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GENERALIZED CLASS OF DIFFERENCE-TYPE RATIO ESTIMATORS FOR ESTIMATING THE POPULATION MEAN USING KNOWN POPULATION PARAMETER OF AUXILIARY VARIABLE

Akingbade, Toluwalase J.[1§] and **Okafor, Fabian C.**[2]
Department of Statistics, University of Nigeria, Nsukka, Nigeria
Email: [1] afuapetoluse@yahoo.com
[2] fabian.okafor@gmail.com
[§] Corresponding author

ABSTRACT

In this paper we have suggested a generalized class of difference-type ratio estimator for estimating the population mean of the study variable in which any suitable choice(s) of population parameters of an auxiliary variable can be applied. The bias and the mean square error of the proposed estimators are derived. Some new and existing estimators that are particular member of this class of estimator were identified. We made correction on the expressions of the bias and the mean square error of ratio-type estimators presented by Raja et al. (2017). The theoretical conditions for which the proposed estimators perform better than the classical ratio and the existing ratio-type estimators were derived. From the empirical study, it is observed that some members of the proposed estimators perform better than the classical ratio and the existing ratio-type estimators considered in the study. Also, at varying values of correlation coefficient, members of the proposed estimator that attain equal efficiency with the classical linear regression without depending on the optimality condition were identified.

KEYWORDS

Bias, Mean Square error, Ratio estimator, Auxiliary variables, Coefficient of variation.

1. INTRODUCTION

In sample surveys, researchers often make use of auxiliary variable either at selection stage or estimation stage or both stages to obtain more precise estimators of finite population mean of the study variable. Ratio, product and regression sampling estimators are the most common and widely discussed in sampling theory literature. Cochran (1940, 1942) proposed the classical ratio and regression estimator and Robson (1952) proposed the classical product estimator using simple random sampling with assumed known population mean. When the correlation between the study variable (Y) and the auxiliary variable (X) is highly positive and the regression line of Y on X is linear and passes through the origin, ratio estimator is appropriate and when the correlation is highly negative, product method of estimation is quite effective in this case. It is well known that ratio estimator is as efficient as the regression estimator when the regression line passes through the origin. In most practical situations, the regression line does not pass through the origin and this makes the ratio estimator having limitation of not performing equally well as regression estimator. To address this limitation, many survey researchers have carried out modification on the existing ratio estimators to provide better estimates. Authors including; Srivastava (1967), Reddy (1973), Tripathi, Das and Khare (1994) and Rao (1991) have modified the classical ratio estimator with the assumed known population mean of the auxiliary variable so as to get more precise estimators.

Over decades, the classical ratio estimator has continuously been improved upon in order to make it as efficient as the classical linear regression estimator. Authors like Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Kakran (1994), Yan and Tian (2010), Singh and Tailor (2003) as well as Subramani and Kumarapandiyan (2013) modified the classical ratio estimator to obtain more precise estimators by utilizing known population parameter(s) such as coefficient of variation, coefficient of kurtosis, deciles etc., of an auxiliary variable. The estimators by these authors can be categorized as class 1 estimators and can be seen in Table 1.

Kadilar and Cingi (2004, 2006), Yan and Tian (2010) and Raja et al. (2017) also suggested the use of linear regression estimator in conjunction with the existing ratio-type estimators by Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Yan and Tian (2010), Singh and Tailor (2003) and Subramani and Kumarapandiyan (2013) to form regression-type ratio estimators. These authors' estimators can be categorized as class 2 estimators and can be seen in Table 2. It is observed that the authors of these existing ratio estimators in class 1 and class 2 do not compare the efficiency of their estimators with the classical linear regression and they do not study the effect of varying the values of the correlation coefficient on the estimators. In this study, a generalized class of difference-type ratio estimator shall be proposed. The biases and the mean square errors of the proposed estimators will be derived. The theoretical conditions for which the proposed estimators perform better than the existing estimators will also be derived. Empirical study shall be conducted to ascertain the performance of the estimators with varying values of correlation coefficient. The proposed estimators will be compared with the classical linear regression estimator.

2. NOTATIONS AND SOME OF THE EXISTING ESTIMATORS

Consider a finite population $S = (s_1, s_2, \dots s_N)$ of size N. Let (Y, X) be the study and auxiliary variables, respectively, taking values (y_i, x_i) on the ith unit $S_i (i = 1,2, \dots, N)$ of the population. Let $(\bar{Y}, \bar{X})$ be the population means of (y, x), respectively. It is assumed that the population mean $\bar{X}$ of the auxiliary variable is known. For estimating the population mean $\bar{Y}$ of the study variable y, a simple random sample of size n is selected without replacement from the population S

The usual sample mean (t_0) and its variance are respectively given as

$$t_0 = \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{1}$$

and

$$\mathrm{V}(t_0) = \frac{(1-f)}{n} \bar{Y}^2 C_y^2 \tag{2}$$

The classical ratio estimator proposed by Cochran (1940) for estimating the population meanof the study variable is as follows:

$$t_r = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{3}$$

The MSE of the estimator $\hat{T}_r$ up to first order approximation is

$$MSE(\hat{T}_r) = \theta \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho_{xy} C_y C_x) \tag{4}$$

$$B(\hat{T}_r) = \frac{\theta(RS_{x-}^2 \rho S_x S_y)}{\bar{X}}$$

The classical regression estimator proposed by Cochran (1942) is given by

$$t_{lr} = \bar{y} + b(\bar{X} - \bar{x}) \tag{5}$$

The MSE of the estimator is

$$MSE(t_{lr}) = \theta(1 - \rho_{xy}^2) S_y^2 \tag{6}$$

where $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ and $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ are the square of the population coefficient of variation of their respective subscripts. $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ and $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ is the population variance of y and x respectively. $S_{xy} = (N-1)^{-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$ is the population covariance between x and y. $\rho_{xy} = \frac{S_{xy}}{S_x S_y}$ is the population correlation coefficient between x and y. $b = \frac{s_{xy}}{s_x^2}$ (sample regression coefficient of y on x), $\theta = \frac{1-f}{n}$, $f = \frac{n}{N}$ (Sampling fraction), $R = \frac{\bar{Y}}{\bar{X}}$ (Ratio of y to x).

The Classical ratio estimator given in (3) is an improvement on the sample mean in (1) when there is a positive correlation and there exist a linear relationship between X and Y. Further improvements were achieved on the classical ratio estimator by introducing known population parameters to form modified ratio-type estimators. The existing modified ratio-type estimators have been categorized into two classes (class 1 and class 2) and are shown in Table 1 and Table 2 respectively.

3. THE PROPOSED GENERALIZED CLASS OF DIFFERENCE-TYPE RATIO ESTIMATOR

Following the existing ratio-type estimators by Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Kadilar and Cingi (2004, 2006) and Subramani and Kumarapandiyan (2013), we suggest a generalized class of difference-type of ratio estimator (t_p) for estimating the population mean $\bar{Y}$ under simple random sampling without replacement as

$$t_p = \frac{\left(\bar{y} - t(\bar{x}^{\gamma} - \bar{X}^{\gamma})\right)\left(A\bar{X} + G\right)^{\alpha}}{\left(A\bar{x} + G\right)^{\alpha}} \tag{7}$$

Let A and G can either be real values or assumed known population parameters such as Kurtosis ($\beta_2(x)$), skewness $(\beta_1(x))$, coefficient of variation (C_x), D_k ; k = 1,2,..., 10 (first decile, second decile,…, 10th decile) of an auxiliary variable, correlation coefficient between x and $y\,(\rho_{xy})$, $0 < \gamma \leq 1$ and t is a suitably chosen constant. The scalar α takes values -1, (for product-type estimator) and +1 (for ratio-type estimator). $\bar{x}$ is the sample mean of the auxiliary variable.

