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ON SOLUTION OF NONLINEAR MODELS: A HYBRID ALGORITHM OF ARTIFICIAL BEE COLONY ALGORITHM AND ARTIFICIAL SHOWERING ALGORITHM

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ABSTRACT

Solving nonlinear systems of equations is a typically difficult computation problem. Classical solvers like Newton-type methods suffer from convergence to a local optimum due to their high sensitivity to the initialization conditions. This study combines the global exploration capabilities of Artificial Bee Colony Algorithm and the high exploitation characteristics of Artificial Showering Algorithm to model a hybrid mechanism for solving systems of nonlinear models. The proposed method is applied to solve three nonlinear systems including the ''Chemical Equilibrium'' and "Geometry Size Thin Wall Girder Section" physical models. Statistical analysis of the results shows that the hybrid approach possesses a high success rate and a quick convergence rate.

KEYWORDS

Artificial Bee Colony algorithm; Artificial Showering Algorithm; Nonlinear Models.

1. INTRODUCTION

Systems involving nonlinear equations are encountered in diverse scientific fields such as chemistry, economics, mechanics, robotics and medicine etc. In the case when at least one member of the system lacks fine polynomial properties, the problem turns to be a nondeterministic polynomial-time hard. However, this problem has been attempted to solve by several approaches such as Jaberipour et al. (2011) used Particle Swarm Algorithm (PSO), Mo et al. (2009) and Luo et al. (2008) combined chaos search with conjugate direction method (CD) and Newton type methods respectively, Raja et al. (2016) hybridised PSO with Nelder Mead Simplex Algorithm and Abdollahi et al. (2013) designed Imperialist Competitive Algorithm (ICA). Certain limitations of the optimization techniques cause some obstacles in solving the nonlinear systems. For example, the convergence and effective performance of Newton-type methods are highly sensitive to the provided initial guess. For these reasons, it is inevitable to design an efficient method for solving nonlinear systems of equations.

In this work, a novel hybrid algorithm, called ABC-MASH, for solving systems of nonlinear equations is proposed. ABC-MASH is based on two nature inspired algorithms, namely, Artificial Bee Colony algorithm [Karaboga (2005)] and Artificial Showering Algorithm [Ali et al. (2015)].

The paper is organized as follows. The Section 2 contains the problem description; Section 3 presents the proposed ABC-MASHA. Parameter settings and details of physical models is provided in Section 4, Computational results and their statistical analyses are presented in Section 5, while concluding remarks are given in Section 6.

2. PROBLEM DESCRIPTION

The general form of the problem of solving nonlinear systems is:

$$Find\ x = (x_1, x_2, x_3, \dots, x_n) \in R^n\ such\ that\ f_j\,(\,x\,) = 0; 1 \le j \le n. \tag{1}$$

where, each f_j is a real valued function defined on $R^n$and each variable is restricted to lie within bounds given as:

$$l_i \le x_i \le u_i;\ 1 \le j \le n \tag{2}$$

The above problem involving (1) and the constraints defined by (2) is transformed to an optimization problem and hence the objective is to minimize the following fitness function:

$$(x) = \frac{1}{n}\sum_{j=1}^{n}\left[\,f_j\,(\,x\,)\right]^2 + M \times \max_i\{0, l_i - x_i, x_i - u_i\} \tag{3}$$

Here M is a sufficiently large positive real number. The objective function (2) has a global minimum value of zero. The global minimum solution of the optimization problem (3) is also the solution of (1) satisfying (2).

3. THE PROPOSED HYBRID ALGORITHM

In this section, formal details of ABC and ASHA are presented. The possible weaknesses of the algorithms are also highlighted which are main reasons and inspirations towards the current study.

3.1 Artificial Bee Colony Algorithm

ABC was proposed by Karaboga (2005, [4]) and is inspired from honey bee smart food foraging activities to discover better solutions in the search space during optimization process. Karaboga and Basturk (2007) showed that it is an efficient and a powerful global search optimizer. In ABC, a colony of NP bees is considered to search at SN $(=\frac{NP}{2})$ food sources which are randomly generated vectors $x_i = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ in the n-dimensional search space with j^{th} coordinate satisfying the following equation:

$$x_{ij} = l_j + rand(0,1) \times (u_j - l_j) \tag{4}$$

The simulated bees work in the form of three phases: (i) employed bees phase (ii) onlooker bees phase and (iii) scout bees phase. In the employed bees phase new food sources are searched by $\frac{NP}{2}$ bees and the information about these food sources is transferred to the rest of $\frac{NP}{2}$ onlooker bees. The governing search equation for i^{th} employed bee is:

$$v_{ij} = x_{ij} + \varphi_{ij} \times \left(x_{kj} - x_{ij}\right) \tag{5}$$

The subscripted k is the number of a neighbouring food source different from i and φ_{ij} is a random number from the interval (– 1, 1).

