Pak. J. Statist.
2018 Vol. 34(5), 397-423

CLASSICAL AND BAYESIAN ESTIMATION OF SCALE PARAMETER IN WEIGHTED NAKAGAMI DISTRIBUTION USING REAL LIFE DATA SETS

Sofi Mudasir[§] and **S.P. Ahmad**
Department of Statistics, University of Kashmir, Srinagar, India
[§]Corresponding author Email: sofimudasir3806@gmail.com

ABSTRACT

The theory of weighted distributions introduced and unified by Fisher (1934) and Rao (1965) gives a consolidated approach of dealing with model specification and data interpretation problems. In this manuscript, we estimate the scale parameter of weighted Nakagami distribution (WND) through classical and Bayesian methods of estimation. The posterior distributions are derived by considering different types of priors. Also the posterior risks and posterior estimates are calculated under the combination of different priors and loss functions. For analysis, we use R-software.

KEYWORDS

Weighted Nakagami distribution, loss functions, priors, real data sets and R-software.

1. INTRODUCTION

The theory of weighted distributions introduced by Fisher (1934) and Rao (1965) gives a consolidated approach for the model specification and data interpretation problems. Fisher (1934) studied how the methods of ascertainment can influence the form of distribution of recorded observations. Then Rao (1965) introduced and formulated it in general terms in connection with modeling statistical data when the usual practice of using standard distributions were found to be unsuitable. Weighted distributions play an important role in research related to reliability, bio-medicine, ecology and several other areas. There are number of authors which have presented the important results on weighted distribution, among them are Priyadarshani (2011) introduced a new class of weighted generalized gamma distribution and related distributions, Das and Roy (2011) discussed the length biased weighted generalized Rayleigh distribution with its properties, Kanchan Jain (2014) introduced the weighted version of gamma distribution, Monsef and Ghoneim (2015) proposed weighted Kumaraswamy distribution for modeling some biological data, Mudasir et al. (2016) estimate the scale parameter of length biased Nakagami distribution, Sofi Mudasir and S.P. Ahmad (2017) introduced the weighted version of Erlang distribution and study its structural properties, Kawsar Fatima and S.P. Ahmad (2017) studied the various properties of weighted inverse Rayleigh distribution, Sofi Mudasir and S.P. Ahmad (2017) proposed the weighted version of Nakagami distribution and find its application to real data using R-software.

The random variable X follows weighted Nakagami distribution if its probability density function is given by

$$f_w(x;\omega,\delta,\theta)=\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)\omega^{\delta+\frac{\theta}{2}}}\,x^{2\delta+\theta-1}\exp\left\{-\left(\frac{\delta x^2}{\omega}\right)\right\},\quad \text{for } x>0;\omega,\delta,\theta>0. \quad (1)$$

where ω, δ and θ are the scale, shape and weighted parameters respectively.

The cumulative distribution function corresponding to (1) is given by

$$F(x;\omega,\delta,\theta)=\frac{1}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\gamma\left(\delta+\frac{\theta}{2},\frac{\delta x^2}{\omega}\right).$$

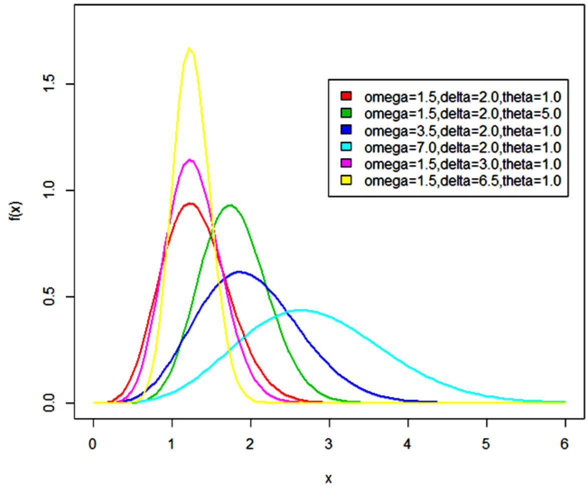

Figure 1: Probability Distribution Function of Weighted Nakagami Distribution

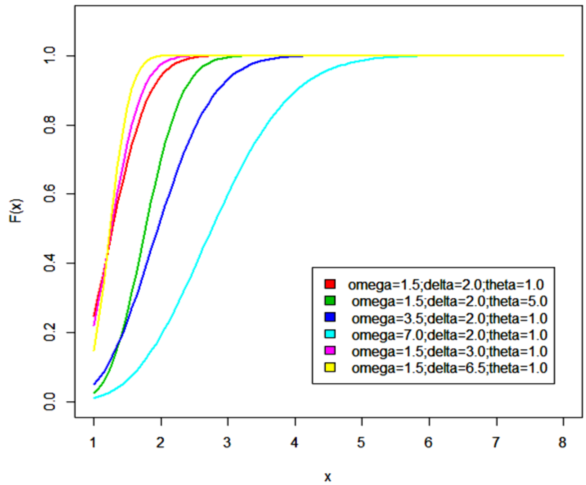

Figure 2: Cumulative Distribution Function of Weighted Nakagami Distribution

2. ESTIMATION OF SCALE PARAMETER

In this section, we estimate the scale parameter of the WND by using maximum likelihood and Bayesian methods of estimation.

2.1 Maximum Likelihood Estimation

Theorem 1:

Let $x_1, x_2, ..., x_n$ be a random sample of size n from WND having pdf given in (1), then the maximum likelihood estimator of the scale parameter ω is given by

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right)}, \text{ where } t = \sum_{i=1}^{n} x_i^2.$$

Proof:

The likelihood function of (1) is given by

$$L(x;\omega,\delta,\theta) = \left(\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\right)^n \left(\frac{1}{\omega^{\delta+\frac{\theta}{2}}}\right)^n \prod_{i=1}^{n} x_i^{2\delta+\theta-1} \exp\left(-\left(\frac{\delta}{\omega}\sum_{i=1}^{n} x_i^2\right)\right).$$

The log-likelihood function is

$$\log L(x;\omega,\delta,\theta) = n\log\left(\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\right) - n\left(\delta+\frac{\theta}{2}\right)\log(\omega) + \left(2\delta+\theta-1\right)\sum_{i=1}^{n}\log x_i - \frac{\delta}{\omega}\sum_{i=1}^{n} x_i^2. \quad (2)$$

On differentiating (2) with respect to ω and equating to zero, we get

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right)}, \quad (3)$$

where $t = \sum_{i=1}^{n} x_i^2$.

2.2 Bayesian Method of Estimation

Here we estimate the scale parameter of the weighted Nakagami distribution by using different priors under various loss functions.