To obtain the Bias and MSE of t_p, up to the first order of approximation, let us define

$$\bar{x} = \bar{X}\left(1+\Delta_x\right),\ \bar{y} = \bar{Y}\left(1+\Delta_y\right),\ \bar{x}^{\gamma} = \bar{X}^{\gamma}\left(1+\Delta_x\right)^{\gamma},$$

Such that $E\left(\Delta_x\right) = E\left(\Delta_y\right) = 0$, $E\left(\Delta_x^2\right) = \theta C_x^2$, $E\left(\Delta_y^2\right) = \theta C_y^2$, $E\left(\Delta_x \Delta_y\right) = \theta\rho_{xy} C_x C_y$. Expressing (7) in terms of Δ_x and Δ_y we have

$$t_p = \frac{\left[\bar{Y} + \bar{Y}\Delta_y - t\left[\left[\bar{X}^{\gamma}\left[1+\Delta_x\right]^{\gamma} - \bar{X}^{\gamma}\right]\right]\right]}{\left[\left(A\bar{X} + G\right)\left[1+\lambda\Delta_x\right]\right]^{\alpha}}\left[A\bar{X} + G\right]^{\alpha}$$

where $\lambda = \dfrac{A\bar{X}}{A\bar{X} + G}$

If we assume, $\left|\Delta_x\right| < 1$ and $\left|\Delta_y\right| < 1$ the expressions $\left(1+\lambda\Delta_x\right)^{-\alpha}$ and $\left(1+\Delta_x\right)^{\gamma}$ are expandable using binomial expansion to a convergent infinite series. Retaining the terms $\Delta' s$ up to power 2, we have

$$t_p = \bar{Y} + \bar{Y}\Delta_y - t\gamma\bar{X}^{\gamma}\Delta_x - \frac{t\gamma\left(\gamma-1\right)\bar{X}^{\gamma}\Delta_x^2}{2}$$
$$-\alpha\lambda\bar{Y}\Delta_x - \alpha\lambda\Delta_x\bar{Y}\Delta_y + t\alpha\gamma\bar{X}^{\gamma}\Delta_x^2 + \frac{\alpha\left(\alpha+1\right)\lambda^2\bar{Y}\Delta_x^2}{2} \tag{8}$$

Subtracting $\bar{Y}$ from both sides of (8) and taking the expectation, the bias of the estimator $\left(t_p\right)$ to the first degree of approximation is

$$B\left(t_p\right) = E\left(t_p - \bar{Y}\right)$$

$$B\left(t_p\right) = \theta\left[-\frac{t\gamma\left(\gamma-1\right)\bar{X}^{\gamma}S_x^2}{2\bar{X}^2} - \frac{\alpha\lambda\bar{Y}\rho_{xy}S_xS_y}{\bar{X}\bar{Y}} + \frac{t\alpha\gamma\lambda\bar{X}^{\gamma}S_x^2}{\bar{X}^2} + \frac{\alpha\left(\alpha+1\right)\lambda^2\bar{Y}S_x^2}{2\bar{X}^2}\right] \tag{9}$$

Squaring both side of (8) and taking the expectation to the first degree of approximation. The mean square error of $\left(t_p\right)$ is

$$MSE\left(\mathrm{t_p}\right)=E\left[t_p-\bar{Y}\right]^2$$

$$MSE\left(t_p\right)=\theta\left[\begin{array}{c}\left[S_y^2+\alpha^2\lambda^2R^2S_X^2-2\alpha\lambda RS_{xy}\right]+t^2\gamma^2\bar{X}^{2\gamma-2}S_x^2 \\ -2t\gamma\bar{X}^{\gamma-1}\rho_{xy}S_xS_y+2t\alpha\gamma\lambda R\bar{X}^{\gamma-1}S_x^2\end{array}\right] \quad (10)$$

where $\lambda R=\dfrac{A\bar{Y}}{A\bar{X}+G}$

To obtain the optimum t, we differentiate (10) with respect to t, equate to zero and solve the equation, we have

$$t_{opt}=\frac{\dfrac{\rho_{xy}S_y}{S_x}-\alpha\lambda R}{\gamma\bar{X}^{\gamma-1}}$$

Substituting t_{opt} in (10) to obtain the optimum $MSE\left(t_p\right)$, we have

$$MSE_{opt}\left(t_p\right)=\theta\left[\left(1-\rho_{xy}^2\right)S_y^2\right] \quad (11)$$

This is equivalent to the approximate variance of the classical linear regression estimator.

3.1 Members of the Proposed Class of Difference-Type Ratio Estimator

In this section, members of the proposed class of difference-type ratio estimator which are categorized as class 1 estimators and class 2 estimators shall be identified.

3.1.1 Identification of Members of the Class 1 Estimators

Estimators in this class is obtained by setting t =0, $\alpha=1$, A=A_j, G=G_j in (7) then we have jth existing ratio-type estimator for estimating the population mean of the variable of interest and is given as follows:

$$t_j=\bar{y}\,\frac{A_j\bar{X}+G_j}{A_j\bar{x}+G_j},\quad j=1,2,3,\ldots,19 \quad (12)$$

This jth existing ratio-type estimator can be can be seen in Table 1.

Substituting these parameters in (9) and (10) we have the bias and MSE are as

$$B\left(t_j\right)=\theta\bar{Y}\left[\lambda_j^2C_x^2-\lambda_j\rho_{xy}C_yC_x\right] \quad (13)$$

$$MSE\left(t_j\right)=\theta\left[S_y^2+\lambda_j^2R^2S_x^2-2\lambda_jR\rho_{xy}S_yS_x\right] \quad (14)$$

where $\lambda_j=\dfrac{A_j\bar{X}}{A_j\bar{X}+G_j}$. $A_j$andG_j are suitable chosen constants or known parameters in jth estimator.

Table 1
Class 1-Existing Members of the Generalized Class of Difference-Type Ratio Estimators

S/N (j)	Existing Estimators t_j	t	α	A_j	G_j
1	Sisodia and Dwivedi (1981) $t_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$	0	1	1	C_x
2	Singh and Kakran (1993) $t_2 = \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$	0	1	1	$\beta_2(x)$
3	Upadhyaya and Singh (1999) $t_3 = \bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right)$	0	1	$\beta_2(x)$	C_x
4	Upadhyaya and Singh (1999) $t_4 = \bar{y}\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\right)$	0	1	C_x	$\beta_2(x)$
5	Yan and Tian (2010) $t_5 = \bar{y}\left(\frac{\bar{X}+\beta_1(x)}{\bar{x}+\beta_1(x)}\right)$	0	1	1	$\beta_1(x)$
6	Yan and Tian (2010) $t_6 = \bar{y}\left(\frac{\beta_1(x)\bar{X}+\beta_2(x)}{\beta_1(x)\bar{x}+\beta_2(x)}\right)$	0	1	$\boldsymbol{\beta_1}$	$\beta_2(x)$
7	Yan and Tian (2010) $t_7 = \bar{y}\left(\frac{\beta_2(x)\bar{X}+\beta_1(x)}{\beta_2(x)\bar{x}+\beta_1(x)}\right)$	0	1	$\beta_2(x)$	$\beta_1(x)$
8	Yan and Tian (2010) $t_8 = \bar{y}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}+\beta_1(x)}\right)$	0	1	C_x	$\beta_1(x)$
9	Singh and Tailor (2003) $t_9 = \bar{y}\left(\frac{\bar{X}+\rho_{xy}}{\bar{x}+\rho_{xy}}\right)$	0	1	1	$\boldsymbol{\rho_{xy}}$
10 . . . 19	Subramani and Kumarapandiyan (2013) $t_j = \bar{y}\left(\frac{\bar{X}+D_k}{\bar{x}+D_k}\right)$, j= 10,11, …,19 k= 1,2, …,10 If k=1, we have estimator $\mathbf{t_{10}}$ k=2: $\mathbf{t_{11}}$, k=3: $\mathbf{t_{12}}$, k=4: $\mathbf{t_{13}}$, k=5: $\mathbf{t_{14}}$, k=6:$\mathbf{t_{15}}$, k=7:$\mathbf{t_{16}}$, k=8:$\mathbf{t_{17}}$, k=9:$\mathbf{t_{18}}$, k=10:$\mathbf{t_{19}}$	0	1	1	$\boldsymbol{D_k}$

3.1.2 Identification of Members of the Class 2 Estimators

The existing ratio-type estimators in this class can be obtained by setting $\gamma = 1$, $\boldsymbol{\alpha = 1}$ (ratio estimator), A=$\boldsymbol{A_j}$, G=$\boldsymbol{G_j}$ and $t = b = \dfrac{s_{xy}}{s_x^2}$ (sample regression coefficient of *y* on *x*) in (7) and the estimator is as follows.

$$t_j = \frac{\left(\bar{y} - b(\bar{x} - \bar{X})\right)\left(A_j \bar{X} + G_j\right)}{\left(A_j \bar{x} + G_j\right)}, \text{ j=20, …,41} \tag{15}$$

This ***j***th existing ratio-type estimator can be seen in the second column of Table 2.