Onlooker bees, which are also $\frac{NP}{2}$ in number, select the food sources depending on the probability proportional to the fitness of food sources, and then exploit the same equation (5) for selected food sources. The fitness and probability of each food source are given by the equations (6) and (7) respectively.

$$fit(x_i) = \begin{cases} \dfrac{1}{1+f(x_i)} \ if\ f(x_i) \geq 0 \\ 1 + abs\big(f(x_i)\big)\ Otherwise \end{cases} \quad (6)$$

$$prob(x_i) = \frac{fit(x_i)}{1 + \sum_{i=1}^{SN} fit(x_i)} \quad (7)$$

Each employed bee is allowed to perform unsuccessful search for maximum number L times. As soon as this limit is abandoned by an employed bee, it is replaced by a scout bee. Each food source is analogous to a possible solution for an optimization problem, whereas the fitness of a food source resembles to the quality of the associated solution. Three phases are continued up to a predefined number of cycles or iterations.

3.2 Artificial Showering Algorithm (ASHA)

Artificial Showering Algorithm, proposed by Ali et al. (2015) is a recent nature inspired algorithm. It is inspired from phenomena of direct and indirect flow of water with a speed F from higher locations to the lower locations. The water units are fetched to the various positions, which are analogous to possible solutions, by overhead sprinklers. The ideas of direct and indirect flows are incorporated by the following equation:

$$x_i^{new} = x_i^{old} + F \times s * \left(x^{lower\ or\ lowest} - x_i^{old}\right) \quad (8)$$

Here F is a positive real number, s is a random number in the interval (0, 1) for direct flow and is an n-dimensional vector of random numbers in (0, 1), and $*$ denotes the special operation of scalar multiple for direct flow and element-wise product for indirect flow. The algorithm works by monitoring the flow of water units based on the probability of the path selection. The water units tend to hoard at the lowest landscape position and the surface resists the absorption. As the resistance level of the surface is surpassed, the water is infiltrated to juncture with the underground water which is again lifted and dispatched to showers at new locations in the field by overhead sprinklers. There are at least 6 control parameters involved in the search mechanism of ASHA. But for the purpose of hybridization only the search equation (8) is focused and all other details are omitted. Interested reader can easily access to the related article.

3.3 Modified Artificial Showering (MASH)

For the proposed hybrid method, first of all the search equation (8) of ASHA is modified to model a new showering mechanism by gun-type sprinklers. The gun-type are sprinklers are often used in irrigation and are capable of throwing water units in the form of thin beams and scattered showers. For each location x_i a different location x_k is selected and the new location of a water unit is monitored according to the following rule:

$$x_i^{new} = \begin{cases} x_i + s * (x_i - x_k) \; if \; f(x_i) \leq f(x_k) \\ x_i + F \times rand(0,1) \times (x_k - x_i) \; if \; f(x_i) > f(x_k) \end{cases} \quad (9)$$

The location x_i is updated to become x_i^{new} if $f(x_i^{new}) < f(x_i)$, otherwise x_i is retained. The value of F is allowed to vary randomly in the interval $[0, F_{max}]$. The larger value of F_{max} is aimed to converge directly along a possible decent direction whereas, its smaller value results in a search in a smaller neighbourhood of the current solution. It should be noted from relation (9) that the emphasis is always on improving the current solution under the effect of other locations. The resulting modified component of ASHA is named as Modified Artificial Showering (MASH). In order to avoid extensive efforts for parameter settings for ASHA, only MASH component is employed in the proposed hybrid algorithm.

