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CODING THEOREMS ON NEW ADDITIVE INFORMATION MEASURE OF ORDER α

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ABSTRACT

In this article we develop a new additive information measure of order α and a new average code-word length and develop the noiseless coding theorems for discrete channel. Also we show that the measures defined in this communication are the generalizations of some well-known measures in the subject of coding and information theory. The results obtained in this article are verified by considering Huffman and Shannon-Fano coding schemes by taking an empirical data. The important properties of the new information measure have also been studied.

KEYWORDS

Shannon's entropy, Mean code-word length, Kraft's inequality, Holder's inequality, Huffman codes, Shannon-Fano codes, Noiseless coding theorem.

AMS Classification 94A17, 94A24

1. INTRODUCTION

The theory of Communication is the early work of Hartley (1928) on the mathematics of information transmission that is recognized by Fisher (1925), which is closely related to Shannon's (1948) entropy. What follows is not intended as a general introduction to information theory through two outstanding contributions to the mathematical theory of communications in 1948 and 1949. Despite several hasty generalization which produces thousands research papers, see for instance the papers Havrda and Charvat (1967), Tsallis (1988), one thing became evident; this scientific theory has stimulated the interest of thousands of scientists around the world. Shannon (1948) introduced the following measure of information and call it as entropy

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \qquad (1.1)$$

Let $p_1, p_2, p_{3,\dots,} p_n$ be the probabilities of n code words to be communicated and let their lengths $l_1, l_2, \dots, l_n$ satisfies the Kraft's (1949) inequality,

$$\sum_{i=1}^{n} D^{-l_i} \le 1 \qquad (1.2)$$

where D is the size of code alphabet.

For uniquely decodable codes, Shannon (1948) provided his noiseless coding theorem, that for all codes satisfying Kraft's inequality (1.2), the minimum value of the mean code-word length,

$$L = \sum_{i=1}^{n} p_i l_i \tag{1.3}$$

lies between $H(P)$ and $H(P) + 1$, where $H(P)$ is Shannon's entropy (1948) defined in (1.1).

Campbell (1965) considered the more general exponentiated mean code word length as

$$L_\alpha = \frac{\alpha}{1-\alpha} \log_D \left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right], \alpha > 0, \alpha \neq 1 \tag{1.4}$$

and showed that subject to (1.2), the minimum value of (1.4) lies between $R_\alpha(P)$ and $R_\alpha(P) + 1$, Where

$$R_\alpha(P) = \frac{1}{1-\alpha} \log_D [\sum_{i=1}^{n} p_i^\alpha] , \alpha > 0, \alpha \neq 1 \tag{1.5}$$

is Renyi's (1961) entropy.

In the last few decades researchers develop various generalized noiseless coding theorems for discrete channel under the condition of uniquely decipherability by taking different generalized information measures, Nath (1968), inaccuracy and coding theory, Longo (1976), also develop noiseless coding theorems for useful mean code-word length in terms of weighted entropy given by Belis and Guiasu (1968), Guiasu and Picard (1971), Gurdial (1977), extended the noiseless coding theorem for useful mean code-word length of order α, also various authors like Jain and Tuteja (1989), Taneja et al. (1985), Bhatia (1995), Hooda and Bhaker (1997), Khan et al. (2005), Bhat and Baig (2016a; 2016b 2016c; 2017a; 2017b, 2018), also develop various generalized coding theorems under the condition of uniquely decipherability.

In this research article we present another new additive information measure of order α and a new average code-word length and characterize these measures in different aspects.

2. NEW ADDITIVE INFORMATION MEASURE OF ORDER A AND ITS CODING THEOREMS

Define a new information measure of order α as:

$$H^\alpha(P) = \frac{1}{1-\alpha} \sum_{i=1}^{n} p_i^\alpha \tag{2.1}$$

where, $0 < \alpha < 1, p_i \geq 0 \ \forall i = 1,2, \dots, n , \sum_{i=1}^{n} p_i = 1$.