Using the large sample approximation as used in the case of the regression estimation of the population mean, where b tends to $\beta = S_{xy} \big/ S_x^2$ (population regression coefficient of *Y* on *X*), the bias and the mean square error of estimator (15) can be obtained from (9) and (10) as follows:

$$B\left(t_j\right) = \theta \frac{S_x^2}{\bar{Y}} (\lambda R)_j^2 \tag{16}$$

The Mean square error is

$$MSE\left(t_j\right) = \theta\left((\lambda R)_j^2 S_x^2 + S_y^2\left(1 - \rho_{xy}^2\right)\right) \tag{17}$$

where $(\lambda R)_j = \dfrac{A_j \bar{Y}}{A_j \bar{X} + G_j}$, $j = 20, ..., 41$. $\boldsymbol{A_j}$ and $\boldsymbol{G_j}$ are suitable chosen constants or parameters in jth estimator.

Also if we set $0 < \gamma < 1$, $\alpha = 1, A = A_j, G = G_j$, $t = b = \dfrac{s_{xy}}{s_x^2}$ in (7) then we have jth new members of class 2 estimators and is given as follows.

$$t_{pj} = \frac{\left(\bar{y} - b(\bar{x}^{\gamma} - \bar{X}^{\gamma})\right)\left(A_j \bar{X} + G_j\right)}{\left(A_j \bar{x} + G_j\right)}; \; j = 20,.., 41; \; 0 < \gamma < 1 \tag{18}$$

This ***j***th estimator can be seen in the third column of Table 2.

Using the large sample approximation, b tends to $\beta = S_{xy} \big/ S_x^2$, the bias and mean square error of the estimator in (18), from (9) and (10) are as follows

$$B\left(t_{pj}\right) = \theta\left[-\frac{\gamma(\gamma - 1)\bar{X}^{\gamma}\rho S_x S_y}{2\bar{X}^2} - \frac{\lambda_j \rho S_x S_y}{\bar{X}\bar{Y}} + \gamma\lambda \bar{X}^{\gamma-2}\rho S_x S_y + \frac{\lambda_j^2 R S_x^2}{\bar{X}}\right] \tag{19}$$

$$MSE\left(t_{pj}\right)=\theta\left[\begin{array}{l}S_y^2+S_y^2\rho^2\left(\gamma^2\bar{X}^{2\gamma-2}-2\gamma\bar{X}^{\gamma-1}\right)\\ \qquad+\left(\lambda R\right)_j^2S_x^2-2\left(\lambda R\right)_j\rho S_xS_y\left(1-\bar{X}^{\gamma-1}\right)\end{array}\right] \quad (20)$$

Also setting A= $\boldsymbol{A_j}$ = $\boldsymbol{C_x}$ and G =$\boldsymbol{G_j}$=$\boldsymbol{D_k}$, t= b, $\boldsymbol{\alpha=1}$, in (7), we have jth member of the class 2 estimator, where j=42,..., 51. See the estimators in the last cell of the third column of Table 2. The biases and mean square errors are the same as (19) and (20) respectively.

Table 2
Class 2-Members of the Generalized Class of Difference-Type Ratio Estimator

$\boldsymbol{j}$	**Existing Ratio-Type Estimators (t_j) at $\boldsymbol{\gamma=1}$**	**New Members (t_{pj}) at $\boldsymbol{0<\gamma<1}$**	$\boldsymbol{A_j}$	$\boldsymbol{G_j}$
20	Kadilar and Cingi (2004) $t_{20}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{\bar{x}}\bar{X}$	$t_{p20}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{\bar{x}}\bar{X}$	1	0
21	Kadilar and Cingi (2004) $t_{21}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}+C_x)}(\bar{X}+C_x)$	$t_{p21}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}+C_x)}(\bar{X}+C_x)$	1	$\boldsymbol{C_X}$
22	Kadilar and Cingi (2004) $\boldsymbol{t_{22}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}+\beta_2(x))}(\bar{X}+\beta_2(x))}$	$\boldsymbol{t_{p22}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}+\beta_2(x))}(\bar{X}+\beta_2(x))}$	1	$\boldsymbol{\beta_2(x)}$
23	Kadilar and Cingi (2004) $\boldsymbol{t_{23}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}\beta_2(x)+C_x)}(\bar{X}\beta_2(x)+C_x)}$	$\boldsymbol{t_{p23}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}\beta_2(x)+C_x)}(\bar{X}\beta_2(x)+C_x)}$	$\boldsymbol{\beta_2(x)}$	$\boldsymbol{C_x}$
24	Kadilar and Cingi (2004) $\boldsymbol{t_{24}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}C_x+\beta_2(x))}(\bar{X}C_x+\beta_2(x))}$	$\boldsymbol{t_{p24}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}C_x+\beta_2(x))}(\bar{X}C_x+\beta_2(x))}$	$\boldsymbol{C_x}$	$\boldsymbol{\beta_2(x)}$
25	Kadilar and Cingi (2006) $\boldsymbol{t_{25}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}+\rho_{xy})}(\bar{X}+\rho_{xy})}$	$\boldsymbol{t_{p25}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}+\rho)}(\bar{X}+\rho_{xy})}$	1	$\boldsymbol{\rho_{xy}}$
26	Kadilar and Cingi (2006) $\boldsymbol{t_{26}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}C_x+\rho_{xy})}(\bar{X}C_x+\rho_{xy})}$	$\boldsymbol{t_{p26}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}C_x+\rho_{xy})}(\bar{X}C_x+\rho_{xy})}$	$\boldsymbol{C_x}$	$\boldsymbol{\rho_{xy}}$
27	Kadilar and Cingi (2006) $\boldsymbol{t_{27}=\dfrac{\bar{y}-b(\bar{x}-\bar{X})}{(\bar{x}\rho_{xy}+C_x)}(\bar{X}\rho_{xy}+C_x)}$	$\boldsymbol{t_{p27}=\dfrac{\bar{y}-b(\bar{x}^{\gamma}-\bar{X}^{\gamma})}{(\bar{x}\rho_{xy}+C_x)}(\bar{X}\rho_{xy}+C_x)}$	$\boldsymbol{\rho_{xy}}$	$\boldsymbol{C_x}$