3.4 Proposed Hybrid of ABC and MASH (ABC-MASH)

Two central mechanisms in a robust search process are exploration and exploitation. Exploration enables an algorithm in finding promising possible solutions whereas exploitation enhances the search process in the neighbourhood of better solutions [Akay and Karaboga (2012)]. It has been reported that ABC is excellent at exploration phase but very poor at the stage of exploitation along with its slow convergence speed some cases [Xiang (2013); Biswas et al. (2014)]. Keeping in view these drawbacks of ABC, a hybrid of ABC with MASH phase of ASHA is presented in this work. The developed hybrid method is named as Artificial Bee Colony algorithm based on Modified Showering phenomena (ABC-MASH). The steps of proposed ABC-MASH are presented in Figure 1.

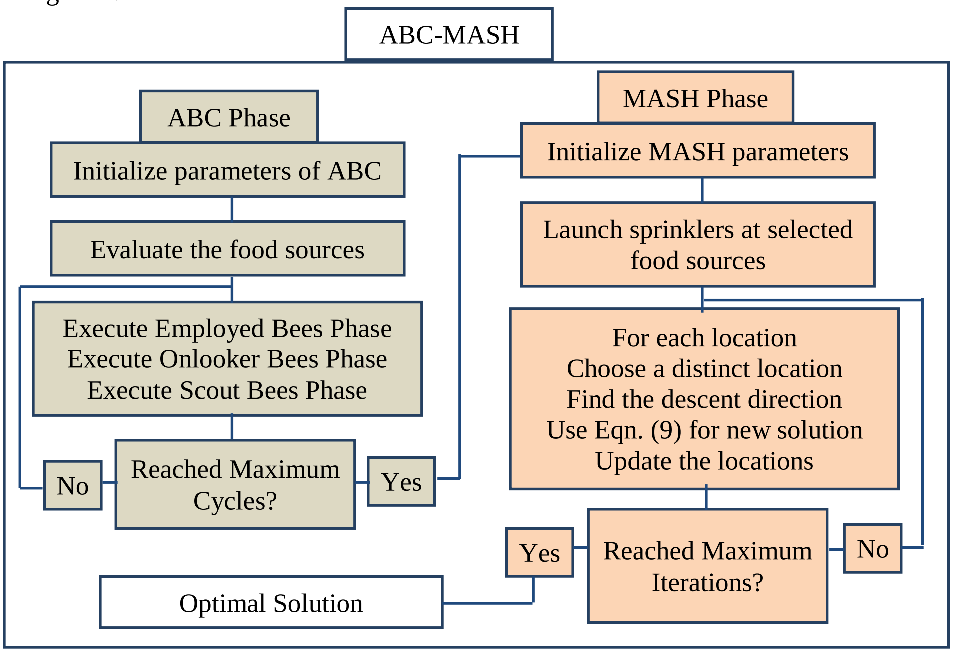

Figure 1: Flow Chart of ABC-MASH

4. PARAMETER SETTINGS AND THE DESCRIPTION OF NONLINEAR MODELS

The proposed ABC-MASH has been applied to solve three challenging nonlinear models, which are (i) Economic Modelling Problem (ii) Chemical Equilibrium Problem (iii) Geometry Size of Thin Wall Girder Section Problem. To evaluate the performance of ABC-MASH, the obtained results are compared with two variants of each of standard ABC and a Global-best Guided ABC (GABC) (Zhu and Kwong 2010). GABC uses a modified search equation given below:

$$v_{ij} = x_{ij} + \varphi_{ij} \times \left(x_{kj} - x_{ij}\right) + \psi_{ij} \times \left(x_j^G - x_{ij}\right) \qquad (10)$$

Here, x_j^G is the j^{th} coordinate of the global best solution found so far, ψ_{ij} is a random number in (0, 1).

The two variants of each of competing algorithms are aimed to test the efficiency of ABC-MASH against the highly potential settings of these algorithms. The variants of ABC and GABC are labelled based on the following parameters settings:

ABC1: NP (*population size*) = 100, L (*limit*) = 100, Cycles (*maximum number of iterations*) = 1000; ABC2: NP = 150, L = 100, Cycles = 1000; GABC1: NP = 100, L = 100, Cycles = 1000; GABC2: NP = 150, L = 100, Cycles = 1000.

With these settings each of the variant utilizes a minimum of 1000000 function evaluations. On the other hand, the parameters of ABC-MASH are set in such a way that it utilizes a maximum of 75000 function evaluations. The parameters for ABC-MASH are defined as under:

ABC-MASH: ABC (NP = 100, L = 100, Cycles = 500); MASH (NP = 50, Iteration = 500, $F_{max} = 10$).