Various interpretations to α can be given. The following is suitable from an application point of view. If we consider the ensemble of events x_i with respective probabilities p_i as a cybernetic system $[x_i, p_i]$, then one can interpret the parameter α as flexibility parameter or as a preassigned number associated with different cybernetic systems. For instance, two cybernetic systems, with the same set of x_i, p_i may have different informations (with respect to the same goal) for different values of α. The parameter α may represent the environment factors, such as temperature, humidity etc.

Moreover, there are many factors like temperature, humidity etc. which affect the diversity in cost. Let α represent such factors upon which the information regarding such a cybernetic system $[x_i, p_i]$, depends.

Remarks for (2.1):

I. When $\alpha \to 1$, (2.1) becomes Shannon's (1948) entropy, i.e.,

$$H(P) = -\sum_{i=1}^{n} p_i log p_i$$

II. When $\alpha \to 1$, and $p_i = \frac{1}{n} \forall i = 1,2, \ldots, n$, then (2.1) reduces to maximum entropy. i.e.,

$$H\left(\frac{1}{n}\right) = \log n$$

Further, we present a new generalized average code-word length of order α as:

$$L^{\alpha}(\mathrm{P}) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha}, 0 < \alpha < 1, p_i \geq 0 \ \forall i = 1,2, \ldots, n, \sum_{i=1}^{n} p_i = 1. \quad (2.2)$$

where, D is the size of code alphabet.

Remarks for (2.2):

I. When $\alpha \to 1$, (3.2) coincides with the optimal code-word length corresponding to Shannon's (1948) entropy. i.e., $L = \sum_{i=1}^{n} p_i l_i$
II. When $\alpha \to 1$, and $l_1 = l_2 = \cdots = l_n = 1$, then (3.2) reduces to 1. i.e., $L = 1$

Now we find the lower and upper bound of new generalized average code-word length defined in (2.2) in terms of new generalized information measure defined in (2.1) under the condition

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \quad (2.3)$$

This is Kraft's (1949) inequality.

Theorem 1:

For all integers $(D > 1)$, if the code-word lengths $l_1, l_2, \ldots, l_n$ satisfy the Kraft's inequality defined in (2.3), then the new generalized average code-word length defined in (2.2) satisfies the inequality

$$L^{\alpha}(P) \geq H^{\alpha}(P), 0 < \alpha < 1. \quad (2.4)$$

where equality holds good iff

$$l_i = -\log_D\left[\frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}}\right] \quad (2.5)$$

Proof:

We know that for all $x_i, y_i > 0, i = 1, 2, 3, \ldots, n$ and $\frac{1}{\gamma} + \frac{1}{\delta} = 1, \gamma < 1(\neq 0), \delta < 0$ or $\delta < 1(\neq 0), \gamma < 0$, then the Holder's inequality

$$\left(\sum_{i=1}^{n} x_i^{\gamma}\right)^{\frac{1}{\gamma}} \left(\sum_{i=1}^{n} y_i^{\delta}\right)^{\frac{1}{\delta}} \leq \sum_{i=1}^{n} x_i y_i \quad (2.6)$$

holds, and equality holds in (2.6) iff there exists a positive constant μ such that

$$x_i^{\gamma} = \mu y_i^{\delta} \tag{2.7}$$

Let's take the following substitution,

$$x_i = p_i^{\frac{\alpha}{\alpha-1}} D^{-l_i}, y_i = p_i^{\frac{\alpha}{1-\alpha}}, \gamma = \frac{\alpha-1}{\alpha}, \text{ and } \delta = 1-\alpha$$

Using the above values in the inequality (2.6), we get

$$\sum_{i=1}^{n} D^{-l_i} \geq \left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{2.8}$$

Now using the inequality (2.3) we get,

$$\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1}{1-\alpha}} \leq 1. \tag{2.9}$$

or equivalently, the inequality (2.9) can be written as

$$\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \leq \left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1}{\alpha-1}} \tag{2.10}$$