j	Existing Ratio-Type Estimators (t_j) at $\gamma = 1$	New Members (t_{pj}) at $0 < \gamma < 1$	A_j	G_j
28	Kadilar and Cingi (2006) $t_{28} = \frac{\bar{y} - b(\bar{x} - \bar{X})}{(\bar{x}\beta_2(x) + \rho_{xy})}(\bar{X}\beta_2(x) + \rho_{xy})$	$t_{p28} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{(\bar{x}\beta_2(x) + \rho_{xy})}(\bar{X}\beta_2(x) + \rho_{xy})$	$\beta_2(x)$	ρ_{xy}
29	Kadilar and Cingi (2006) $t_{29} = \frac{\bar{y} - b(\bar{x} - \bar{X})}{(\bar{x}\rho + \beta_2(x))}(\bar{X}\rho + \beta_2(x))$	$t_{p29} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{(\bar{x}\rho + \beta_2(x))}(\bar{X}\rho + \beta_2(x))$	ρ_{xy}	$\beta_2(x)$
30	Yan and Tian (2010) $t_{30} = \frac{\bar{y} - b(\bar{x} - \bar{X})}{\bar{x} + \beta_1(x)}(\bar{X} + \beta_1(x))$	$t_{p30} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{\bar{x} + \beta_1(x)}(\bar{X} + \beta_1(x))$	1	$\beta_1(x)$
31	Yan and Tian (2010) $t_{31} = \frac{\bar{y} - b(\bar{x} - \bar{X})}{(\bar{x}\beta_1(x) + \beta_2(x))}(\bar{X}\beta_1(x) + \beta_2(x))$	$t_{p31} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{(\bar{x}\beta_1(x) + \beta_2(x))}(\bar{X}\beta_1(x) + \beta_2(x))$	$\beta_1(x)$	$\beta_2(x)$
32 . . . 41	Raja et al. (2017) $t_j = \frac{\bar{y} - b(\bar{x} - \bar{X})}{(\bar{x}D_k + C_x)}(\bar{X}D_k + C_x)$ j= 32,33, ---,41 k=1, 2, 3,---,10 where k=1, we have estimator t_{32} k=2: t_{33}, k=3: t_{34}, k=4: t_{35}, k=5: $\hat{T}_{36}$, k=6:$\hat{T}_{37}$, k=7:$\hat{T}_{38}$, k=8:t, k=9:t_{40}, k=10:t_{41}	$t_{pj} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{(\bar{x}D_k + C_x)}(\bar{X}D_k + C_x)$ j= 32,33, ---,41 k=1, 2, 3,---,10 where k=1, we have estimator t_{p32} k=2: t_{p33}, k=3: t_{p34}, k=4: t_{p35}, k=5: t_{p36}, k=6:t_{p37}, k=7:t_{p38}, k=8:t_{p39}, k=9:t_{p40}, k=10:t_{p41}	D_k	C_x
42 . . . 51	$t_j = \frac{\bar{y} - b(\bar{x} - \bar{X})}{(\bar{x}C_x + D_k)}(\bar{X}C_x + D_k)$ j= 42,43, ---,51 k=1, 2, 3,---,10 where k=1, we have estimator t_{42} k=2: t_{43}, k=3: t_{44}, k=4: t_{45}, k=5: t_{46}, k=6:t_{47}, k=7:t_{48}, k=8:t_{49}, k=9:t_{50}, k=10:t_{51}	$t_{pj} = \frac{\bar{y} - b(\bar{x}^\gamma - \bar{X}^\gamma)}{(\bar{x}C_x + D_k)}(\bar{X}C_x + D_k)$ j= 42,43, ---,5 k=1, 2, 3,---,10 were k=1, we have estimator t_{p42} k=2: t_{p43}, k=3: t_{p44}, k=4: t_{p45}, k=5: t_{p46}, k=6:t_{p47}, k=7:t_{p48}, k=8:t_{p49}, k=9:t_{p50}, k=10:t_{p51}	C_x	D_k

Note: The stated expressions for the biases and MSEs of all the existing ratio-type estimators in Table 2 given by these authors in their papers go in accordance with the proof of the proposed estimator except for Raja et al. (2017) estimators. The correct expressions of their bias and MSE can be seen below.

Correction on Raja et al. (2017) ratio type estimators

Having studied Raja et al. (2017) generalized ratio estimators; it is observed that the expressions for the bias and mean square errors are wrong. Based on the proof of the suggested generalized class of estimator in (7), at the set of $t = b = \frac{s_{xy}}{s_x^2}$, $\boldsymbol{\gamma = 1}$, $\boldsymbol{\alpha = 1}$, A= $\boldsymbol{A_j}$=$\boldsymbol{D_k}$ (first decile, second decile,…, tenth decile) and G= $\boldsymbol{G_j}$=$\boldsymbol{C_X}$ in (7), their estimators $\boldsymbol{\widehat{T}_j}$, j=32,…,41 are obtained, which are shown in second column of Table 2. Hence, the correct expressions for the bias and mean square error are from (9) and (10). Using the large sample approximation, b tends to $\beta = S_{xy}/S_x^2$, the bias and the mean square error of the estimator after proper substitution, we have

$$B\left(t_j\right) = \theta \frac{S_x^2}{\bar{Y}} \left(\lambda R\right)_j^2$$

$$MSE\left(t_j\right) = \theta\left[\left(\lambda R\right)_j^2 S_x^2 + S_y^2\left(1-\rho_{xy}^2\right)\right]$$

where $\left(\lambda R\right)_j = \frac{D_k \bar{Y}}{\left(D_k \bar{X} + C_x\right)}$ and not $\frac{\bar{X} D_k}{\bar{X} D_k + C_x}$

as stated in their paper.

4. EFFICIENCY COMPARISON

In this section, the theoretical conditions for which the new members of the generalized class of ratio estimators in (18) perform better than the sample mean, classical ratio estimator and existing estimators given in class 1 (Table 1) and class2 (Table 2) considered in this study are given below.

i) The proposed ratio-type estimators $\boldsymbol{t_{pj}}$, j=20,...,51 will be better than the sample mean $\boldsymbol{t_0}$, iff

$\boldsymbol{MSE(t_{pj}) < V(t_0)}$,that is if

$$\left(S_y^2 \rho_{xy}^2 \left(\gamma^2 \bar{X}^{2\gamma-2} - 2\gamma \bar{X}^{\gamma-1}\right) + \left(\lambda R\right)_j^2 S_x^2 - 2\left(\lambda R\right)_j \rho_{xy} S_x S_y \left(1 - \bar{X}^{\gamma-1}\right)\right) \le 0$$

ii) The proposed ratio-type estimators $\boldsymbol{t_{pj}}$, j=20,...,51 will be better than the classical ratio estimator $\boldsymbol{t_r}$, iff

$\boldsymbol{MSE(t_{pj}) \le MSE(t_r)}$, that is if

$$\left(S_y^2 \rho_{xy}^2 \left(\gamma^2 \bar{X}^{2\gamma-2} - 2\gamma \bar{X}^{\gamma-1}\right) + R^2 S_x^2 \left(\lambda_j^2 - 1\right) - 2R\rho_{xy} S_x S_y \left(\lambda_j - \lambda_j \bar{X}^{\gamma-1} + 1\right)\right) \le 0$$

iii) The proposed ratio-type estimators $\boldsymbol{t_{pj}}$, j=20,..., 51 will be more efficient than the existing ratio-type estimators $\boldsymbol{t_j}$, j=1,2,3,…,19 in class 1. Iff

$MSE\left(t_{pj}\right) \le MSE\left(t_j\right)$, that is if

$$\left(\rho_{xy} S_y \left(\gamma^2 \bar{X}^{2\gamma-2} - 2\gamma \bar{X}^{\gamma-1}\right) + 2(\lambda R)_j \rho_{xy} S_x \bar{X}^{\gamma-1}\right) \le 0$$

iv) The proposed ratio-type estimators $\boldsymbol{t_{pj}}$, j=20,..., 51 will be more efficient than the existing ratio-type estimators $\boldsymbol{t_{j}}$, j=20,…,41 in class 2. Iff

$\boldsymbol{MSE}(\boldsymbol{t_{pj}}) \le \boldsymbol{MSE}(\boldsymbol{t_{j}})$, this boils down to

$$\left(\rho_{xy} S_y \left(\gamma^2 \bar{X}^{2\gamma-2} - 2\gamma \bar{X}^{\gamma-1} + 1\right) - 2(\lambda R)_j S_x (1 - \bar{X}^{\gamma-1})\right) \le 0$$

In general, looking at MSE in (11), it is observed that the proposed generalized class of estimator at optimum t is more efficient than the MSE of the existing estimator in (15) with $t = b$ (sample regression coefficient).

The smaller the mean square error, the more efficient the estimator becomes.

5. EMPIRICAL STUDY

In this section, the real life data set by Singh and Chaudhary (1986) are considered and the parameters are shown below.

