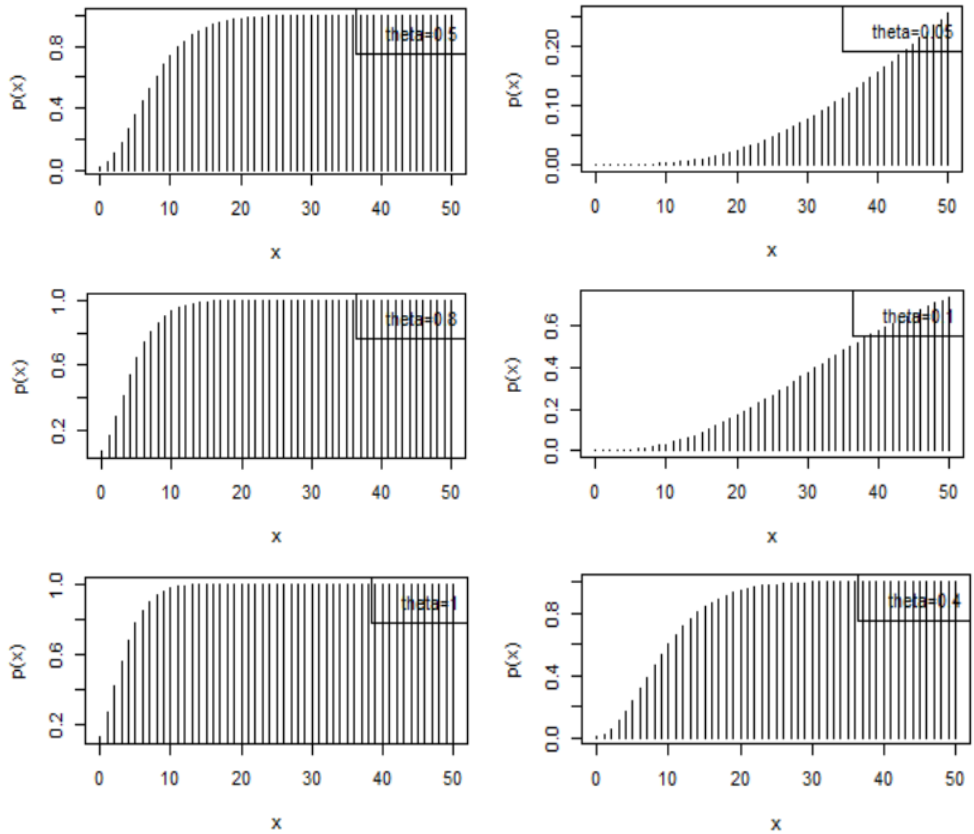
$$1-\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7+3\theta^6+3\theta^5\\+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\quad z>0,\theta>0$$

Graphs of p.m.f and c.d.f of Poisson-Pranav distribution are given below as:





The above six graphs represent p.m.f of Poisson-Pranav distribution for different values of θ (0.05, 1, 0.4, 0.5, 0.1, and 0.8)



Above six graphs represent c.d.f of Poisson-Pranav distribution for different values of θ (0.5, 0.05, 0.8, 0.1, 1, and 0.4).

3. STATISTICAL PROPERTIES OF POISSON-PRANAV DISTRIBUTION

In this part some vital structural properties of the Poisson-Pranav model are obtained. These include moment, generating functions (m.g.f and p.g.f).

3.1 Moments of Poisson-Pranav Distribution

3.1.1 Factorial Moments

Using (2.1), the rth factorial moment about origin of the PPD (2.1)can be obtained as

$$\mu'_{(r)} = E[E(Z^{(r)} \mid \lambda)], \text{ where } Z^{(r)} = Z(Z-1)(Z-2)...(Z-r+1)$$

$$\mu'_{(r)} = \int_0^\infty \left[\sum_{z=0}^\infty z^{(r)} \frac{e^{-\lambda}\lambda^z}{(z)!} \right] . \frac{\theta^4}{\theta^4+6}(\theta+\lambda^3)e^{-\theta\lambda} d\lambda$$

$$\mu'_{(r)} = \frac{\theta^4}{\theta^4+6}\int_0^\infty \left[\lambda^r \left(\sum_{z=r}^\infty \frac{e^{-\lambda}\lambda^{z-r}}{(z-r)!} \right) \right] (\theta+\lambda^3)e^{-\theta\lambda} d\lambda$$