As $0 < \alpha < 1$, then $(\alpha - 1) < 0$, raising the power $(\alpha - 1) < 0$, to the inequality (2.10), we get

$$\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \left[\sum_{i=1}^{n} p_i^{\alpha}\right] \tag{2.11}$$

As $0 < \alpha < 1$, then $(1 - \alpha) > 0$ and $\frac{1}{(1-\alpha)} > 0$, now multiply inequality (2.11) both sides by $\frac{1}{(1-\alpha)} > 0$, we get

$$\frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha}\right] \tag{2.12}$$

or equivalently we can write (2.12) as

$L^{\alpha}(P) \geq H^{\alpha}(P)$, hence the result for $0 < \alpha < 1$.

Now we will see that the equality in (2.4) is satisfied if and only if

$$l_i = -\log_D \left[\frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}}\right]$$

or equivalently the above equation can be written as

$$D^{-l_i} = \frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}} \tag{2.13}$$

Raising to the power $\left(\frac{\alpha-1}{\alpha}\right)$, throughout the equation (2.13) and by suitable simplification we get

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} = p_i^{(\alpha-1)}\left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1-\alpha}{\alpha}} \tag{2.14}$$

Multiply equation (2.14) both sides by p_i then taking sum over $i = 1,2, \ldots, n$, and by suitable simplification, we get

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} p_i D^{-l_{\mathrm{i}}\left(\frac{\alpha-1}{\alpha}\right)} = \left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1}{\alpha}} \tag{2.15}$$

Raising to the power α both sides to equation (2.15), then multiply both sides by $\frac{1}{1-\alpha}$, we get

$L^{\alpha}(P) = H^{\alpha}(P)$, Hence the result.

Theorem 2:

If for every code with lengths $l_1, l_2, \dots, l_n$satisfies Kraft's inequality defined in (2.3), then the new generalized average code-word length $L^{\alpha}(P)$ defined in (2.2) satisfy the inequality

$$L^{\alpha}(P) < H^{\alpha}(P) D^{(1-\alpha)}, 0 < \alpha < 1. \tag{2.16}$$

Proof:

From the theorem 1 we have,

$$L^{\alpha}(P) = H^{\alpha}(P)$$

Holds if and only if

$$D^{-l_i} = \frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}}, 0 < \alpha < 1.$$

or, equivalently, the above equation can be written as

$$l_i = -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right],$$

We choose the code-word lengths $l_i, i = 1,2, \dots, n$ in such a manner that they satisfy the inequality

$$-\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] \le l_i < -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] + 1 \tag{2.17}$$

Consider the interval

$$\delta_i = \left[-\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right], -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] + 1\right]$$

of length unity. In every interval δ_i, there lies exactly one positive integer l_i, such that, the following inequality holds

$$0 < -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] \le l_i < -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] + 1 \tag{2.18}$$

Now we will first see that the defined sequence $l_1, l_2, \dots, l_n$, of code-word lengths satisfies the Kraft's (1949) inequality.

The left side of the inequality (2.18), gives

$$-\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] \le l_i$$

or, equivalently the above expression can be written as

$$D^{-l_i} \le \frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}} \tag{2.19}$$

Taking summation over $i = 1,2, \dots, n$, on both sides to the inequality (2.9), we get,

$$\sum_{i=1}^{n} D^{-l_i} \leq 1,$$

which is Kraft's (1949) inequality.

The last inequality of (2.18), gives

$$l_i < -\log_D p_i^{\alpha} + \log_D\left[\sum_{i=1}^{n} p_i^{\alpha}\right] + 1$$

or equivalently the above can be written as

$$D^{l_i} < \left[\frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}}\right]^{-1} D \tag{2.20}$$

As $0 < \alpha < 1$, then $(1-\alpha) > 0$, and $\left(\frac{1-\alpha}{\alpha}\right) > 0$, raise throughout to the power $\left(\frac{1-\alpha}{\alpha}\right) > 0$, to the inequality (2.20), we get

$$D^{l_i\left(\frac{1-\alpha}{\alpha}\right)} < \left[\frac{p_i^{\alpha}}{\sum_{i=1}^{n} p_i^{\alpha}}\right]^{\frac{\alpha-1}{\alpha}} D^{\frac{1-\alpha}{\alpha}}$$

or, equivalently the above expression can be written as

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < p_i^{(\alpha-1)}\left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1-\alpha}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \tag{2.21}$$