2.2.1 Posterior Distribution under Uniform Prior

The uniform prior relating to the scale parameter ω is given by

$$\pi_1(\omega) = k.$$

Now the posterior distribution under uniform prior is given as

$$P_1(\omega \mid \underline{x}) \propto L(x;\omega,\delta,\theta)\pi_1(\omega)$$

$$\Rightarrow P_1(\omega \mid \underline{x}) = B\left(\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\right)^n \left(\frac{1}{\omega^{\delta+\frac{\theta}{2}}}\right)^n \prod_{i=1}^{n} x_i^{2\delta+\theta-1} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right)k. \tag{4}$$

where B is the normalizing constant and is given by

$$B = \left(\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\right). \tag{5}$$

By substituting the value of eq. (5) in eq. (4), we get

$$P_1(\omega \mid \underline{x}) = \frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)} \omega^{-n(\delta+\frac{\theta}{2})} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right). \tag{6}$$

This is the required posterior distribution under uniform prior.

2.2.2 Posterior Distribution under Jeffrey's Prior

The Jeffrey's prior for the scale parameter ω is given by

$$\pi_2(\omega) = \frac{1}{\omega}, \quad \omega > 0.$$

The posterior distribution under Jeffrey's prior is given as

$$P_2(\omega \mid \underline{x}) \propto L(x;\omega,\delta,\theta)\pi_2(\omega)$$

$$\Rightarrow P_2(\omega \mid \underline{x}) = B\left(\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\right)^n \left(\frac{1}{\omega^{\delta+\frac{\theta}{2}}}\right)^n \prod_{i=1}^{n} x_i^{2\delta+\theta-1} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right)\frac{1}{\omega}. \tag{7}$$

where B is the normalizing constant and is given by

$$B = \left(\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\right). \tag{8}$$

By substituting the value of eq. (8) in eq. (7), we get

$$P_2(\omega\,|\,\underline{x})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\,\omega^{-n\left(\delta+\frac{\theta}{2}\right)-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right). \tag{9}$$

2.2.3 Posterior Distribution under Extension of Jeffrey's Prior

The extension of Jeffrey's prior proposed by Al-Kutubi relating to the scale parameter is given by

$$\pi_3(\omega)=\frac{1}{\omega^{2c_1}},\qquad c_1\in R.$$

Now the posterior distribution under extension of Jeffrey's prior is given as

$$P_3(\omega\,|\,\underline{x})\propto L(x;\omega,\delta,\theta)\pi_3(\omega)$$

$$\Rightarrow P_3(\omega\,|\,\underline{x})=B\left(\frac{2\delta^{\delta+\frac{\theta}{2}}}{\Gamma\left(\delta+\frac{\theta}{2}\right)}\right)^n\left(\frac{1}{\omega^{\delta+\frac{\theta}{2}}}\right)^n\prod_{i=1}^{n}x_i^{2\delta+\theta-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)\frac{1}{\omega^{2c_1}}.$$

$$\Rightarrow P_3(\omega\,|\,\underline{x})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)+2c_1-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}\,\omega^{-n\left(\delta+\frac{\theta}{2}\right)-2c_1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right). \tag{10}$$

Remark 1:

If $c_1=1/2$, the posterior distribution given in (10) coincides with the posterior distribution given in (9).

2.2.1.1 Bayesian Estimation by Using Uniform Prior under Different Loss Functions

Theorem 2:

Assuming the squared error loss function (SELF) $L(\omega,\widehat{\omega})=b(\omega-\widehat{\omega})^2$, the Bayes estimator of ω for known values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-2},\ \text{where}\ t=\sum_{i=1}^{n}x_i^2.$$

Proof:

Under SELF, the risk function is given by

$$R(\hat{\omega})=\int_0^{\infty}b(\omega-\hat{\omega})^2\,P_1(\omega\,|\,\underline{x})\,d\omega. \tag{11}$$

Using (6) in (11), we get

$$R(\hat{\omega})=\frac{b(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\int_0^{\infty}\left(\omega^2+\hat{\omega}^2-2\omega\hat{\omega}\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

By substituting $y=\frac{\delta}{\omega}t$, we get

$$R(\hat{\omega})=\frac{b(\delta t)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)-3\right)}+b\hat{\omega}^2-\frac{2b\delta t}{n\left(\delta+\frac{\theta}{2}\right)-2}\hat{\omega}.$$

Now the solution of $\frac{\partial(R(\hat{\omega}))}{\partial\hat{\omega}}=0$ is the required Bayes estimator and is given by

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-2}.$$

Theorem 3:

Assuming the loss function $L(\omega,\widehat{\omega})=\left(\frac{\omega-\hat{\omega}}{\omega}\right)^2$ which is the quadratic loss function (QLF) the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{\left(n\left(\delta+\frac{\theta}{2}\right)\right)}, \text{ where } t=\sum_{i=1}^{n}x_i^2.$$

Proof:

The risk function under QLF is given by

$$R(\hat{\omega})=\int_0^{\infty}\left(\frac{\omega-\hat{\omega}}{\omega}\right)^2 P_1(\omega\,|\,\underline{x})\,d\omega\ . \tag{12}$$

Using (6) in (12), we get

$$R(\hat{\omega})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\int_0^{\infty}\left(1+\frac{\hat{\omega}}{\omega^2}-2\frac{\hat{\omega}}{\omega}\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After solving the above equation, we get

$$R(\hat{\omega}) = 1 + \frac{\left(n\left(\delta+\frac{\theta}{2}\right)\right)\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}{(\delta t)^2}\hat{\omega}^2 - \frac{2\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}{\delta t}\hat{\omega}.$$

The required Bayes estimator will be obtained by solving $\frac{\partial(R(\hat{\omega}))}{\partial\hat{\omega}} = 0$ and is given by

$$\hat{\omega} = \frac{\delta t}{\left(n\left(\delta+\frac{\theta}{2}\right)\right)}.$$

Theorem 4:

Assuming the loss function $L(\omega,\hat{\omega}) = \frac{(\omega-\hat{\omega})^2}{\hat{\omega}}$ which is the precautionary loss function (PLF) the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega} = \frac{\delta t}{\left(\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)-3\right)\right)^{\frac{1}{2}}}.$$

Proof:

Under PLF, the risk function is given by

$$R(\hat{\omega}) = \int_0^\infty (\omega-\hat{\omega})^2 P_1(\omega \mid \underline{x})\, d\omega. \tag{13}$$

Using (6) in (13), we get

$$R(\hat{\omega}) = \frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\int_0^\infty \left(\frac{\omega^2}{\hat{\omega}} + \hat{\omega} - 2\omega\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right) d\omega.$$