Taking $u = z - r$, we get

$$\mu'_{(r)} = \frac{\theta^4}{\theta^4+6}\int_0^\infty \left[\lambda^r \left(\sum_{u=0}^\infty \frac{e^{-\lambda}\lambda^{u}}{u!} \right) \right] (\theta+\lambda^3)e^{-\theta\lambda} d\lambda$$

$$\mu'_{(r)} = \frac{r!}{\theta^4+6}\left[\frac{\theta^4 + (r+3)(r+2)(r+1)}{\theta^r} \right] \tag{3.1.1.1}$$

Taking r = 1,2,3,4 in (3.1.1.1), the first 4 factorial moments about origin of Poisson-Pranav distribution can be obtained as

$$\mu'_{(1)} = \frac{\theta^4+24}{\theta(\theta^4+6)}$$

$$\mu'_{(2)} = \frac{2(\theta^4+60)}{\theta^2(\theta^4+6)}$$

$$\mu'_{(3)} = \frac{6(\theta^4+120)}{\theta^3(\theta^4+6)}$$

$$\mu'_{(4)} = \frac{24(\theta^4+210)}{\theta^4(\theta^4+6)}.$$

3.1.2 Moments about Origin (Raw Moments)

The first four moments about origin, using the relationship between factorial moments about origin and the moments about origin of PPD (2.1) are generated as

$$\mu_1' = \frac{\theta^4+24}{\theta(\theta^4+6)}$$

which is the mean of Poisson-Pranav Distribution

$$\mu_2' = \frac{2(\theta^4+60)+\theta(\theta^4+24)}{\theta^2(\theta^4+6)}$$

$$\mu_3' = \frac{6(\theta^4+120)+6\theta(\theta^4+60)-2\theta^2(\theta^4+24)}{\theta^3(\theta^4+6)}$$

$$\mu_4' = \frac{24(\theta^4+210)+18\theta(\theta^4+120)-22\theta^2(\theta^4+60)+6\theta^3(\theta^4+24)}{\theta^4(\theta^4+6)}.$$

3.1.3 Moments about the Mean (Central Moments)

Using the relationship $\mu_r = E(Z-\mu_1')^r = \sum_{K=0}^{r}\binom{r}{k}\mu_k'(-\mu_1')^{r-k}$ between moments about the mean and the moments about origin, the moments about the mean of the PPD (2.1) can be obtained as

$$\mu_2 = \frac{\{2(\theta^4+60)+\theta(\theta^4+24)\}(\theta^4+6)-(\theta^4+24)^2}{\theta^2(\theta^4+6)^2}$$

$$\mu_3 = \frac{[\theta^4+6]^2\{6(\theta^4+120)+6\theta(\theta^4+60)-2\theta^2(\theta^4+24)\} - 3[\theta^4+6]\{(2(\theta^4+60)+\theta(\theta^4+24))(\theta^4+24)\}+2(\theta^4+24)^3}{\theta^3(\theta^4+6)^3}$$

$$\mu_4 = \frac{\begin{array}{l}(\theta^4+6)^3\{24(\theta^4+210)+18\theta(\theta^4+120)-22\theta^2(\theta^4+60)+6\theta^3(\theta^4+24)\} \\ -4(\theta^4+6)^2\{(6(\theta^4+120)+6\theta(\theta^4+60)-2\theta^2(\theta^4+24))(\theta^4+24)\} \\ +6(\theta^4+6)\{(2(\theta^4+60)+\theta(\theta^4+24))(\theta^4+24)^2\}-3(\theta^4+24)^4\end{array}}{\theta^4(\theta^4+6)^4}.$$