Multiply the inequality (2.21) throughout by p_i, then taking sum over $i = 1,2,\dots,n$, throughout to the resulted expression and after simplification, we get

$$\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < \left[\sum_{i=1}^{n} p_i^{\alpha}\right]^{\frac{1}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \tag{2.22}$$

As $0 < \alpha < 1$, then $(1-\alpha) > 0$ and $\left(\frac{1}{1-\alpha}\right) > 0$, raise to the power α throughout to the inequality (2.22), then multiply the resulted expression throughout by $\left(\frac{1}{1-\alpha}\right) > 0$, we get

$$\frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} < \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha}\right] D^{(1-\alpha)}$$

or equivalently the above can be written as

$L^{\alpha}(P) < H^{\alpha}(P) D^{(1-\alpha)}$, Hence the result for $0 < \alpha < 1$.

Thus from above two coding theorems we have shown that

$H^{\alpha}(P) \leq L^{\alpha}(P) < H^{\alpha}(P) D^{(1-\alpha)}$, Where $0 < \alpha < 1$.

3. ILLUSTRATION

In this section we show the validity of the theorems 1 and 2 by taking an empirical data as given in table 1 and table 2.

By taking Huffman coding scheme into consideration the different values of $H^{\alpha}(P)$, $H^{\alpha}(P)D^{(1-\alpha)}$, $L^{\alpha}(P)$ and η for various values of α are shown in the table 1 as:

Table 1
Values of $H^{\alpha}(P)$, $L^{\alpha}(P)$, $H^{\alpha}(P)D^{(1-\alpha)}$ and η for different values of α using Huffman coding scheme, here D=2 in this case, as we use here binary code

p_i	*Huffman Code-words*	l_i	α	$H^{\alpha}(P)$	$L^{\alpha}(P)$	$\eta = \frac{H^{\alpha}(P)}{L^{\alpha}(P)} \times 100$	$H^{\alpha}(P)D^{(1-\alpha)}$
0.41	1	1	0.9	11.711	11.776	99.5320%	12.551
0.18	000	3	0.8	6.886	6.993	98.469%	7.909
0.15	001	3					
0.13	010	3					
0.1	0110	4					
0.03	0111	4					

Now we take Shannon-Fano coding scheme into consideration the different values of $H^{\alpha}(P)$, $H^{\alpha}(P)D^{(1-\alpha)}$, $L^{\alpha}(P)$ and η for various values of α are shown in the table 2 as:

Table 2
Values of $H^{\alpha}(P)$, $L^{\alpha}(P)$, $H^{\alpha}(P)D^{(1-\alpha)}$ and η for different values of α using Shannon-Fano coding scheme, here D=2 in this case, as we use here binary code

p_i	*Shannon-Fano Code-words*	l_i	α	$H^{\alpha}(P)$	$L^{\alpha}(P)$	$\eta = \frac{H^{\alpha}(P)}{L^{\alpha}(P)} \times 100$	$H^{\alpha}(P)D^{(1-\alpha)}$
0.41	00	2	0.9	11.711	11.825	99.032%	12.551
0.18	01	2	0.8	6.886	7.003	98.329%	7.909
0.15	100	3					
0.13	101	3					
0.1	110	3					
0.03	111	3					

From the tables 1 and 2 we can infer the following results:

I. Using Shannon-Fano coding and Huffman coding schemes theorems 1 and 2 holds in both the cases i.e.

$$H^{\alpha}(P) \leq L^{\alpha}(P) < H^{\alpha}(P)D^{(1-\alpha)}, \text{ where } 0 < \alpha < 1.$$

II. Using the above two coding schemes of Huffman and Shannon-Fano we see that our new generalized mean code-word length has less code-word length in case of Huffman coding scheme as compared to using Shannon-Fano coding scheme

III. Using the above two coding schemes of Huffman and Shannon-Fano we see that the efficiency (η) of our generalized mean code-word length is greater in case of Huffman coding scheme as compared to using Shannon-Fano coding scheme, so we conclude that Huffman coding scheme is more efficient than Shannon-Fano coding scheme.