After simplification, we get

$$R(\hat{\omega}) = \frac{(\delta t)^2}{\hat{\omega}\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)-3\right)} + \hat{\omega} - \frac{2\delta t}{n\left(\delta+\frac{\theta}{2}\right)-2}.$$

Differentiating the above equation with respect to $\hat{\omega}$ and equate to zero, we get the Bayes estimator as

$$\hat{\omega} = \frac{\delta t}{\left(\left(n\left(\delta + \frac{\theta}{2}\right) - 2\right)\left(n\left(\delta + \frac{\theta}{2}\right) - 3\right)\right)^{\frac{1}{2}}}$$

Theorem 5:

Assuming the Al-Bayyati's loss function (ALF) $L(\omega, \hat{\omega}) = \omega^{c_2}\left(\omega - \hat{\omega}\right)^2$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) - c_2 - 2}.$$

Proof:

Under Al-Bayyati's loss function (ALF), the risk function is given by

$$R(\hat{\omega}) = \int_0^\infty \omega^{c_2}\left(\omega - \hat{\omega}\right)^2 P_1\left(\omega \mid \underline{x}\right) d\omega. \tag{14}$$

By substituting the value of equation (6) in equation (14), we get

$$R(\hat{\omega}) = \frac{\left(\delta t\right)^{n\left(\delta + \frac{\theta}{2}\right) - 1}}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - 1\right)} \int_0^\infty \omega^{c_2}\left(\omega^2 + \hat{\omega}^2 - 2\omega\hat{\omega}\right)^2 \omega^{-n\left(\delta + \frac{\theta}{2}\right)} \exp\left(-\left(\frac{\delta}{\omega} t\right)\right) d\omega.$$

After solving the above integral, we get

$$R(\hat{\omega}) = \left(\delta t\right)^{c_2 + 2} \frac{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - c_2 - 3\right)}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - 1\right)} + \left(\delta t\right)^{c_2} \frac{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - c_2 - 1\right)}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - 1\right)} \hat{\omega}^2$$
$$- 2\left(\delta t\right)^{c_2 + 1} \frac{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - c_2 - 2\right)}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) - 1\right)} \hat{\omega}.$$

The solution of $\frac{\partial(R(\hat{\omega}))}{\partial\hat{\omega}} = 0$ is the required Bayes estimator and is given by

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) - c_2 - 2}.$$

2.2.1.2 Bayesian Estimation by using Jeffrey's Prior under Different Loss Functions

Theorem 6:

Assuming the SELF $L(\omega,\hat{\omega})=b(\omega-\hat{\omega})^2$, the Bayes estimator of ω is of the form

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-1}.$$

Proof:

Under SELF the risk function is given by

$$R(\hat{\omega})=\int_0^\infty b\left(\omega-\hat{\omega}\right)^2 P_2\left(\omega\,|\,\underline{x}\right)d\omega\,. \tag{15}$$

Using (9) in (15), we get

$$R(\hat{\omega})=\frac{b\left(\delta t\right)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\int_0^\infty(\omega^2+\hat{\omega}^2-2\omega\hat{\omega})\omega^{-n\left(\delta+\frac{\theta}{2}\right)-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

By substituting $y=\frac{\delta}{\omega}t,$ we get

$$R(\hat{\omega})=\frac{b\left(\delta t\right)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)}+b\hat{\omega}^2-\frac{2b\delta t}{n\left(\delta+\frac{\theta}{2}\right)-1}\hat{\omega}.$$

On differentiating the above equation w.r.t. $\hat{\omega}$ and equate to zero, we get

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-1}.$$

Theorem 7:

Assuming the QLF $L(\omega,\hat{\omega})=\left(\frac{\omega-\hat{\omega}}{\omega}\right)^2$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)+1}.$$

Proof:

The risk function under QLF is given by

$$R(\hat{\omega})=\int_0^\infty\left(\frac{\omega-\hat{\omega}}{\omega}\right)^2 P_2\left(\omega\,|\,\underline{x}\right)d\omega\;. \tag{16}$$

Using (9) in (16), we get

$$R(\hat{\omega})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\int_0^\infty\left(1+\frac{\hat{\omega}^2}{\omega^2}-2\frac{\hat{\omega}}{\omega}\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After solving the above integral, we get

$$R(\hat{\omega})=1+\frac{\left(n\left(\delta+\frac{\theta}{2}\right)+1\right)\left(n\left(\delta+\frac{\theta}{2}\right)\right)}{(\delta t)^2}\hat{\omega}^2-\frac{2n\left(\delta+\frac{\theta}{2}\right)}{\delta t}\hat{\omega}.$$

Now, the solution of $\frac{\partial(R(\hat{\omega}))}{\partial\hat{\omega}}=0$ is the required Bayes estimator and is given by

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)+1}\;.$$

Theorem 8:

Assuming the PLF $L(\omega,\hat{\omega})=\frac{(\omega-\hat{\omega})^2}{\hat{\omega}}$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{\left(\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\right)^{\frac{1}{2}}}\;.$$

Proof:

Under PLF, the risk function is given by

$$R(\hat{\omega})=\int_0^\infty\frac{(\omega-\hat{\omega})^2}{\hat{\omega}}P_2\left(\omega\,|\,\underline{x}\right)d\omega. \tag{17}$$

Using (9) in (17), we get

$$R(\hat{\omega})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\int_0^\infty\left(\frac{\omega^2}{\hat{\omega}}+\hat{\omega}-2\omega\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After simplification, we get

$$R(\hat{\omega})=\frac{\left(\delta t\right)^{2}}{\hat{\omega}\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)}+\hat{\omega}-\frac{2\delta t}{n\left(\delta+\frac{\theta}{2}\right)-1}.$$

Differentiating the above equation with respect to $\hat{\omega}$ and equate to zero, we get the Bayes estimator as

$$\hat{\omega}=\frac{\delta t}{\left(\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\right)^{\frac{1}{2}}}.$$

Theorem 9:

Assuming the ALF $L(\omega,\hat{\omega})=\omega^{c_2}\left(\omega-\hat{\omega}\right)^{2}$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-c_2-1}.$$

Proof:

Under Al-Bayyati's loss function (ALF), the risk function is given by

$$R(\hat{\omega})=\int_0^\infty \omega^{c_2}\left(\omega-\hat{\omega}\right)^{2} P_2\left(\omega\,|\,\underline{x}\right)d\omega. \tag{18}$$