3.2 Coefficient of Variation, Skewness, Kurtosis and Index of Dispersion

The coefficient of variation (C.V), coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis and index of dispersion (γ) of the PPD are thus determined as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\left[\{2(\theta^4+60)+\theta(\theta^4+24)\}(\theta^4+6)-(\theta^4+24)^2\right]^{\frac{1}{2}}}{\theta^4+24}$$

$$\sqrt{\beta_1}=\frac{\mu_3}{\mu_2^{3/2}}=\left\{\frac{\begin{array}{l}[\theta^4+6]^2\{6(\theta^4+120)+6\theta(\theta^4+60)-2\theta^2(\theta^4+24)\}\\ -3[\theta^4+6]\{(2(\theta^4+60)+\theta(\theta^4+24))(\theta^4+24)\}+2(\theta^4+24)^3\end{array}}{(\{2(\theta^4+60)+\theta(\theta^4+24)\}(\theta^4+6)-(\theta^4+24)^2)^{3/2}}\right\}$$

$$\beta_2=\frac{\mu_4}{\mu_2^2}=\frac{\begin{array}{l}(\theta^4+6)^3\{24(\theta^4+210)+18\theta(\theta^4+120)-22\theta^2(\theta^4+60)\\ +6\theta^3(\theta^4+24)\}-4(\theta^4+6)^2\{(6(\theta^4+120)+6\theta(\theta^4+60)\\ -2\theta^2(\theta^4+24))(\theta^4+24)\}+6(\theta^4+6)\{(2(\theta^4+60)\\ +\theta(\theta^4+24))(\theta^4+24)^2\}-3(\theta^4+24)^4\end{array}}{\left(\{2(\theta^4+60)+\theta(\theta^4+24)\}(\theta^4+6)-(\theta^4+24)^2\right)^2}$$

$$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\{2(\theta^4+60)+\theta(\theta^4+24)\}(\theta^4+6)-(\theta^4+24)^2}{\theta(\theta^4+24)(\theta^4+6)}.$$

3.3 Generating functions (m.g.f, c.g.f, p.g.f) of Poisson-Pranav Distribution

We will obtain generating functions (m.g.f, c.g.f, p.g.f) of PPD in this section.

Theorem 3.3.1:

If Z ~PPD (θ), then the Probability generating function $P_Z(t)$ is obtained as

$$P_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6]}{(\theta+1-t)}+\frac{6t^3}{(\theta+1-t)^4}+\frac{9(t(\theta+1-t)+2t^2)}{(\theta+1-t)^3}+\\ \frac{9t}{(\theta+1-t)^2}\end{array}\right].$$

Proof:

We begin with the well-known definition of the probability generating function given by

$$P_Z(t)=\sum_{z=0}^{\infty}t^z\left(\frac{\theta^4}{(\theta^4+6)}\left(\frac{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}{(1+\theta)^{z+4}}\right)\right)$$

$$P_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^4}\left[\begin{array}{l}\theta(1+\theta)^3\sum_{z=0}^{\infty}\left(\frac{t}{1+\theta}\right)^z+\sum_{z=0}^{\infty}z^3\left(\frac{t}{1+\theta}\right)^z\\ +6\sum_{z=0}^{\infty}\left(\frac{t}{1+\theta}\right)^z z^2+11\sum_{z=0}^{\infty}\left(\frac{t}{1+\theta}\right)^z z+6\sum_{z=0}^{\infty}\left(\frac{t}{1+\theta}\right)^z\end{array}\right]$$

$$P_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^4}\left[\begin{array}{l}\frac{\theta(1+\theta)^4}{(\theta+1-t)}+\frac{6t^3(1+\theta)}{(\theta+1-t)^4}+\frac{9(t(\theta+1-t)+2t^2)(1+\theta)}{(\theta+1-t)^3}\\ +\frac{9t(1+\theta)}{(\theta+1-t)^2}+\frac{6(1+\theta)}{(\theta+1-t)}\end{array}\right]$$

$$P_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6}{(\theta+1-t)}+\frac{6t^3}{(\theta+1-t)^4}\\ +\frac{9(t(1+\theta-t)+2t^2)}{(\theta+1-t)^3}+\frac{9t}{(\theta+1-t)^2}\end{array}\right].$$