$\boldsymbol{Y}$- Number of animals in the ith unit
$\boldsymbol{X}$- The size of the ith unit
$\boldsymbol{N = 34}$ $\boldsymbol{n = 20}$ $\boldsymbol{\bar{Y} = 856.4117}$ $\boldsymbol{\bar{X} = 199.4412}$ $\boldsymbol{\rho_{xy} = 0.4453}$
$\boldsymbol{S_y = 733.1407}$ $\boldsymbol{S_x = 150.2150}$ $\boldsymbol{C_y = 0.8561}$ $\boldsymbol{C_x = 0.7531}$
$\boldsymbol{\beta_{2(x)} = 1.0445}$ $\boldsymbol{\beta_{1(x)} = 1.1823}$ $\boldsymbol{D_1 = 60.600}$ $\boldsymbol{D_2 = 83.0000}$
$\boldsymbol{D_3 = 102.700}$ $\boldsymbol{D_4 = 111.200}$ $\boldsymbol{D_5 = 142.500}$ $\boldsymbol{D_6 = 210.200}$
$\boldsymbol{D_7 = 264.5000}$ $\boldsymbol{D_8 = 304.400}$ $\boldsymbol{D_9 = 373.2000}$ $\boldsymbol{D_{10} = 634.0000}$

The correlation coefficient of the data set is 0.4453, we shall vary the values of the population correlation coefficient to obtain the pattern of behaviours of the MSEs and the biases of the proposed estimators and the existing estimators. In the literature, it is well known that the performance of classical ratio estimator depends largely on how positively high the correlation coefficient is. The correlation coefficients of 0.2, 0.4453, 0.65 and 0.8 shall be considered for the population. The results of the computation of the biases and MSEs for class 1 and class 2 estimators are presented in Tables 3 to Table 7.

Notes: Values in bold represents estimators that have smaller MSEs compared to the classical ratio estimator. Value(s) in bold and asterisk (*) depicts estimators that have approximate equal value of MSE to the classical linear regression estimator.

Table 3
The MSE and Bias Values of the Existing Estimators (Class 1) in Table 1 with Varying Values of ρ_{xy}

Estima-tors	$\rho_{xy} = \mathbf{0.2}$		$\rho_{xy} = \mathbf{0.4453}$		$\rho_{xy} = \mathbf{0.65}$		$\rho_{xy} = \mathbf{0.8}$	
	MSE	**Bias**	**MSE**	**Bias**	**MSE**	**Bias**	**MSE**	**Bias**
t_0	20476.017	0	20476.017	0	20476.017	0	20476.017	0
t_l	10623.42	-	8871.748	-	6390.65	-	3983.782	-
t_r	11066.06	10.047	10961.1	4.9398	6975.127	2.6127	4054.286	0.9074
t_1	15687.97	10.00	**10929.39**	4.8837	**6958.414**	2.5653	**4048.560**	0.8665
t_2	15668.92	9.9271	**10917.25**	4.8622	**6952.045**	2.5472	**4046.421**	0.8508
t_3	15690.08	9.8983	**10930.74**	4.8861	**6959.118**	2.5673	**4048.798**	0.8682
t_4	15646.64	9.9303	**10903.08**	4.8371	**6944.630**	2.5260	**4043.960**	0.8325
t_5	15659.94	9.8645	**10911.54**	4.8521	**6949.052**	2.5387	**4045.424**	0.8434
t_6	15679.44	9.9577	**10923.95**	4.8741	**6955.558**	2.5572	**4047.598**	0.8595
t_7	15663.22	9.9432	**10913.62**	4.8558	**6950.145**	2.5418	**4045.787**	0.8461
t_8	15634.79	9.8347	**10895.54**	4.8237	**6940.699**	2.5147	**4042.669**	0.8228
t_9	15724.4	9.9596	**10942.3**	4.9066	**6965.2**	2.5846	**4050.866**	0.8831
t_{10}	13117.95	9.7710	**9454.532**	2.0009	**6397.449**	0.2161	4157.28	1.0917
t_{11}	12587.3	9.8847	**9214.414**	1.4126	**6399.784**	0.2306	4337.28	1.4347
t_{12}	12227.78	9.9142	**9074.812**	1.0165	**6443.699**	0.5195	4515.673	1.6452
t_{13}	12096.65	10.0010	**9029.963**	0.8727	**6470.844**	0.6213	4595.575	1.7161
t_{14}	11708.69	10.0013	**8922.715**	0.4500	**6597.848**	0.9073	4894.234	1.9019
t_{15}	11200.47	10.0015	**8874.925**	0.0937	**6934.282**	1.2267	5512.217	2.0570
t_{16}	**10974.91**	10.0015	**8921.541**	0.3278	7208.032	1.3282	5952.407	2.0612
t_{17}	**10866.69**	10.0017	**8975.935**	0.4366	7398.121	1.3578	6241.931	2.0328
t_{18}	**10748.76**	10.0018	**9085.167**	0.5498	7696.92	1.3603	6679.641	1.9542
t_{19}	**10635.65**	10.0019	**9481.626**	0.6386	8527.789	1.1955	7828.836	1.6036

Table 4
The MSE and Bias Values of the Existing and New Estimators (Class 2) in Table 2 and the Optimum MSE at $\rho_{xy} = \mathbf{0.2}$

j	MSE and Bias of the proposed estimators $\boldsymbol{t_{pj}}$ at $0<\gamma<1$, $\boldsymbol{\alpha=1}$ and t = b								MSE and Bias of the existing estimators $\boldsymbol{t_j}$ $\boldsymbol{\gamma=1}$	
	$\boldsymbol{\gamma=0.1}$		$\boldsymbol{\gamma=0.3}$		$\boldsymbol{\gamma=0.5}$		$\boldsymbol{\gamma=0.8}$		$\boldsymbol{\gamma=1}$	
	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
20	15740.2	7.73	15759.8	7.75	15844.7	7.83	16606.5	8.42	19189.5	10
21	15690.5	7.66	15710.1	7.68	15794.5	7.76	16552.8	8.35	19125.1	9.93
22	15671.5	7.64	15691	7.66	15775.3	7.74	16532.1	8.33	19100.4	9.9
23	15692.6	7.67	15712.2	7.69	15796.7	7.77	16555	8.36	19127.9	9.93
24	15649.2	7.61	15668.6	7.63	15752.8	7.71	16508	8.3	19071.6	9.86
25	15710.8	7.69	15730.3	7.71	15815	7.79	16574.6	8.38	19151.3	9.96
26	15701.1	7.68	15720.7	7.7	15805.3	7.78	16564.3	8.37	19138.9	9.94
27	15629.5	7.58	15648.9	7.6	15732.9	7.68	16486.7	8.27	19046	9.83
28	15712	7.69	15731.6	7.71	15816.3	7.79	16576	8.38	19153	9.96
29	15587.5	7.53	15606.8	7.55	15690.4	7.62	16441.2	8.21	18991.5	9.77
30	15662.5	7.63	15682	7.65	15766.2	7.72	16522.4	8.31	19088.8	9.88
31	15682	7.65	15701.5	7.67	15785.9	7.75	16543.5	8.34	19114.1	9.91
32	15739.4	7.73	15759	7.75	15843.9	7.83	16605.6	8.42	19188.4	10.00
33	15739.6	7.73	15759.2	7.75	15844.1	7.83	16605.8	8.42	19188.7	10.00
34	15739.7	7.73	15759.4	7.75	15844.3	7.83	16606	8.42	19188.8	10.00
35	15739.8	7.73	15759.4	7.75	15844.3	7.83	16606	8.42	19188.9	10.00
36	15739.9	7.73	15759.5	7.75	15844.4	7.83	16606.1	8.42	19189	10.00
37	15740	7.73	15759.6	7.75	15844.5	7.83	16606.2	8.42	19189.2	10.00
38	15740	7.73	15759.7	7.75	15844.6	7.83	16606.3	8.42	19189.2	10.00
39	15740	7.73	15759.7	7.75	15844.6	7.83	16606.3	8.42	19189.3	10.00
40	15740.1	7.73	15759.7	7.75	15844.6	7.83	16606.4	8.42	19189.3	10.00
41	15740.1	7.73	15759.8	7.75	15844.7	7.83	16606.4	8.42	19189.4	10.00
42	12641.7	3.46	12654	3.48	12707.5	3.54	13198.3	3.97	14972.3	5.08
43	12112.6	2.69	12123.2	2.7	12169.3	2.76	12595.6	3.15	14177	4.15
44	11775.8	2.18	11785.1	2.19	11825.7	2.25	12204.7	2.62	13644.9	3.53
45	11657.7	2.00	11666.5	2.01	11704.9	2.06	12065.8	2.42	13451.6	3.3
46	11324.2	1.47	11331.5	1.48	11363.2	1.53	11666.1	1.85	12879.1	2.63
47	**10931.5**	0.79	**10936.3**	0.8	**10957.5**	0.84	11169.6	1.12	12111.2	1.74
48	**10779.7**	0.49	**10783.1**	0.5	**10798.4**	0.54	**10959**	0.78	11747.1	1.31
49	**10714.8**	0.34	**10717.4**	0.35	**10729.2**	0.39	**10859.9**	0.61	11558.5	1.09
50	**10654.1**	0.17	**10655.6**	0.18	**10662.7**	0.21	**10752.4**	0.42	11328.8	0.82
51	**10634.3**	0.07	**10633.4**	0.06	***10630**	0.03	**10629.8**	0.12	10937.7	0.37
$MSE(t_p)_{opt}$	10623.42									