By substituting the value of equation (9) in equation (18), we get

$$R(\hat{\omega})=\frac{\left(\delta t\right)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\int_0^\infty \omega^{c_2}\left(\omega^{2}+\hat{\omega}^{2}-2\omega\hat{\omega}\right)^{2}\omega^{-n\left(\delta+\frac{\theta}{2}\right)-1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After solving the above integral, we get

$$R(\hat{\omega})=\left(\delta t\right)^{c_2+2}\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-c_2-2\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}+\left(\delta t\right)^{c_2}\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-c_2\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\hat{\omega}^{2}$$
$$-2\left(\delta t\right)^{c_2+1}\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-c_2-1\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\hat{\omega}.$$

The solution of $\frac{\partial (R(\hat{\omega}))}{\partial \hat{\omega}} = 0$ is the required Bayes estimator and is given by

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) - c_2 - 1}.$$

2.2.1.3 Bayesian Estimation by using Extension of Jeffrey's Prior under Different Loss Functions

Theorem 10:

Assuming the SELF $L(\omega, \hat{\omega}) = b(\omega - \hat{\omega})^2$, the Bayes estimator of ω is of the form

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 2}.$$

Proof:

Under SELF the risk function is given by

$$R(\hat{\omega}) = \int_0^\infty b\left(\omega - \hat{\omega}\right)^2 P_3\left(\omega \mid \underline{x}\right) d\omega. \tag{19}$$

Using (10) in (19), we get

$$R(\hat{\omega}) = \frac{b\left(\delta t\right)^{n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1}}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1\right)} \int_0^\infty \left(\omega^2 + \hat{\omega}^2 - 2\omega\hat{\omega}\right) \omega^{-n\left(\delta + \frac{\theta}{2}\right) - 2c_1} \exp\left(-\left(\frac{\delta}{\omega} t\right)\right) d\omega.$$

$$\Rightarrow R(\hat{\omega}) = \frac{b\left(\delta t\right)^2}{\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 2\right)\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 3\right)} + b\hat{\omega}^2 - \frac{2b\delta t}{n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 2}\hat{\omega}.$$

On differentiating the above equation w.r.t. $\hat{\omega}$ and equate to zero, we get

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 2}.$$

Theorem 11:

Assuming the QLF $L(\omega, \hat{\omega}) = \left(\frac{\omega - \hat{\omega}}{\omega}\right)^2$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) + 2c_1}.$$

Proof:

Under QLF the risk function is given by

$$R(\hat{\omega}) = \int_0^\infty \left(\frac{\omega - \hat{\omega}}{\omega}\right)^2 P_3\left(\omega \mid \underline{x}\right) d\omega . \qquad (20)$$

Using (10) in (20), we get

$$R(\hat{\omega}) = \frac{\left(\delta t\right)^{n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1}}{\Gamma\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1\right)} \int_0^\infty \left(1 + \frac{\hat{\omega}^2}{\omega^2} - 2\frac{\hat{\omega}}{\omega}\right) \omega^{-n\left(\delta + \frac{\theta}{2}\right) - 2c_1} \exp\left(-\left(\frac{\delta}{\omega} t\right)\right) d\omega.$$

After solving the above integral, we get

$$R(\hat{\omega}) = 1 + \frac{\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1\right)\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1\right)}{\left(\delta t\right)^2} \hat{\omega}^2 - \frac{2\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 1\right)}{\delta t} \hat{\omega}.$$

Now, the solution of $\frac{\partial(R(\hat{\omega}))}{\partial \hat{\omega}} = 0$ is the required Bayes estimator and is given by

$$\hat{\omega} = \frac{\delta t}{n\left(\delta + \frac{\theta}{2}\right) + 2c_1}.$$

Theorem 12:

Assuming the PLF $L(\omega, \hat{\omega}) = \frac{\left(\omega - \hat{\omega}\right)^2}{\hat{\omega}}$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega} = \frac{\delta t}{\left(\left(n(\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 2\right)\left(n\left(\delta + \frac{\theta}{2}\right) + 2c_1 - 3\right)\right)^{\frac{1}{2}}}.$$

Proof:

Under PLF, the risk function is given by

$$R(\hat{\omega}) = \int_0^\infty \frac{\left(\omega - \hat{\omega}\right)^2}{\hat{\omega}} P_3\left(\omega \mid \underline{x}\right) d\omega . \qquad (21)$$

Using (10) in (21), we get

$$R(\hat{\omega})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)+2c_1-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}\int_0^\infty\left(\frac{\omega^2}{\hat{\omega}}+\hat{\omega}-2\omega\right)\omega^{-n\left(\delta+\frac{\theta}{2}\right)-2c_1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After simplification, we get

$$R(\hat{\omega})=\frac{(\delta t)^2}{\hat{\omega}\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-3\right)}+\hat{\omega}-\frac{2\delta t}{n\left(\delta+\frac{\theta}{2}\right)+2c_1-2}.$$

Differentiating the above equation with respect to $\hat{\omega}$ and equate to zero, we get the Bayes estimator as

$$\hat{\omega}=\frac{\delta t}{\left(\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-3\right)\right)^{\frac{1}{2}}}.$$

Theorem 13:

Assuming the ALF $L(\omega,\hat{\omega})=\omega^{c_2}(\omega-\hat{\omega})^2$, the Bayes estimator of ω for the given values of other parameters is of the form

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)+2c_1-c_2-2}.$$

Proof:

Under ALF, the risk function is given by

$$R(\hat{\omega})=\int_0^\infty \omega^{c_2}(\omega-\hat{\omega})^2 P_3(\omega\mid \underline{x})\,d\omega. \tag{22}$$

By substituting the value of equation (10) in equation (22), we get

$$R(\hat{\omega})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)+2c_1-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}\int_0^\infty \omega^{c_2}\left(\omega^2+\hat{\omega}^2-2\omega\hat{\omega}\right)^2\omega^{-n\left(\delta+\frac{\theta}{2}\right)-2c_1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After solving the above integral, we get

$$R(\hat{\omega})=\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-c_2-3\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}(\delta t)^{c_2+2}+\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-c_2-1\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}(\delta t)^{c_2}\hat{\omega}^2$$

$$-2\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-c_2-2\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}(\delta t)^{c_2+1}\hat{\omega}.$$

The solution of $\frac{\partial(R(\hat{\omega}))}{\partial\hat{\omega}}=0$ is the required Bayes estimator and is given by

$$\hat{\omega}=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)+2c_1-c_2-2}.$$

Remark 2:

If $c_1=1/2$, the estimators obtained under extension of Jeffrey's prior coincides with the estimators obtained under Jeffrey's prior.

Remark 3:

If $c_2=0$, the estimators obtained using ALF coincides with the estimators obtained using SELF.