Theorem 3.3.2:

If Z ~PPD (θ), then the m.g.f $M_Z(t)$ is obtained as

$$M_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6}{(\theta+1-e^t)}+\frac{6e^{3t}}{(\theta+1-e^t)^4}+\frac{9(e^t(1+\theta-e^t)+2e^{2t})}{(\theta+1-e^t)^3}\\ +\frac{9e^t}{(\theta+1-e^t)^2}\end{array}\right]$$

And characteristic function has the form

$$\varphi_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6}{(\theta+1-e^{it})}+\frac{6e^{3it}}{(\theta+1-e^{it})^4}+\frac{9(e^{it}(1+\theta-e^{it})+2e^{2it})}{(\theta+1-e^{it})^3}\\ +\frac{9e^{it}}{(\theta+1-e^{it})^2}\end{array}\right]$$

Proof:

We know that relation between m.g.f $M_Z(t)$ and p.g.f $P_Z(s)$ is that $s=e^t$

So

$$M_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6}{(\theta+1-e^t)}+\frac{6e^{3t}}{(\theta+1-e^t)^4}+\frac{9(e^t(1+\theta-e^t)+2e^{2t})}{(\theta+1-e^t)^3}\\ +\frac{9e^t}{(\theta+1-e^t)^2}\end{array}\right]$$

Also $M_Z(t)=\varphi_Z(it)$ so

$$\varphi_Z(t)=\frac{\theta^4}{(\theta^4+6)(1+\theta)^3}\left[\begin{array}{l}\frac{\theta(1+\theta)^3+6}{(\theta+1-e^{it})}+\frac{6e^{3it}}{(\theta+1-e^{it})^4}+\frac{9(e^{it}(1+\theta-e^{it})+2e^{2it})}{(\theta+1-e^{it})^3}\\+\frac{9e^{it}}{(\theta+1-e^{it})^2}\end{array}\right].$$

4. RELIABILITY ANALYSIS OF POISSON-PRANAV DISTRIBUTION

In this part, we obtained important measures reliability, hazard rate, reverse hazard rate and Mills ratio of the proposed Poisson-Pranav model

4.1 Reliability Function R(x)

The reliability of most of the systems is a decreasing function of time. So the probability that a system which is surviving till time "t" will fail beyond that time is called reliability of that system. It can be computed from cumulative distribution function of the model by using a well-known relation between reliability ($R(z)$) and cumulative distribution function (F(z)) as

$$R(z)\ =1-F(z)$$

The reliability function of Poisson-Pranav distribution is determined as:

$$R(z,\theta)=\left(\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7+3\theta^6+3\theta^5+\theta^4+24\theta^3\\+36\theta^2+6z\theta+24\theta+6)\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\right).$$

4.2 Hazard Function

The hazard function is defined as measure of the tendency of the system to fail within a very narrow time frame. It is obtained as:

$$\text{H.R=h}\,(z,\theta)=\frac{f(z,\theta)}{R(z,\theta)}=\left\{\frac{\theta^4[\theta(1+\theta)^3+(z+3)(z+2)(z+1)]}{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7+3\theta^6\\+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}\right\}.$$

4.3 Reverse Hazard Rate and Mills Ratio

The reverse hazard rate and the mills ratio of Poisson-Pranav distribution are respectively given as:

$$\text{R.H.R}= h_r(z,\theta)=\left\{\frac{\theta^4[\theta(1+\theta)^3+(z+3)(z+2)(z+1)]}{\begin{array}{l}(1+\theta)^{z+4}(\theta^4+6)-(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z\\ +21\theta^2z+\theta^7+3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}\right\}$$

$$\text{Mills ratio}=\left\{\frac{\begin{array}{l}(1+\theta)^{z+4}(\theta^4+6)-(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z\\ +\theta^7+3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}{\theta^4[\theta(1+\theta)^3+(z+3)(z+2)(z+1)]}\right\}.$$