Table 5
The MSE and Bias Values of the Existing and New Estimators (Class 2) in Table 2 and the Optimum MSE at $\rho_{xy} = 0.4453$

j	MSE and Bias of the proposed estimators t_{pj} at $0<\gamma<1$, $\alpha = 1$ and t = b								MSE and Bias of the existing estimators t_j $\gamma = 1$	
	$\gamma = 0.1$		$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.8$		$\gamma = 1$	
	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
20	10964.75	4.94	10992.8	4.99	11115.47	5.16	12317.96	6.48	17437.79	10
21	**10933.01**	4.89	10960.8	4.93	11082.61	5.10	12277.2	6.42	17373.47	9.93
22	**10920.86**	4.86	10948.6	4.91	11070.02	5.08	12261.58	6.39	17348.77	9.9
23	**10934.36**	4.89	10962.2	4.94	11084.00	5.10	12278.93	6.42	17376.19	9.93
24	**10906.67**	4.84	10934.3	4.89	11055.33	5.05	12243.32	6.37	17319.89	9.86
25	**10945.93**	4.91	10973.8	4.96	11095.98	5.13	12293.79	6.44	17399.67	9.96
26	**10939.79**	4.90	10967.6	4.95	11089.63	5.11	12285.91	6.43	17387.23	9.94
27	**10894.15**	4.82	10921.7	4.86	11042.35	5.03	12227.19	6.34	17294.34	9.83
28	**10946.73**	4.91	10974.6	4.96	11096.81	5.13	12294.82	6.44	17401.29	9.96
29	**10867.47**	4.77	10894.8	4.82	11014.71	4.98	12192.8	6.29	17239.8	9.77
30	**10915.14**	4.85	10942.8	4.9	11064.1	5.07	12254.22	6.38	17337.13	9.88
31	**10927.57**	4.88	10955.3	4.92	11076.97	5.09	12270.2	6.41	17362.41	9.91
32	10964.22	4.94	10992.2	4.99	11114.92	5.16	12317.28	6.48	17436.73	10.00
33	10964.36	4.94	10992.4	4.99	11115.07	5.16	12317.47	6.48	17437.01	10.00
34	10964.44	4.94	10992.5	4.99	11115.15	5.16	12317.56	6.48	17437.16	10.00
35	10964.46	4.94	10992.5	4.99	11115.17	5.16	12317.59	6.48	17437.21	10.00
36	10964.53	4.94	10992.5	4.99	11115.24	5.16	12317.67	6.48	17437.34	10.00
37	10964.60	4.94	10992.6	4.99	11115.31	5.16	12317.77	6.48	17437.49	10.00
38	10964.63	4.94	10992.6	4.99	11115.34	5.16	12317.81	6.48	17437.55	10.00
39	10964.64	4.94	10992.7	4.99	11115.36	5.16	12317.83	6.48	17437.58	10.00
40	10964.66	4.94	10992.7	4.99	11115.38	5.16	12317.85	6.48	17437.62	10.00
41	10964.70	4.94	10992.7	4.99	11115.42	5.16	12317.9	6.48	17437.69	10.00
42	**9238.182**	1.47	**9249.96**	1.51	**9302.77**	1.64	**9902.002**	2.61	13220.63	5.08
43	**9035.793**	0.89	**9043.7**	0.93	**9079.875**	1.04	**9535.486**	1.93	12425.29	4.15
44	**8938.406**	0.52	**8943.48**	0.56	**8967.456**	0.67	**9317.784**	1.49	11893.23	3.53
45	**8912.415**	0.39	**8916.4**	0.43	**8935.678**	0.54	**9245.48**	1.34	11699.93	3.3
46	**8872.224**	0.04	**8872.74**	0.07	**8877.078**	0.17	**9057.941**	0.89	11127.4	2.63
47	**8939.505**	0.36	**8934.57**	0.34	**8915.478**	0.25	**8894.061**	0.35	**10359.56**	1.74
48	**9048.16**	0.51	**9040.15**	0.49	**9007.783**	0.41	***8871.85**	0.12	**9995.445**	1.31
49	**9134.973**	0.57	**9125.16**	0.56	**9085.071**	0.47	**8882.435**	0.02	**9806.848**	1.09
50	**9281.56**	0.62	**9269.3**	0.61	**9218.647**	0.53	**8924.873**	0.08	**9577.171**	0.82
51	**9717.206**	0.59	**9699.55**	0.58	**9625.699**	0.52	**9131.656**	0.19	**9185.989**	0.37
$MSE(t_p)_{opt}$	8871.748									

Table 6
The MSE and Bias Values of the Existing and New Estimators (Class 2) in Table 2 and the Optimum MSE at $\rho_{xy} = 0.65$

j	MSE and Bias of the proposed estimators t_{pj} at $0 < \gamma < 1$, $\alpha = 1$ and t = b								MSE and Bias of the existing estimators t_j $\gamma = 1$	
	$\gamma = 0.1$		$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.8$		$\gamma = 1$	
	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
20	6977.94	2.62	6999.74	2.69	7098.04	2.94	8252.10	4.87	14956.7	10.00
21	**6961.19**	2.57	6982.67	2.64	7079.64	2.89	8222.18	4.81	14892.37	9.93
22	**6954.80**	2.56	6976.17	2.62	7072.62	2.87	8210.73	4.79	14867.67	9.90
23	**6961.89**	2.58	6983.39	2.64	7080.41	2.89	8223.45	4.81	14895.09	9.93
24	**6947.37**	2.54	6968.59	2.60	7064.45	2.85	8197.36	4.77	14838.79	9.86
25	**6967.99**	2.58	6984.99	2.65	7087.11	2.9	8234.36	4.82	14918.57	9.94
26	**6964.76**	2.57	6980.21	2.63	7083.56	2.88	8228.57	4.8	14906.13	9.92
27	**6940.83**	2.52	6961.93	2.58	7057.2	2.83	8185.56	4.75	14813.25	9.83
28	**6968.41**	2.59	6990.03	2.66	7087.57	2.91	8235.11	4.84	14920.19	9.96
29	**6926.95**	2.52	6963.77	2.59	7041.97	2.84	8160.43	4.75	14758.71	9.84
30	**6951.80**	2.55	6973.11	2.61	7069.32	2.86	8205.34	4.78	14856.03	9.88
31	**6958.32**	2.57	6979.75	2.63	7076.49	2.88	8217.05	4.8	14881.31	9.91
32	6977.66	2.62	6999.45	2.69	7097.73	2.94	8251.61	4.87	14955.63	10.00
33	6977.74	2.62	6999.53	2.69	7097.81	2.94	8251.74	4.87	14955.92	10.00
34	6977.78	2.62	6999.57	2.69	7097.86	2.94	8251.81	4.87	14956.07	10.00
35	6977.79	2.62	6999.58	2.69	7097.87	2.94	8251.83	4.87	14956.11	10.00
36	6977.82	2.62	6999.61	2.69	7097.91	2.94	8251.89	4.87	14956.24	10.00
37	6977.86	2.62	6999.65	2.69	7097.95	2.94	8251.96	4.87	14956.39	10.00
38	6977.88	2.62	6999.67	2.69	7097.97	2.94	8251.99	4.87	14956.45	10.00
39	6977.89	2.62	6999.68	2.69	7097.98	2.94	8252.01	4.87	14956.48	10.00
40	6977.9	2.62	6999.69	2.69	7097.99	2.94	8252.02	4.87	14956.52	10.00
41	6977.91	2.62	6999.71	2.69	7098.01	2.94	8252.06	4.87	14956.59	10.00
42	**6396.28**	0.18	**6394.36**	0.13	***6390.65**	0.06	**6664.14**	1.48	10739.53	5.08
43	**6466.46**	0.6	**6458.91**	0.56	**6430.90**	0.38	**6494.75**	0.92	9944.19	4.15
44	**6568.89**	0.85	**6557.2**	0.81	**6511.38**	0.64	**6421.56**	0.56	9412.13	3.53
45	**6619.81**	0.94	**6606.52**	0.89	**6553.86**	0.73	**6404.8**	0.44	9218.83	3.30
46	**6824.33**	1.15	**6805.98**	1.11	**6731.50**	0.96	**6394.31**	0.1	8646.30	2.63
47	7275.51	1.34	7249.21	1.3	7140.5	1.17	**6508.07**	0.28	7878.46	1.74
48	7601.49	1.36	7570.69	1.33	7442.64	1.2	**6643.03**	0.42	7514.34	1.31
49	7814.90	1.34	7781.48	1.31	7642.16	1.2	**6745.17**	0.47	7325.75	1.09
50	8134.46	1.29	8097.45	1.26	7942.71	1.16	**6912.70**	0.5	7096.07	0.82
51	8950.19	1.04	8905.32	1.02	8716.71	0.93	7394.36	0.45	**6704.89**	0.37
$MSE(t_p)_{opt}$	6390.65									