3. POSTERIOR MEAN AND POSTERIOR VARIANCE UNDER DIFFERENT PRIORS

3.1 Posterior Mean and Posterior Variance under Uniform Prior

Under uniform prior, the posterior distribution is given as

$$P_1(\omega\mid \underline{x})=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\omega^{-n\left(\delta+\frac{\theta}{2}\right)}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right).$$

Now,

$$E(\omega^r)=\int_0^\infty \omega^r P_1(\omega\mid \underline{x})d\omega.$$

$$\Rightarrow E(\omega^r)=\frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}\int_0^\infty \omega^{-n\left(\delta+\frac{\theta}{2}\right)+r}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

$$\Rightarrow E(\omega^r) = \frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-r-1\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)}(\delta t)^r . \tag{23}$$

If r=1 in (23), we get

$$E(\omega) = \frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-2} .$$

This is the posterior mean

If r=2 in (23), we get

$$E(\omega^2) = \frac{(\delta t)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)-3\right)} .$$

Thus the posterior variance is given as

$$v(\omega) = \frac{(\delta t)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)^2\left(n\left(\delta+\frac{\theta}{2}\right)-3\right)} .$$

3.2 Posterior Mean and Posterior Variance under Jeffrey's Prior

Under Jeffrey's prior, the posterior distribution is given as

$$P_2(\omega \mid \underline{x}) = \frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)} \omega^{-n\left(\delta+\frac{\theta}{2}\right)-1} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right) . \tag{24}$$

Now,

$$E(\omega^r) = \int_0^\infty \omega^r P_2(\omega \mid \underline{x}) d\omega . \tag{25}$$

By substituting the value of (24) in (25), we get

$$E(\omega^r) = \frac{(\delta t)^{n\left(\delta+\frac{\theta}{2}\right)}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)} \int_0^\infty \omega^{-n\left(\delta+\frac{\theta}{2}\right)+r-1} \exp\left(-\left(\frac{\delta}{\omega}t\right)\right) d\omega.$$

$$\Rightarrow E(\omega^r)=\frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)-r\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)\right)}\left(\delta t\right)^r . \tag{26}$$

If r=1 in (26), we get

$$E(\omega)=\frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)-1} .$$

This is the posterior mean

If r=2 in (26), we get

$$E(\omega^2)=\frac{\left(\delta t\right)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)} .$$

Thus the posterior variance is given as

$$v(\omega)=\frac{\left(\delta t\right)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)-1\right)^2\left(n\left(\delta+\frac{\theta}{2}\right)-2\right)} .$$

3.3 Posterior Mean and Posterior Variance under Extension of Jeffrey's Prior

Under extension of Jeffrey's prior, the posterior distribution is given as

$$P_3(\omega\,|\,\underline{x})=\frac{\left(\delta t\right)^{n\left(\delta+\frac{\theta}{2}\right)+2c_1-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}\omega^{-n\left(\delta+\frac{\theta}{2}\right)-2c_1}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right). \tag{27}$$

Now,

$$E(\omega^r)=\int_0^\infty \omega^r P_3(\omega\,|\,\underline{x})d\omega . \tag{28}$$

By substituting the value of (27) in (28), we get

$$E(\omega^r)=\frac{\left(\delta t\right)^{n\left(\delta+\frac{\theta}{2}\right)+2c_1-1}}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}\int_0^\infty \omega^{-n\left(\delta+\frac{\theta}{2}\right)-2c_1+r}\exp\left(-\left(\frac{\delta}{\omega}t\right)\right)d\omega.$$

After solving the above expression, we get

$$E(\omega^r) = \frac{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-r-1\right)}{\Gamma\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-1\right)}(\delta t)^r. \tag{29}$$

If r=1 in (29), we get

$$E(\omega) = \frac{\delta t}{n\left(\delta+\frac{\theta}{2}\right)+2c_1-2}.$$

This is the posterior mean

If r=2 in (29), we get

$$E(\omega^2) = \frac{(\delta t)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-2\right)\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-3\right)}.$$

Thus the posterior variance is given as

$$v(\omega) = \frac{(\delta t)^2}{\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-2\right)^2\left(n\left(\delta+\frac{\theta}{2}\right)+2c_1-3\right)}.$$

Remark 4:

If $c_1 = 1/2$, the posterior mean and posterior variance obtained under extension of Jeffrey's prior coincides with the posterior mean and posterior variance obtained under Jeffrey's prior.

4. FITTING TO REAL LIFE DATA SETS

In this section, weighted Nakagami distribution is fitted to the real life data sets and the results are presented in the tables below.

Data Set I: The first data set below are from an accelerated life test of 59 conductors, failure times (hours), and there are no censored observations Lawless (2003).

2.997, 4.137, 4.288, 4.531, 4.700, 4.706, 5.009, 5.381, 5.434, 5.459, 5.589, 5.640, 5.807, 5.923, 6.033, 6.071, 6.087, 6.129, 6.352, 6.369, 6.476, 6.492, 6.515, 6.522, 6.538, 6.545, 6.573, 6.725, 6.869, 6.923, 6.948, 6.956, 6.958, 7.024, 7.224, 7.365, 7.398, 7.459, 7.489, 7.495, 7.496, 7.543, 7.683, 7.937, 7.945, 7.974, 8.120, 8.336, 8.532, 8.591, 8.687, 8.799, 9.218, 9.254, 9.289, 9.663, 10.092, 10.491, 11.038.

Table 1
Descriptive Statistics of Data Set I

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Skewness	Kurtosis
2.997	6.052	6.923	6.980	7.941	11.040	0.1931723	3.087389

Data Set II: The second data set given below is taken from Murthy et al. (2004), Weibull models (vol. 505) John Willy and sons page 180 represents 50 items put into use at t=0 and failure times are in weeks.

0.013, 0.065, 0.111, 0.111, 0.163, 0.309, 0.426, 0.535, 0.684, 0.747, 0.997, 1.284, 1.304, 1.647, 1.829, 2.336, 2.838, 3.269, 3.977, 3.981, 4.520, 4.789, 4.849, 5.202, 5.291, 5.349, 5.911, 6.018, 6.427, 6.456, 6.572, 7.023, 7.291, 7.087, 7.787, 8.596, 9.388, 10.261, 10.713, 11.658, 13.006, 13.388, 13.842, 17.152, 17.283, 19.418, 23.471, 24.777, 32.795, 48.105.