5. ORDER STATISTICS OF POISSON-PRANAV DISTRIBUTION

Suppose $Z_1, Z_2, Z_3, \ldots. Z_n$ be a random sample obtained from the Poisson-Pranav distribution with cumulative distribution function $F_Z(z)$ and probability mass function $P_Z(z)$ with $Z_{(1)}, Z_{(2)}, Z_{(3)}\ldots, Z_{(n)}$ being ordered statistic of the drawn random sample. Then the probability mass function of k^{th} order statistics $Z_{(k)}$ is given by:

$$f_{z(k)}(z,c,\theta)=\frac{n!}{(k-1)!(n-k)!}P(z)\left[F(z)\right]^{k-1}\left[1-F(z)\right]^{n-k}.\ \text{k}=1,2,3,\ldots,n \quad (5.1)$$

Using the equations (2.1) and (2.2), the probability density function of k^{th} order statistics of Poisson-Pranav distribution is given by:

$$f_{(r)}(z,,\theta)=\frac{n!}{(r-1)!(n-r)!}\frac{\theta^4(\theta(1+\theta)^3+(z+1)(z+2)(z+1))}{(\theta^4+6)(1+\theta)^{z+4}}$$

$$\left[1-\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7\\ +3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\right]^{k-1}$$

$$\left[\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7\\ +3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\right]^{n-k} \quad \ldots. \quad (5.2)$$

Put value of k=1 in (5.2) the p.m.f of first order $Z_{(1)}$ Poisson-Pranavis given as:

$$f_1(z,\theta) = n\frac{\theta^4(\theta(1+\theta)^3+(z+1)(z+2)(z+3))}{(\theta^4+6)(1+\theta)^{z+4}}$$

$$\left[\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7\\ \quad+3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6)\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\right]^{n-1}.$$

Put k=n in (5.2) the p.m.f of nth order $Z_{(n)}$ Poisson-Pranav model is given as:

$$f_{w(n)}(z,\theta) = n\frac{\theta^4(\theta(1+\theta)^3+(z+1)(z+2)(z+1))}{(\theta^4+6)(1+\theta)^{z+4}}$$

$$\left[1-\frac{\begin{array}{l}(\theta^3z^3+9\theta^3z^2+3\theta^2z^2+26\theta^3z+21\theta^2z+\theta^7\\ \quad+3\theta^6+3\theta^5+\theta^4+24\theta^3+36\theta^2+6z\theta+24\theta+6\end{array}}{(1+\theta)^{z+4}(6+\theta^4)}\right]^{n-1}$$

6. RECURRENCE RELATION FOR PROBABILITIES OF PPD

If $Z \sim \text{PPD}(\theta)$ then p.m.f of Z is

$$P(Z=z) = \frac{\theta^4}{(\theta^4+6)}\left[\frac{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}{(1+\theta)^{z+4}}\right]; z=0,1,2,3,...,;\theta>0$$

The recurrence relation of Poisson-Pranav Distribution is obtained as

$$\frac{P(z+1)}{P(z)} = \left(\frac{\theta(1+\theta)^3+(z+4)(z+3)(z+2)}{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}\right)\left(\frac{(1+\theta)^{z+4}}{(1+\theta)^{z+5}}\right)$$

$$P(z+1) = \left(\frac{\theta(1+\theta)^3+(z+4)(z+3)(z+2)}{\theta(1+\theta)^3+(z+3)(z+2)(z+1)}\right)\left(\frac{1}{(1+\theta)}\right)P(z)$$

This is the required recurrence relation of PPD.

7. ESTIMATION OF PARAMETERS

In this section, we estimate the unknown parameter of the Poisson-Pranav distribution by using method of maximum likelihood estimation.