Table 7
The MSE and Bias Values of the Existing and New Estimators (Class 2) inTable 2 and the Optimum MSE at $\rho_{xy} = \mathbf{0.8}$

j	MSE and Bias of the proposed estimators t_{pj} at $0<\gamma<1$, $\alpha = 1$ and t = b								MSE and Bias of the existing estimators t_j $\gamma = 1$	
	$\gamma = 0.1$		$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.8$		$\gamma = 1$	
	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
20	4055.49	0.92	4065.08	1	4113.2	1.31	4158.87	3.68	12549.83	10.0
21	**4049.71**	0.88	4058.92	0.96	4105.39	1.27	4969.4	3.63	12485.5	9.93
22	**4047.56**	0.86	4056.62	0.94	4102.46	1.25	4961	3.61	12460.8	9.90
23	**4049.95**	0.88	4059.18	0.96	4105.72	1.27	4970.33	3.63	12488.23	9.93
24	**4045.07**	0.84	4053.96	0.92	4099.06	1.23	4951.21	3.59	12431.92	9.86
25	**4052.04**	0.88	4058.55	0.95	4108.55	1.27	4968.04	3.63	12511.7	9.92
26	**4050.93**	0.86	4056.48	0.94	4107.05	1.25	4960.49	3.61	12499.26	9.90
27	**4042.91**	0.87	4057.43	0.95	4096.09	1.26	4963.97	3.62	12406.38	9.91
28	**4052.10**	0.88	4058.82	0.96	4108.74	1.27	4969.03	3.63	12513.32	9.93
29	**4038.43**	0.85	4054.59	0.93	4089.87	1.24	4953.53	3.60	12351.84	9.87
30	**4046.55**	0.85	4055.54	0.93	4101.08	1.24	4957.05	3.6	12449.16	9.88
31	**4048.74**	0.87	4057.89	0.95	4104.08	1.26	4965.64	3.62	12474.44	9.91
32	4055.37	0.92	4064.98	1.00	4113.06	1.31	4991.02	3.68	12548.76	10.00
33	4055.42	0.92	4065.00	1.00	4113.10	1.31	4991.11	3.68	12549.05	10.00
34	4055.43	0.92	4065.02	1.00	4113.12	1.31	4991.17	3.68	12549.2	10.00
35	4055.44	0.92	4065.02	1.00	4113.12	1.31	4991.18	3.68	12549.25	10.00
36	4055.45	0.92	4065.04	1.00	4113.14	1.31	4991.23	3.68	12549.37	10.00
37	4055.46	0.92	4065.05	1.00	4113.16	1.31	4991.28	3.68	12549.52	10.00
38	4055.47	0.92	4065.06	1.00	4113.17	1.31	4991.3	3.68	12549.58	10.00
39	4055.47	0.92	4065.06	1.00	4113.17	1.31	4991.31	3.68	12549.62	10.00
40	4055.47	0.92	4065.06	1.00	4113.17	1.31	4991.32	3.68	12549.65	10.00
41	4055.48	0.92	4065.07	1.00	4113.18	1.31	4991.35	3.68	12549.73	10.00
42	4312.78	1.39	4293.20	1.33	4215.75	1.09	**4010.16**	0.65	8332.66	5.08
43	4582.71	1.70	4556.18	1.64	4448.87	1.42	***3985.2**	0.17	7537.32	4.15
44	4831.55	1.87	4799.93	1.81	4670.67	1.60	**4017.91**	0.12	7005.26	3.53
45	4938.83	1.92	4905.25	1.86	4767.56	1.66	**4041.99**	0.22	6811.96	3.30
46	5322.67	2.03	5282.85	1.98	5118.32	1.79	4161.11	0.49	6239.46	2.63
47	6055.16	2.05	6005.56	2.00	5798.93	1.84	4478.31	0.75	5471.59	1.74
48	6540.40	1.98	6485.26	1.93	6254.79	1.78	4728.45	0.82	5107.47	1.31
49	6846.58	1.91	6788.22	1.87	6543.87	1.73	4897.68	0.83	4918.88	1.09
50	7292.88	1.78	7230.11	1.74	6966.79	1.61	5156.87	0.81	4689.20	0.82
51	8387.13	1.37	8314.67	1.34	8009.67	1.23	5839.96	0.64	4298.02	0.37
$MSE(t_p)_{opt}$	3983.782									

5. RESULTS AND DISCUSSION

From Table 3 Result (Class 1 estimators)

At ρ_{xy} equal to 0.2, 0.4453, 0.65 and 0.8 the existing ratio-type estimators in class 1, t_j, j = 16, 17, 18, 19; t_j, j = 1, …, 19; t_j, j = 1, …, 15 and t_j, j = 1,…, 9 respectively, are more efficient than the classical ratio estimator because they have smaller MSEs but they are not as efficient as the classical linear regression estimator. At ρ_{xy} equal to 0.2 and 0.65, all the existing ratio-type estimators t_j, j = 1, …, 19 have smaller bias values compared to the bias of the classical ratio estimator.

From Table 4 Result (Class 2 estimators) at $\rho_{xy} = \mathbf{0.2}$

From Table 4, none of the existing estimators t_j, j = 20, …, 50 by Kadilar and Cingi (2004, 2006), Yan and Tian (2010) and Raja et al. (2017) perform better than the classical ratio estimator. The MSEs of the proposed estimators t_{pj}, j = 20, …, 50 increases as $\boldsymbol{\gamma}$ increases from 0.1 to 0.8 while the MSE of estimator t_{p51}, decreases as $\boldsymbol{\gamma}$ increases. The proposed estimators t_{pj}, j = 47, …, 51 at $\boldsymbol{\gamma = 0.1}$ to 0.8 are more efficient than the classical ratio estimator, existing estimators by Kadilar and Cingi (2004, 2006), Yan and Tian (2010) and Raja et al. (2017). The proposed estimators t_{p51} at $\boldsymbol{\gamma = 0.5}$ has MSE close to the MSE of the proposed generalized class of estimator at t optimum. This estimator t_{p51} utilizes the tenth decile and the coefficient of variation of the auxiliary variable for symbol A and G respectively. The proposed generalized class of estimator t_p at t optimum has equal efficiency with the classical linear regression estimator. As $\boldsymbol{\gamma}$ increases from 0.1 to 0.8, the biases of the proposed estimator increases.