Table 2
Descriptive Statistics of Data Set II

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Skewness	Kurtosis
0.013	1.390	5.320	7.821	10.040	48.100	2.306048	9.408282

Table 3
Posterior Mean and (Posterior Variance) under Different Priors using Data Set I

θ	δ	c_1	Uniform Prior	Jeffrey's Prior	Extension of Jeffrey's Prior
1.0	0.5	0.5	26.54085 (12.57887)	26.08325 (11.935719)	26.08325 (11.935719)
	1.0	1.0	34.97869 (14.31005)	34.57894 (13.823156)	34.18822 (13.358104)
	2.0	2.0	41.58979 (11.97031)	41.30590 (11.726304)	40.47702 (11.032922)
	3.5	2.5	45.25555 (8.78998)	45.06298 (8.678085)	44.30879 (8.249027)
2.0	0.5	0.5	17.48935 (3.577512)	17.28947 (3.455789)	17.28947 (3.455789)
	1.0	1.0	26.08325 (5.915965)	25.86032 (5.765138)	25.64116 (5.619395)
	2.0	2.0	34.57894 (6.871856)	34.38247 (6.755166)	33.80622 (6.420566)
	3.5	2.5	40.18899 (6.152972)	40.03705 (6.083360)	39.44060 (5.815180)
4.0	0.5	0.5	10.39745 (0.7481447)	10.32647 (0.732894)	10.32647 (0.732894)
	1.0	1.0	17.28947 (1.7179640)	17.19123 (1.688791)	17.09411 (1.660276)
	2.0	2.0	25.86032 (2.8701974)	25.75027 (2.833661)	25.42569 (2.727703)
	3.5	2.5	32.83659 (3.3537840)	32.73508 (3.322747)	32.33527 (3.202357)

Table 4
Posterior Mean and (Posterior Variance) under Different Priors using Data Set II

θ	δ	c_1	**Uniform Prior**	**Jeffrey's Prior**	**Extension of Jeffrey's Prior**
1.0	0.5	0.5	75.11938 (120.06214)	73.58633 (112.81141)	73.58633 (112.81141)
	1.0	1.0	98.78712 (135.54022)	97.45216 (130.09485)	96.15280 (124.93731)
	2.0	2.0	117.25951 (112.70323)	116.31387 (109.99120)	113.56630 (102.35956)
	3.5	2.5	127.47531 (82.48707)	126.83473 (81.24772)	124.33552 (76.53129)
2.0	0.5	0.5	49.39356 (33.88506)	48.72608 (32.52371)	48.72608 (32.52371)
	1.0	1.0	73.58633 (55.82420)	72.84303 (54.14395)	72.11460 (52.53046)
	2.0	2.0	97.45216 (64.60493)	96.79812 (63.30998)	94.88763 (59.62691)
	3.5	2.5	113.18435 (57.70584)	112.67906 (56.93530)	110.70224 (53.98672)
4.0	0.5	0.5	29.31488 (7.043952)	29.07847 (6.87445)	29.07847 (6.87445)
	1.0	1.0	48.72608 (16.151232)	48.39906 (15.82749)	48.07640 (15.51235)
	2.0	2.0	72.84303 (26.934555)	72.47699 (26.52987)	71.40060 (25.36341)
	3.5	2.5	92.45462 (31.425942)	92.11719 (31.08270)	90.79176 (29.75864)

Table 5
Estimates and (Posterior Risk) under Uniform Prior using Different Loss Functions using Data Set I

θ	δ	b	c_2	MLE	SELF	QLF	PLF	ALF
1.0	0.5	1.0	0.5	25.64116	26.54085 (12.57887)	25.64116 (0.016949153)	26.77677 (0.4718466)	26.77573 (16.98797)
	1.0	1.5	1.0	34.18822	34.97869 (21.46507)	34.18822 (0.011299435)	35.18265 (0.4079183)	35.38780 (28.44957)
	2.0	2.5	1.5	41.02586	41.58979 (29.92579)	41.02586 (0.006779661)	41.73345 (0.2873224)	42.02302 (39.28586)
	3.5	3.0	2.0	44.87203	45.25555 (26.36994)	44.87203 (0.004237288)	45.35257 (0.1940219)	45.64569 (34.53866)
2.0	0.5	1.0	0.5	17.09411	17.48935 (3.577512)	17.09411 (0.011299435)	17.59133 (0.2039591)	17.59103 (4.930602)
	1.0	1.5	1.0	25.64116	26.08325 (8.873947)	25.64116 (0.008474576)	26.19641 (0.2263199)	26.31006 (11.84136)
	2.0	2.5	1.5	34.18822	34.57894 (17.179640)	34.18822 (0.005649718)	34.67816 (0.1984448)	34.87789 (22.59885)
	3.5	3.0	2.0	39.88625	40.18899 (18.458917)	39.88625 (0.003766478)	40.26547 (0.1529554)	40.49637 (24.20033)
4.0	0.5	1.0	0.5	10.25646	10.39745 (0.7481447)	10.25646 (0.006779661)	10.43336 (0.07183059)	10.43330 (1.074781)
	1.0	1.5	1.0	17.09411	17.28947 (2.5769460)	17.09411 (0.005649718)	17.33908 (0.09922241)	17.38883 (3.486364)
	2.0	2.5	1.5	25.64116	25.86032 (7.1754935)	25.64116 (0.004237288)	25.91575 (0.11086965)	26.02716 (9.477781)
	3.5	3.0	2.0	32.63421	32.83659 (10.0613519)	32.63421 (0.003081664)	32.88762 (0.10205628)	33.04150 (13.21644)

Table 6
Estimates and (Posterior Risk) under Jeffrey's Prior using Different Loss Functions using Data Set I

θ	δ	**b**	c_2	**MLE**	**SELF**	**QLF**	**PLF**	**ALF**
1.0	0.5	1.0	0.5	25.64116	26.08325 (11.93572)	25.21381 (0.016666667)	26.31106 (0.4556113)	26.31006 (16.1304)
	1.0	1.5	1.0	34.18822	34.57894 (20.73473)	33.80622 (0.011173184)	34.77824 (0.3986078)	34.97869 (27.48786)
	2.0	2.5	1.5	41.02586	41.30590 (29.31576)	40.74959 (0.006734007)	41.44760 (0.2834032)	41.73320 (38.48767)
	3.5	3.0	2.0	44.87203	45.06298 (26.03426)	44.68270 (0.004219409)	45.15916 (0.1923715)	45.44978 (34.10011)
2.0	0.5	1.0	0.5	17.09411	17.28947 (3.455789)	16.90311 (0.011173184)	17.38912 (0.1993039)	17.38883 (4.766146)
	1.0	1.5	1.0	25.64116	25.86032 (8.647707)	25.42569 (0.008403361)	25.97154 (0.2224554)	26.08325 (11.54214)
	2.0	2.5	1.5	34.18822	34.38247 (16.887915)	33.99615 (0.005617978)	34.48056 (0.1961914)	34.67802 (22.21664)
	3.5	3.0	2.0	39.88625	40.03705 (18.250080)	39.73658 (0.003752345)	40.11295 (0.1517994)	40.34209 (23.92732)
4.0	0.5	1.0	0.5	10.25646	10.32647 (0.732894)	10.18740 (0.006734007)	10.36190 (0.07085081)	10.36184 (1.053622)
	1.0	1.5	1.0	17.09411	17.19123 (2.533187)	16.99807 (0.005617978)	17.24028 (0.09809568)	17.28947 (3.427971)
	2.0	2.5	1.5	25.64116	25.75027 (7.084151)	25.53297 (0.004219409)	25.80524 (0.10992658)	25.91569 (9.357786)
	3.5	3.0	2.0	32.63421	32.73508 (9.968240)	32.53395 (0.003072197)	32.78580 (0.10142557)	32.93872 (13.09456)