7.1 Method of Maximum Likelihood Estimation

Method of Maximum Likelihood Estimation is simple and most efficient method of estimation. In this method unknown parameters are obtained by maximizing likelihood function. Suppose $Z_1, Z_2, Z_3, ..., Z_n$ is a random sample of size n taken from Poisson-Pranav Distribution (PPD), then the likelihood function of PPD is given as

$$L(z\,|\,\theta) = \frac{\theta^{4n}}{(\theta^4+6)^n}\prod_{i=1}^{n}\left(\frac{(\theta(1+\theta)^3+(z_i+3)(z_i+2)(z_i+1))}{(1+\theta)^{z_i+4}}\right)$$

The log likelihood function is

$$\log L = \sum_{i=1}^{n}\log(\theta(1+\theta)^3+(z_i+3)(z_i+2)(z_i+1))$$
$$-(\sum_{i=1}^{n} z_i + 4n)\log(1+\theta) - n\log(\theta^4+6) + 4n\log\theta$$

differentiating log likelihood function with respect to θ we get

$$\frac{\delta}{\delta\theta}\log L = \sum_{i=1}^{n}\frac{((1+\theta)^3+3\theta(1+\theta)^2)}{(\theta(1+\theta)^3+(z_i+3)(z_i+2)(z+1))} - \frac{4n\theta^3}{(\theta^4+6)} - \frac{\sum_{i=1}^{n} z_i + 4n}{(1+\theta)} + \frac{4n}{\theta} = 0$$

The maximum likelihood of θ is obtained by solving above equation through R software.

8. APPLICATIONS OF POISSON-PRANAV DISTRIBUTION

We fitted our proposed model on two data sets and showed that it gives better results than some other related models.

Data set (1):

The data representing distribution of mistakes in copying groups of random digits used by and due to Kemp and Kemp (1965) as below.

No. of Errors per Group	**0**	**1**	**2**	**3**	**4**
Observed Frequency	**35**	**11**	**8**	**4**	**2**

Now we analyzed the data set 1 by computing AIC, BIC, chi-square value, p value by fitting our proposed model and compared our model with other related models.

Table 1

Criterion	**Poisson Distribution**	**Poisson Lindley Distribution**	**Poisson Pranav Distribution**
-logL	77.95	73.35	73.20
AIC	157.9	148.70	148.40
BIC	156.6	147.4	147.1

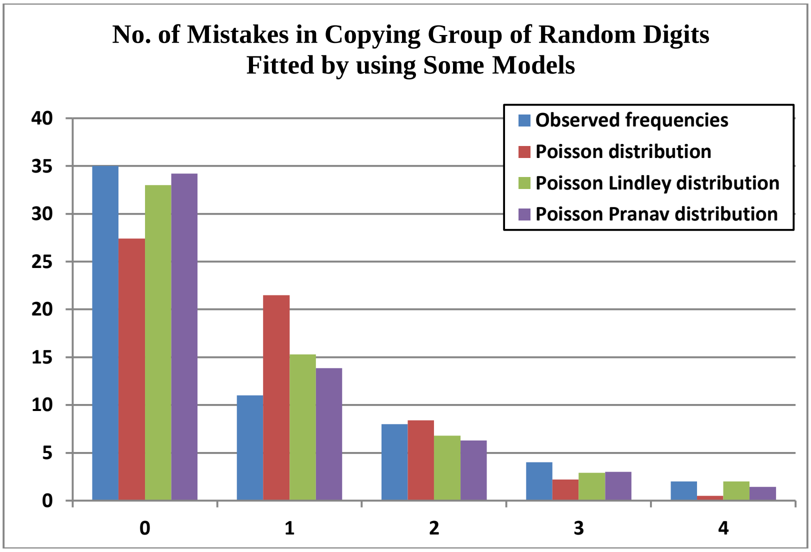
The above table reveals that our newly formulated Poisson-Pranav model possesses least value of AIC and BIC as compared to Poisson distribution and Poisson Lindley distribution for the data set (1). This means that loss of information by fitting Poisson-Pranav model to data set (1) is less as compared to other mentioned related models.

Table 2

Number of Errors(X)	Observed Frequencies	Poisson Distribution	Poisson Lindley Distribution	Poisson-Pranav Distribution
0	35	27.4	33	34.20
1	11	21.5	15.3	13.84
2	8	8.4	6.8	6.28
3	4	2.2	2.9	3.0
4	2	0.5	2.0	1.45
Total	60			
ML estimates (Standard error)		θ =0.78 (0.11)	θ =1.74 (0.28)	θ =2.11 (0.18)
Chi-squared		7.98	2.20	1.60
d.f		1	1	1
p-value		0.0047	0.13	0.20

The above table reveals that p-value is maximum and chi-square value is minimum for our formulated Poisson Pranav model as compared to Poisson model and Poisson Lindley model which shows that our proposed model fits well to data set (1) as compared to other mentioned related models.