4.1 Economic Modelling Problem.

The general econometric modelling problem has been studied by Grosan and Abraham (2008), and Morgan (1987). Mathematical form of the 5-dimensional problem can be stated as [Raja et al. (2016)]:

$$\begin{cases} f_1(x) = x_1 + x_2 + x_3 + x_4 - 1 = 0 \\ f_2(x) = x_1x_5 + x_1x_2x_5 + x_2x_3x_5 + x_3x_4x_5 - 1 = 0 \\ f_3(x) = x_2x_5 + x_1x_3x_5 + x_2x_4x_5 - 1 = 0 \\ f_4(x) = x_3x_5 + x_1x_4x_5 - 1 = 0 \\ f_5(x) = x_4x_5 - 1 = 0 \end{cases} \qquad (11)$$

The associated objective function is:

$$f_{EM}(x) = \frac{1}{5}\sum_{i=1}^{5}[f_i(x)]^2$$

4.2 Chemical Equilibrium Problem

Chemical equilibrium problem involves five nonlinear equations in five unknowns and is described as [Meintjes and Morgan (1990)]:

$$\begin{cases} f_1(x) = x_1x_2 + x_1 - 3x_5 = 0 \\ f_2(x) = 2x_1x_2 + x_1 + x_2x_3^2 + r_8x_2 - r\,x_5 + 2\,r_{10}x_2^2 + r_7x_2x_3 + r_9x_4x_2 = 0 \\ f_3(x) = 2x_2x_3^2 + r_7x_2x_3 + 2r_5x_3^2 + r_6x_3 - 8x_5 = 0 \\ f_4(x) = r_9x_2x_4 + 2x_4^2 + 4\,rx_5 = 0 \\ f_5(x) = x_1x_2 + x_1 + r_{10}x_2^2 + x_2x_3^2 + r_7x_2x_3 + r\,x_2x_4 + r_8x_2 + r_5x_3^2 + r_6x_3 + x_4^2 - 1 = 0 \end{cases} \quad (14)$$

where, r_8= 0.000000449; r = 10; r_{10}= 0.000000961; r_7= 0.000545; r_9= 0.0000340; r_5= 0.193; r_6= 0.000410; r_7= 0.000545. The objective function related to chemical equilibrium problem constructed as:

$$f_{CE}(x) = \frac{1}{5}\sum_{i=1}^{5}[f_i(x)]^2$$

4.3 Geometry Size of Thin Wall Girder Section Problem

Physically meaningful Geometry Size of Thin Wall Girder Section Problem can be described as [Lou et al. (2008)]:

$$\begin{cases} f_1(b,h,t) = bh - (b-2t)(h-2t) - 165 = 0 \\ f_2(b,h,t) = \frac{bh^3}{12} - \frac{(b-2t)(h-2t)^3}{12} - 9369 = 0 \\ f_1(b,h,t) = \frac{2(b-t)^2(h-t)^2t}{h+b-2t} - 6835 = 0 \end{cases} \quad (12)$$

Physical constraints on the design variables b, h and t are:

$$\begin{cases} g_1(b,h,t) = b - h < 0 \\ g_2(b,h,t) = t - b < 0 \\ g_3(b,h,t) = t < 0 \end{cases} \quad (13)$$

The associated objective function is constructed based on penalty function approach [Chaudhry et al. (2009)] and is given by:

$$f_{GS}(x) = \frac{1}{3}\sum_{i=1}^{3}[f_i(b,h,t)]^2 + M \times \max\{0, g_1(b,h,t), g_2(b,h,t), g_3(b,h,t)\}.$$

Here M is a sufficiently large real number and is called penalty factor.

5. ANALYSIS OF NUMERICAL RESULTS

Results of the proposed ABC-MASH and variants of ABC and GABC to find approximate solutions of systems of nonlinear models for three considered problems are present in this section. To check the effectiveness of the developed approach in comparison with those of the other four variants, each algorithm is simulated for 31 times on each problem. The obtained results are exhibited in Table 1 in terms of best, mean, median, worst and standard deviation. To check the computational efficiency aggregated CPU times for each algorithm on each problem are reported in the table.

From the Table 1 and Figures 1-3 one can easily conclude that the proposed ABC-MASH outperforms the original ABC and its modified version GABC which was acclaimed to be superior to ABC. It is worth mentioning that all the considered variants were allowed to consume significantly high computational costs in order to fully expose their efficiencies. For a clear conclusion about the performances of all the algorithms, the measures focused for detailed analysis are (i) the best solution found (ii) the mean solution (iii) standard deviation and (iv) the CPU time.