In the next section, the important properties of our new generalized information measure of order α have been studied.

4. VARIOUS PROPERTIES OF OUR NEW GENERALIZED INFORMATION MEASURE $H^{\alpha}(P)$:

Here we discuss some important properties of our new generalized information measure $H_{\alpha}^{\beta}(P)$ defined in (2.1)

Property 1:

$H^{\alpha}(P)$ is non-negative, for given values of α.

Proof:

From the equation (2.1), we have

$$H^{\alpha}(P) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha}\right], 0 < \alpha < 1.$$

It is easy to see that for given values of α, $\sum_{i=1}^{n} p_i^{\alpha} \geq 1$, and then $\frac{1}{1-\alpha} > 0$, therefore we conclude that $\frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha}\right] \geq 0$. Also we see from the tables 1 and 2, for various values of α in the defined range the values of $H^{\alpha}(P)$ are non-negative. Hence, $H^{\alpha}(P)$ is non-negative, for given values of α.

Property 2:

$H^{\alpha}(P)$ is a symmetric function on every $p_i, i = 1, 2, 3, \dots, n$.

Proof:

It is obvious that $H^{\alpha}(P)$ is a symmetric function on every $p_i, i = 1, 2, 3, \dots, n$. i.e.,

$$H^{\alpha}(p_1, p_2, \dots, p_{n-1}, p_n) = H^{\alpha}(p_n, p_1, p_2, \dots, p_{n-1})$$

Property 3:

$H_{\alpha}^{\beta}(P)$ attains its maximum value when $\alpha \to 1$ and all the events are equally likely.

Proof:

Let $p_i = \frac{1}{n} \forall i = 1, 2, \dots, n$ and $\alpha \to 1$, then $H^{\alpha}(P) = \log n$, which is maximum entropy.

Property 4:

For $\alpha \to 1$, $H^{\alpha}(P)$ is a concave function for $p_1, p_2, \dots, p_n$.

Proof:

From the equation (2.1) we have

$$H^{\alpha}(P) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha}\right], 0 < \alpha < 1.$$

$$\lim_{\alpha \to 1} H^{\alpha}(P) = -\sum_{i=1}^{n} p_i \log p_i$$

Now differentiate partially above equation with respect p_i, we get

$$\left[\frac{\partial}{\partial p_i}\left(\lim_{\alpha \to 1} H^{\alpha}(P)\right)\right] = -1 - \log p_i$$

And the second derivative is given as

$$\left[\frac{\partial^2}{\partial p_i^2}\left(\lim_{\alpha \to 1} H^{\alpha}(P)\right)\right] = -\left(\frac{1}{p_i}\right) < 0.$$ For all $p_i \epsilon [0,1]$ and $i = 1, 2, \dots, n$.

Since the second derivative of $\lim_{\alpha \to 1} H^{\alpha}(P)$ with respect to p_i is negative on given interval $p_i \epsilon [0,1], i = 1, 2, \dots, n,$ therefore,

$H^{\alpha}(P)$ is a concave function for $p_1, p_2, \dots, p_n$.

CONCLUSION

In this communication we present a new information measure of order α and a new average code-word length and develop the noiseless coding theorems for discrete channel. Also we show that the measures defined in this communication are the generalizations of some well-known measures in the subject of coding and information theory. The coding theorems for discrete channel proved in this article are verified by taking an empirical data and see that our generalized mean code-word length has less code word length in case of Huffman coding scheme as compared to using Shannon-Fano coding scheme and conclude that Huffman coding is more efficient than Shannon-Fano coding scheme. The important properties of our new generalized information have also been discussed.

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