From Table 5 Result (Class 2 estimators) at $\rho_{xy} = \mathbf{0.4453}$

It is observed that as the value of gamma ($\boldsymbol{\gamma}$) increases from 0.1 to 0.8, the MSE of the proposed estimators t_j, j = 20, …, 41 increases while estimators t_{pj}, j = 42,…,51 have their MSE decreases as value of $\boldsymbol{\gamma}$increases from 0.1 to 0.8. All the proposed estimators t_{pj}, j = 20, …, 51 at γ = 0.1 to 0.8 perform better than the existing estimators t_j, j = 20, …, 41 at γ = 1 by Kadilar and Cingi (2004, 2006), Yan and Tian (2010) and Raja et al. (2017) because they have smaller MSEs and higher PREs. The proposed estimators t_{pj}, j = 20, …, 31 at γ = 0.1 to 0.3 and t_{pj}, j = 42, …, 51 at γ = 0.1 to 0.8 perform better than the classical ratio estimator. The proposed generalized class of difference-type ratio estimator, t_p at t optimum is equally efficient as the classical linear regression estimator. The proposed estimator t_{p48} at γ = 0.8 attain equal efficiency as estimator t_p at t optimum without depending on the optimality condition. It is also observed that the proposed estimators t_{pj}, j = 20, …, 51 at γ = 0.1 to 0.8 have smaller biases compared to the biases of the existing estimators t_j, j = 20, …, 41 at γ = 1.

Tables 6 Results (Class 2 estimators) at $\rho_{xy} = \mathbf{0.65}$

It is observed that as values of gamma ($\boldsymbol{\gamma}$) increases from 0.1 to 0.8, the MSEs of all the proposed estimators t_{pj}, j = 20, …, 41 increases while estimators t_{pj}, j = 42, …, 51 decreases as value of $\boldsymbol{\gamma}$ increases. The proposed estimators t_{pj}, j = 21, …, 31 at $\boldsymbol{\gamma = 0.1}$; t_{pj}, j = 42, …, 47 at $\boldsymbol{\gamma = 0.1}$ to 0.8 and t_{pj}, j = 47, …, 50 at $\boldsymbol{\gamma = 0.8}$ are more efficient than the classical ratio estimator. Estimator t_{p42} at $\boldsymbol{\gamma = 0.5}$ is the most efficient estimator because it has least value of MSE and is also equivalent to the MSE of the proposed generalized class of ratio estimator at t optimum and the classical linear regression estimator. This implies that the proposed estimator t_{p42} at $\boldsymbol{\gamma = 0.5}$ attain optimum result without depending on the optimality condition.

Tables 7 Results (Class 2 estimators) at $\rho_{xy} = \mathbf{0.8}$

It is observed that the proposed estimators t_{pj}, j = 21,…,31 at $\boldsymbol{\gamma = 0.1}$ and $\boldsymbol{t_{p43}}$, at $\boldsymbol{\gamma = 0.8}$ are more efficient than the classical ratio estimator. The MSE of the Proposed estimator $\boldsymbol{t_{p43}}$, at $\boldsymbol{\gamma = 0.8}$ is almost equivalent to the MSE of the proposed generalized class of ratio estimator at t optimum.

General observation- It is observed that for a very low correlation (0.2), the proposed estimator $\boldsymbol{t_{p51}}$ at $\boldsymbol{\gamma = 0.5}$ with known coefficient of variation and highest decile of the auxiliary variables perform better than the existing and classical ratio estimator. The estimator is almost as good as the classical linear regression estimator. That is, $\boldsymbol{\gamma}$ provides an alternative to obtain more efficient estimators, even if t deviates from its optimum value. The proposed generalized class of ratio estimator at t optimum is equal to the variance of the classical linear regression estimator.

6. CONCLUSION

We have proposed a generalized class of difference-type ratio estimator for estimating the population mean of the variable of interest based on simple random sampling without replacement such that any suitable known population parameters of the auxiliary variable can be applied. Corrections were effected on Raja et al. (2017) ratio-type estimators and new members of the generalized class of ratio type estimators were identified. From the result obtained, it is observed that the biases and mean square errors of the proposed estimators are smaller than the existing ratio-type estimators considered in this study. Based on the summary results presented in section 5, we therefore conclude that the proposed estimators $\boldsymbol{t_{p51}}$ at $\boldsymbol{\gamma = 0.5}$ when $\rho_{xy} = \mathbf{0.2}$, $\boldsymbol{t_{p48}}$ at $\boldsymbol{\gamma =}$ 0.8 when $\rho_{xy} = \mathbf{0.4453}$, $\boldsymbol{t_{p42}}$ at $\boldsymbol{\gamma = 0.5}$ when $\rho_{xy} = \mathbf{0.65}$ and $\boldsymbol{t_{p43}}$ at $\boldsymbol{\gamma = 0.8}$ when $\rho_{xy} = \mathbf{0.8}$ are the most efficient estimators for this data set at their various level of correlation coefficient because they have greater improvement over the classical ratio estimator and all the existing estimators considered in this study. The proposed generalized class of difference-type ratio estimator $\boldsymbol{t_p}$ at t optimum performs better than all the existing estimators considered in this study and is suitable for all level of correlation data set. Therefore, we strongly recommend the proposed estimators over the existing ratio-type estimators for practical applications.

REFERENCES

1. Cochran, W.G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *Journal of Agricultural Sciences,* 59, 1225-1226.
2. Cochran, W.G. (1942). Sampling theory when the sampling units are unequal sizes. *Journal of American Statistics Association*, 37, 199-212.
3. Kadilar, C. and Cingi, H. (2004). Ratio estimators in simple random sampling. *Applied Mathematics and Computation*, 151(3), 893-902.
4. Kadilar, C. and Cingi, H. (2006). An improvement in estimating the population mean by using the correlation coefficient. *Hacettepe Journal of Mathematics and Statistics*, 35(1), 103-109.
5. Rao, T.J. (1991). On certain methods of improving ratio and regression estimators. *Communications in Statistics: Theory and Methods*, 20(10), 3325-3340.
6. Raja, T.A., Subair, M., Maqbool, S. and Hakak, A. (2017). Enhancing the mean ratio estimator for estimating population mean using conventional parameters. *International Journal of Mathematics and Statistics Invention,* 5(1), 58-61.
7. Reddy, N.V. (1973). On ratio and product methods of estimation. *Sankhya Series. B*(35), 307-316.
8. Robson, D.S. (1952). Multiple Sampling of Attributes. *Journal of the American Statistical Association,* 47, 203-215.
9. Singh, D. and Chaudhary, F.S. (1986). *Theory and analysis of sample survey designs*. (1st ed.), New Age International Publisher, India.
10. Sisodia, B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of the Indian Society of Agricultural Statistics,* 33(1), 13-18.
11. Singh, H.P. and Kakran, M.S. (1993). A modified ratio estimator using known coefficient of kurtosis of an auxiliary character. *Revised version submitted to Journal of Indian Society of Agricultural Statistics,* New Delhi, India.
12. Singh, H.P. and Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population means. *Statistics in Transition,* 6(4), 555-560.
13. Srivastava, S.K. (1967). An estimator using auxiliary information. *Calcutta Statist. Assoc. Bull.,* 16, 121-132.
14. Subramani, J. and Kumarapandiyan, G. (2013). Estimation of finite population mean using deciles of an auxiliary variable. *Statistics in Transition-new series, Spring 2013,* 14(1), 75-88.
15. Tripathi, T.P., Das, A.K. and Khare, B.B. (1994). Use of auxiliary information in sample surveys: A review. *Aligarh Journal of Statistics*, 14, 79-134.
16. Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical Journal,* 41(5), 627-636.
17. Yan, Z. and Tian, B. (2010). Ratio method to the mean estimation using coefficient of skewness of auxiliary variable. *International Conference on Information Computing and Application 2010, Part II, Communications in Computer and Information Science,* 106, 103-110.