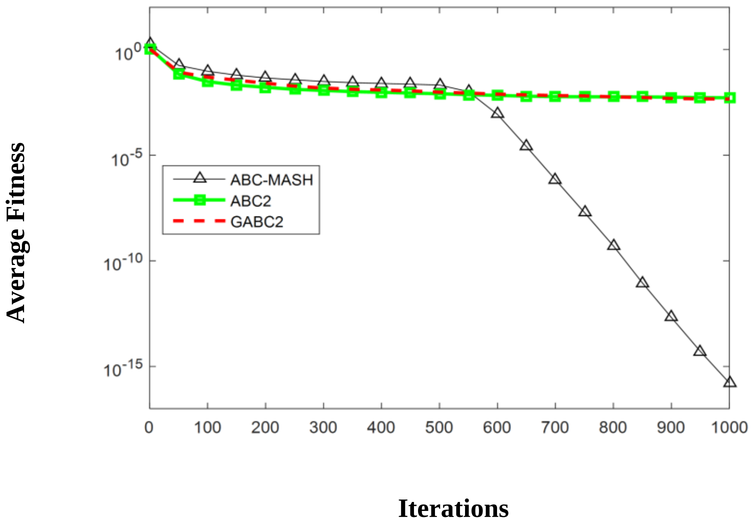

Figure 2: Average Convergence Curves of ABC2, GABC2 and ABC-MASH on Economic Modelling Problem

Table 1
Statistical Results of ABC-MASH and Variants of ABC and GABC on Three Benchmarks

Model	Performance Measure	ABC1	ABC2	GABC1	GABC2	Proposed ABC-MASH
f_{EM}	Best	0.002269015	0.00118973	0.0028915	0.001267795	3.84593E-17
	Mean	0.0065726^{5}	0.0052515^{3}	0.0057262^{4}	0.0045747^{2}	1.658E-16^{1}
	Median	0.0065579	0.0057699	0.0055676	0.004111	1.4218 E -16
	Worst	0.010312	0.0075661	0.0089928	0.0078411	8.7925 E -16
	Std.	0.0023313	0.0017807	0.0017871	0.0019264	1.5504 E -16
	CPU Time	8.00212^{2}	11.9836	8.07994^{3}	12.1804	4.98392^{1}
f_{CE}	Best	1.4366e-10	8.4862e-10	3.721e-09	6.4829e-09	5.2033 E-18
	Mean	4.1915e-09^{3}	3.1199e-09^{2}	1.294e-07^{5}	8.2251e-08^{4}	2.1602 E-17^{1}
	Median	4.284e-09	2.7911e-09	1.2033e-07	8.2701e-08	2.3806 E-17
	Worst	9.4142e-09	6.8031e-09	2.7036e-07	1.7906e-07	3.3142 E-17
	Std.	2.4756e-09	1.7933e-09	7.2534e-08	5.1844e-08	8.1569 E-18
	CPU Time	5.2953^{2}	7.8782	5.8669^{3}	7.989	2.9663^{1}
f_{GS}	Best	48.9855	18.6334	9.32468	4.25397	4.22406 E-19
	Mean	355.3181^{5}	239.9425^{4}	69.00217^{3}	49.54725^{2}	5.6671 E-18^{1}
	Median	390.9069	227.8435	61.27662	41.10099	4.65991 E-18
	Worst	660.0262	459.7189	170.2906	110.1735	1.37581 E-17
	Std.	160.1559	133.5222	40.38766	34.80927	3.67464 E-18
	CPU Time	4.2595^{3}	6.3695	4.2276^{2}	6.4577	2.3824^{1}
Overall Ranks	Mean Value	2.6	1.8	2.4	1.6	1
	CPU Times	2.33		2.67		1

It is evident from Table 1 that the best, mean and worst values of Economic Modelling Problem found by the proposed ABC-MASH are extremely small as compared to those of ABC1, ABC2, GABC1 and GABC2. The convergence curves in Figure 2 expose that as soon as the MASH phase is initiated the convergence speed to the minimum rapidly rise. The mean value, standard deviation and the convergence prove that the developed method possesses unmatchable performance in solving Economic Modelling Problem.