Table 7
Estimates and (Posterior Risk) under Extension of Jeffrey's Prior using Different Loss Functions using Data Set I

θ	δ	b	c_1	c_2	MLE	SELF	QLF	PLF	ALF
1.0	0.5	1.0	0.5	0.5	25.64116	26.08325 (11.93572)	25.21381 (0.016666667)	26.31106 (0.4556113)	26.31006 (16.1304)
	1.0	1.5	1.0	1.0	34.18822	34.18822 (20.03716)	33.43268 (0.011049724)	34.38302 (0.3896124)	34.57894 (26.56917)
	2.0	2.5	2.0	1.5	41.02586	40.47702 (27.58231)	39.94267 (0.006600660)	40.61308 (0.2721152)	40.88726 (36.21933)
	3.5	3.0	2.5	2.0	44.87203	44.30879 (24.74708)	43.94108 (0.004149378)	44.40177 (0.1859762)	44.68270 (32.41837)
2.0	0.5	1.0	0.5	0.5	17.09411	17.28947 (3.455789)	16.90311 (0.011173184)	17.38912 (0.1993039)	17.38883 (4.766146)
	1.0	1.5	1.0	1.0	25.64116	25.64116 (8.429092)	25.21381 (0.008333333)	25.75051 (0.2186889)	25.86032 (11.25295)
	2.0	2.5	2.0	1.5	34.18822	33.80622 (16.051416)	33.43268 (0.005524862)	33.90105 (0.1896566)	34.09191 (21.12058)
	3.5	3.0	2.5	2.0	39.88625	39.44060 (17.445539)	39.14898 (0.003696858)	39.51425 (0.1473039)	39.73658 (22.8755)
4.0	0.5	1.0	0.5	0.5	10.25646	10.32647 (0.732894)	10.18740 (0.006734007)	10.36190 (0.07085081)	10.36184 (1.053622)
	1.0	1.5	1.0	1.0	17.09411	17.09411 (2.490414)	16.90311 (0.005586592)	17.14260 (0.09698804)	17.19123 (3.370885)
	2.0	2.5	2.0	1.5	25.64116	25.42569 (6.819258)	25.21381 (0.004166667)	25.47927 (0.10716846)	25.58695 (9.009771)
	3.5	3.0	2.5	2.0	32.63421	32.33527 (9.607070)	32.13900 (0.003034901)	32.38475 (0.09896032)	32.53395 (12.62178)

Table 8
Estimates and (Posterior Risk) under Uniform Prior using Different Loss Functions using Data Set II

θ	δ	b	c_2	MLE	SELF	QLF	PLF	ALF
1.0	0.5	1.0	0.5	72.1146	75.11938 (12.00621)	72.1146 (0.02000000)	75.91431 (0.15898723)	75.91011 (15.84076)
	1.0	1.5	1.0	96.1528	98.78712 (20.33103)	96.1528 (0.01333333)	99.47078 (0.13673122)	100.15917 (26.62542)
	2.0	2.5	1.5	115.3834	117.25951 (28.17581)	115.3834 (0.00800000)	117.73911 (0.09591820)	118.70716 (36.76365)
	3.5	3.0	2.0	126.2006	127.47531 (24.74612)	126.2006 (0.00500000)	127.79844 (0.06462637)	128.77607 (32.26047)
2.0	0.5	1.0	0.5	48.0764	49.39356 (3.388506)	48.0764 (0.0013333333)	49.73539 (0.06836561)	49.73421 (4.495666)
	1.0	1.5	1.0	72.1146	73.58633 (8.373630)	72.1146 (0.0010000000)	73.96467 (0.07566767)	74.34495 (10.98539)
	2.0	2.5	1.5	96.1528	97.45216 (16.151232)	96.1528 (0.0006666667)	97.78307 (0.06618163)	98.44997 (21.0835)
	3.5	3.0	2.0	112.1783	113.18435 (17.311753)	112.1783 (0.0004444444)	113.43898 (0.05092666)	114.20864 (22.57206)
4.0	0.5	1.0	0.5	28.84584	29.31488 (0.7043952)	28.84584 (0.0008000000)	29.43478 (0.02397955)	29.43453 (0.9479272)
	1.0	1.5	1.0	48.07640	48.72608 (2.4226848)	48.07640 (0.0006666667)	48.89154 (0.03309081)	49.05755 (3.193375)
	2.0	2.5	1.5	72.11460	72.84303 (6.7336386)	72.11460 (0.0005000000)	73.02768 (0.03692935)	73.39909 (8.802388)
	3.5	3.0	2.0	91.78222	92.45462 (9.4277826)	91.78222 (0.0003636364)	92.62441 (0.03395948)	93.13694 (12.30074)

Table 9
Estimates and (Posterior Risk) under Jeffrey's Prior using Different Loss Functions using Data Set II