Below histogram represents data set 1 for different models and for our proposed model.



Data set (2):

The data set second regarding distribution of Pyrausta Nublilalis is due to Beall (1940) and is given as below:

No. of Insects	**0**	**1**	**2**	**3**	**4**	**5**
Observed Frequency	**33**	**12**	**6**	**3**	**1**	**1**

Now we analyzed the data set 2 by computing AIC, BIC, chi-square value, p value by fitting our proposed model and compare this model with other related models.

Table 3

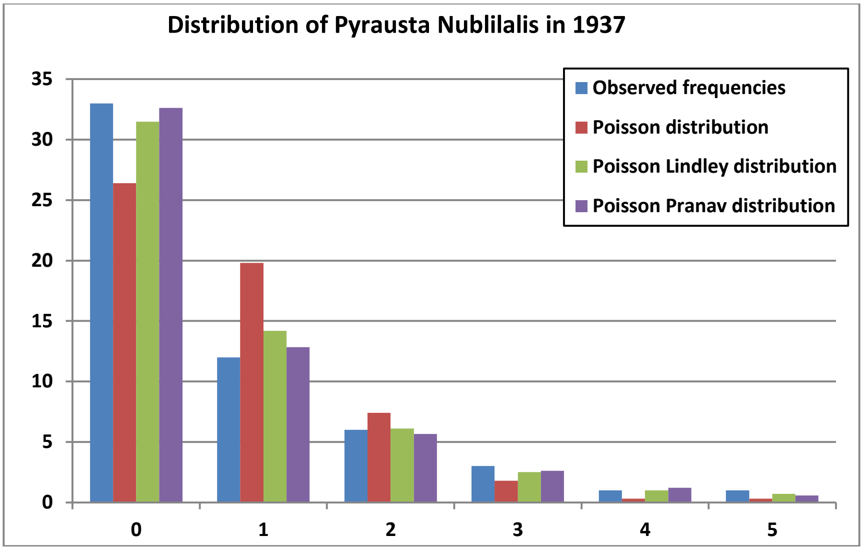
Criterion	**Poisson Distribution**	**Poisson Lindley Distribution**	**Poisson Pranav Distribution**
-logl	71.58	67	66.8
AIC	145.16	136	135.6
BIC	143.93	134.77	134.37

The above table reveals that our newly formulated Poisson -Pranav model possesses least value of AIC and BIC as compared to Poisson distribution and Poisson Lindley distribution for the data set (2). This means that loss of information by fitting Poisson-Pranav model to data set (2) is less as compared to other mentioned related models.

Table 4

Number of insects (X)	**Observed Frequencies**	**Poisson Distribution**	**Poisson Lindley Distribution**	**Poisson Pranav Distribution**
0	33	26.4	31.5	32.64
1	12	19.8	14.2	12.83
2	6	7.4	6.1	5.65
3	3	1.8	2.5	2.62
4	1	0.3	1.0	1.22
5	1	0.3	0.7	0.56
Total	56			
ML estimates (Standard error)		θ =0.75 (0.11)	θ =1.81 (0.30)	θ =2.16 (0.20)
Chi-squared		4.87	0.53	0.14
d.f		1	1	1
p-value		0.0273	0.4666	0.70

The above table reveals that p-value is maximum and chi-square value is minimum for our formulated Poisson Pranav model as compared to Poisson model and Poisson Lindley model which shows that our proposed model fits well to data set (2) as compared to other mentioned related models.



Above histogram represents data set 2 for different models and for our proposed model.

9. CONCLUSION

We formulated a new probability model known as Poisson-Pranav distribution for count data by mechanism of compounding. Then we obtained its vital statistical properties and fitted our model to count data sets and compared with related models and showed our model gives better fit to count data as compared to Poisson distribution and Poisson Lindley distribution.

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