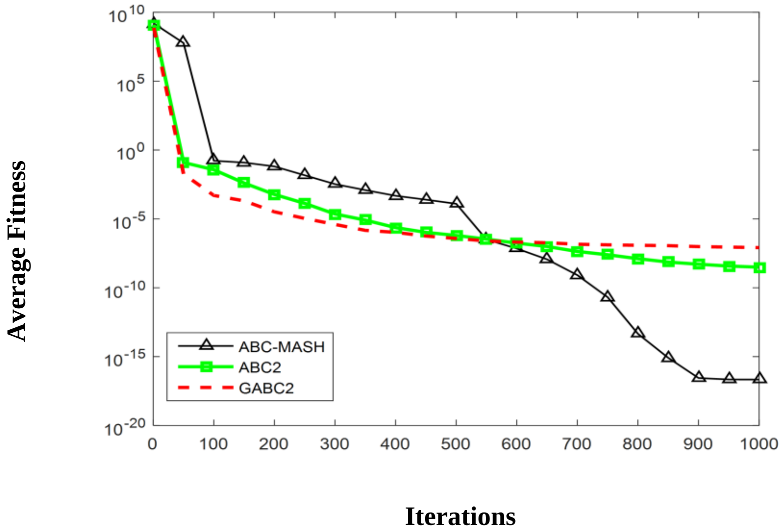

Figure 3: Average Convergence Curves of ABC2, GABC2 and ABC-MASH on Chemical Equilibrium Problem

For Chemical Equilibrium Problem, the worst minimum value found by the developed hybrid in this study is even better than the best minimum values found by four competing approaches. However Figure 3 describes that the variants of ABC find acceptable solutions but rest around their mean solutions for iterations 500 to 1000. This evidently demonstrates the weaknesses of original ABC in its exploitation part. On the other hand, the MASH phase increases the exploitation capability of the algorithm during the assigned span of iterations.

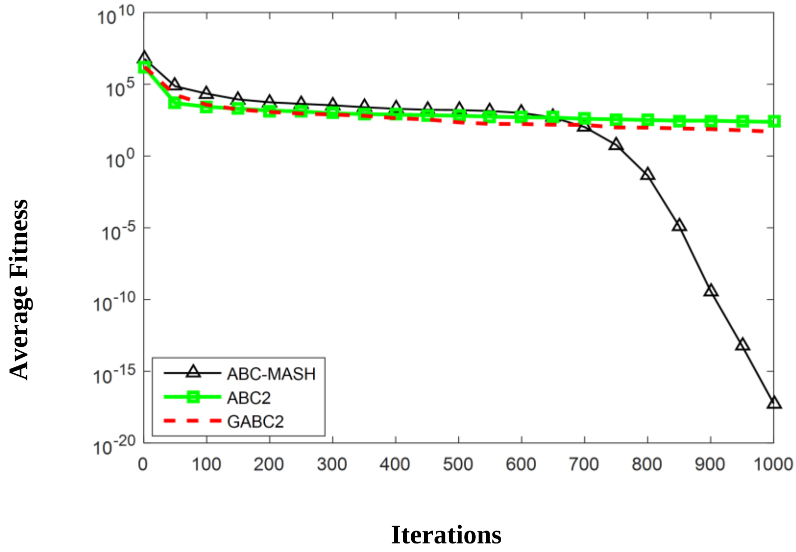

Figure 4: Average Convergence Curves of ABC2, GABC2 and ABC-MASH on Girder Section Problem

It can be noticed from relevant performances presented in Table 1 and the convergence curves in Figure 4 that ABC, GABC and their variants were unable to detect acceptable solutions for the Girder Section problem. On the other hand the proposed ABC-MASH consistently found the high quality solution for the problem.

To evaluate the overall performances of all the considered problems, Wilcoxon Ranks of the mean solutions found by all the algorithms and the computational times of ABC1, GABC1 and ABC-MASH. Overall ranks clearly witness that the proposed ABC-MASH significantly outperforms other approaches.

6. CONCLUSION

In this study, a hybrid algorithm based on a nature inspired algorithm (ABC) and a modified artificial showering (MASH) phenomenon is presented. The proposed method efficiently solves physical nonlinear models in smaller computational times and at smaller computational cost. The proposed ABC-MASH outperforms its competitors in the sense of solution quality, computational efficiency, consistency and speed of convergence.

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