θ	δ	b	c_2	MLE	SELF	QLF	PLF	ALF
1.0	0.5	1.0	0.5	72.1146	73.58633 (11.28114)	70.70059 (0.0019607843)	74.34890 (0.15251460)	74.34495 (14.8663)
	1.0	1.5	1.0	96.1528	97.45216 (19.51423)	94.88763 (0.0013157895)	98.11737 (0.13304204)	98.78712 (25.54316)
	2.0	2.5	1.5	115.3834	116.31387 (27.49780)	114.46762 (0.0007936508)	116.78574 (0.09437270)	117.73812 (35.87086)
	3.5	3.0	2.0	126.2006	126.83473 (24.37431)	125.57269 (0.0004975124)	127.15461 (0.06397726)	128.12239 (31.77042)
2.0	0.5	1.0	0.5	48.0764	48.72608 (3.252371)	47.44382 (0.0013157895)	49.05869 (1.4562485)	49.05755 (6.122916)
	1.0	1.5	1.0	72.1146	72.84303 (8.121593)	71.40060 (0.0009900990)	73.21374 (0.9801385)	73.58633 (11.83354)
	2.0	2.5	1.5	96.1528	96.79812 (15.827494)	95.51603 (0.0006622517)	97.12459 (0.4923776)	97.78251 (21.21669)
	3.5	3.0	2.0	112.1783	112.67906 (17.080590)	111.68190 (0.0004424779)	112.93143 (0.2431115)	113.69419 (22.52139)
4.0	0.5	1.0	0.5	28.84584	29.07847 (0.687445)	28.61691 (0.0007936508)	29.19643 (0.02359317)	29.19619 (0.9253814)
	1.0	1.5	1.0	48.07640	48.39906 (2.374124)	47.75801 (0.0006622517)	48.56230 (0.03264701)	48.72608 (3.129663)
	2.0	2.5	1.5	72.11460	72.47699 (6.632467)	71.75582 (0.0004975124)	72.65978 (0.03655844)	73.02744 (8.67038)
	3.5	3.0	2.0	91.78222	92.11719 (9.324810)	91.44968 (0.0003623188)	92.28575 (0.03371172)	92.79452 (12.16655)

Table 10
Estimates and (Posterior Risk) under Extension of Jeffrey's Prior using Different Loss Functions using Data Set II

θ	δ	**b**	c_1	c_2	**MLE**	**SELF**	**QLF**	**PLF**	**ALF**
1.0	0.5	1.0	0.5	0.5	72.1146	73.58633 (11.28114)	70.70059 (0.0019607843)	74.3489 (0.15251460)	74.34495 (14.8663)
	1.0	1.5	1.0	1.0	96.1528	96.15280 (18.74060)	93.65533 (0.0012987013)	96.8003 (0.12950019)	97.45216 (24.53282)
	2.0	2.5	2.0	1.5	115.3834	113.56630 (25.58989)	111.80558 (0.0007751938)	114.0161 (0.08995386)	114.92367 (33.3848)
	3.5	3.0	2.5	2.0	126.2006	124.33552 (22.95939)	123.12249 (0.0004878049)	124.6429 (0.06147625)	125.57269 (29.92776)
2.0	0.5	1.0	0.5	0.5	48.0764	48.72608 (3.252371)	47.44382 (0.0013157895)	49.05869 (0.06652102)	49.05755 (4.31627)
	1.0	1.5	1.0	1.0	72.1146	72.11460 (7.879569)	70.70059 (0.0009803922)	72.47790 (0.07266001)	72.84303 (10.33917)
	2.0	2.5	2.0	1.5	96.1528	94.88763 (14.906727)	93.65533 (0.0006493506)	95.20131 (0.06273580)	95.83336 (19.46115)
	3.5	3.0	2.5	2.0	112.1783	110.70224 (16.196016)	109.73961 (0.0004347826)	110.94581 (0.04871392)	111.68190 (21.11871)
4.0	0.5	1.0	0.5	0.5	28.84584	29.07847 (0.687445)	28.61691 (0.0007936508)	29.19643 (0.02359317)	29.19619 (0.9253814)
	1.0	1.5	1.0	1.0	48.07640	48.07640 (2.326853)	47.44382 (0.0006578947)	48.23746 (0.03221208)	48.39906 (3.06764)
	2.0	2.5	2.0	1.5	72.11460	71.40060 (6.340852)	70.70059 (0.0004901961)	71.57799 (0.03547861)	71.93476 (8.289867)
	3.5	3.0	2.5	2.0	91.78222	90.79176 (8.927593)	90.14325 (0.0003571429)	90.95550 (0.03274728)	91.44968 (11.64891)

5. CONCLUSIONS

In this paper, we estimate the scale parameter of the weighted Nakagami distribution through maximum likelihood and Bayesian methods of estimation. For the comparison of prior distributions and loss functions two real life data sets are used and the results which are obtained through R-Software are presented in the tables above. Table 3 and table 4 represents the posterior mean and posterior variance under uniform, Jeffrey's and extension of Jeffrey's priors using data set I and data set II respectively. In both the tables posterior mean and posterior variance decreases with the increase in the value of θ and it is the extension of Jeffrey's prior which is preferable one as posterior distribution under this prior has the minimum variance in both the tables. Table (5,6,7) and table (8,9,10) represents the Bayes estimates and (posterior risk) of $\hat{\omega}$ under different loss functions using different priors for data set I and data set II respectively. From these tables it is clear that posterior risk decreases with the increase in the value of θ and it is also observed that quadratic loss function (QLF) totally dominates the other loss functions in terms of having least posterior risk. It is also revealed that on fitting weighted Nakagami distribution to the considered real life data sets, it is better to consider the combination of extension of Jeffrey's prior and quadratic loss function due to its least possession of posterior risk.

6. REFERENCES

1. Das, K.K. and Roy, T.D. (2011). Applicability of length biased weighted generalized Rayleigh distribution. *Advances in Applied Science Research*, 2(4), 320-327.
2. Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. *The Annals of Eugenics* 6, 13-25.
3. Jain, K. (2014). A weighted version of Gamma distribution. *Discussiones mathematicae probability and statistics*, 34, 89-111.
4. Kawsar Fatima and Ahmad, S.P. (2017). Weighted inverse Rayleigh distribution. *International journal of Statistics and systems*, 12,119-137.
5. Monsef, M.M.E. and Ghoneim, S.A.E. (2015). The weighted kumaraswamy distribution. *International information institute*, 18, 3289-3300.
6. Priyadarshani, H.A. (2011). Statistical properties of weighted generalized Gamma distribution. *Mathematics Subject Classification*, 62N05, 62B10.
7. Rao, C.R. (1965). On discrete distributions arising out of methods of ascertainment, in classical and contagious discrete distributions. *Pergamon press and statistical publishing society, Calcutta*, 320-332.
8. Sofi Mudasir and Ahmad, S.P. (2017). Characterization and information measures of weighted Erlang distribution. *Journal of Statistics applications & probability letters,* 4, 91-95.
9. Sofi Mudasir and Ahmad, S.P. (2017). Weighted Nakagami distribution with applications using R-Software. *AGU International journal of management studies & research*, 5, 941-952.
10. Sofi Mudasir, Ahmad, S.P. and Ahmad, A. (2016). Bayesian estimation of length biased Nakagami distribution. *International Journal of Modern Mathematical Sciences*, 14(2), 147-159.