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BAYESIAN ANALYSIS OF UNIT ROOT TESTING FOR PANEL DATA TIME SERIES MODEL IN THE PRESENCE OF TIME EFFECT

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ABSTRACT

Univariate time series models are variable centric and exclude the information of other associated variables. However, panel data time series models are frequently applied when there are several similar variables under study or a variable is recorded over time at multiple places. Present work deals with Bayesian analysis of panel data unit root test for AR (1) time series model involving time effect. Simple panel data time series model doesn't consider the time effect, which is an important variable and may affect the time series model properties. The posterior odds ratio for testing the unit root hypothesis is derived under appropriate prior assumptions. To get the justification and illustration of the theorem, practical performance of posterior odds ratio has been explored through simulated and real data. Simulation and real data analysis results justify the theoretical results.

KEY WORDS

AR (1) model, panel data, Time Effect, Unit Root Hypothesis, Prior and Posterior distribution, New Pension Scheme, Net Asset Value.

1. INTRODUCTION

Time series refers to a sequence of numerical data points ordered in chronologically. Nowadays time series analysis is challenging and demanding research area because of its versatile applicability in different fields**.** Analysis of panel data time series model became more popular during last few decades because of its ergonomic behavior including some cross-section information. This area gained important during recent research in both theoretical and empirical concept through exploring the non-stationary processes in panel data. Importance of this area of research is evident from the increasing tendency of researchers to employ panels of non-stationary processes in empirical studies.

Testing of unit root test in univariate time series is common in practice and become a primary part of econometrics literature. If unit root time trend is present in the series, it may reduce the degree of polynomial by one and model becomes difference stationary (Dickey and Fuller (1979, 1981)). Boumahdi and Thomas (1991) proposed generalized Dickey Fuller (DF) unit root test for panel data to provide the efficiency of French capital

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market. Levin and Lin (1992) extended the panel model for fixed effect, deterministic trends and serially correlated heterogeneous errors. Quah (1994) advocated unit root panel data test when fixed effects is not measured. Breitung and Meyer (1994) applied modified DF statistic to test panel unit root for contracted wages in Western Germany.

Since last few decades, literature on panel data unit root test has been thoroughly explored. Main inspiration behind the panel data unit root testing is to increase the power of the test by increasing the sample size through panels (Kim and Maddala (1998)). Hsiao *et al.* (1999) presented classical and Bayesian setup for estimating the parameters in a dynamic panel data model and establish the relative rates of convergence for number of time points T and number of panels n. However, testing of unit root in panel data time series is more challenging see Im *et al.* (1997), Harris and Tzavalis (1998), Maddala and Wu (1999), Choi (1999) and Hadri (1999). The survey paper by Baltagi and Kao (2000) categorized various applications such as purchasing power parity (PPP), growth convergence and international R&D spillovers. Oaxaca and Geisler (2003) discussed the relationship between two-stage GLS and pooled OLS estimator in an unbalanced fixed effect panel with time invariant covariates. They also established equivalence between GLS and OLS coefficient estimates on the time invariant covariates from a pooled cross-section time-series model.

Joakim (2014) introduced a panel data unit root test when errors are unconditionally heteroscedastic with serial or cross-correlation. The classical testing is predominantly based on the assumption that parameters are fixed and population is finite. However Bayesian analysis is free from such assumptions. Recently Kumar *et al.* (2016) derived the posterior odds ratio for Panel data AR(1) (P-AR(1)) model with linear time trend with or without augmentation term and tested the unit root hypothesis.

In the present paper, we have considered a panel data time series incorporating individual and time specific effects along with augmentation term in the model. A posterior odds ratio is derived for testing the unit root hypothesis considering prior assumptions see Schotman and van Dijk (1991 a,b). A simulation study at all combinations of assumed values of different parameters is done using Matlab. For illustration purpose an empirical analysis of NAV series of new pension scheme has been carried out. Both simulation as well as real data analysis properly justifies the theoretical results.

2. MODEL WITH TIME EFFECT

Let $\{y_{it} : i = 1, 2, \ldots, n;\ t = 1, 2, \ldots, T\}$ be a T time period panel data model containing n cross sectional units. If we include linear trend component in the model, then it can be written as

$$y_{it} = \varphi_i + \delta_i t + u_{it} \tag{1}$$

where error term u_{it} follows panel autoregressive process of order one

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \tag{2}$$

Here ε_{it}'s are *iid* normal random variables having zero mean and unknown variance τ^{-1}.

Incorporating equation (2) in (1) along with individual and time specific effects and augmentation term, the model can be expressed as

$$y_{it} = \rho y_{it-1} + \left[(1-\rho)\varphi_i + \rho\delta_i\right] + (1-\rho)\delta_i t + w_t + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{it-j} + \varepsilon_{it} \tag{3}$$

Here, w_t represent the time effect and assumed to be normally distributed. We may rewrite the model (3) as

$$y_{it} = \rho y_{it-1} + \alpha_i + \beta_i t + w_t + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{it-j} + \varepsilon_{it} \tag{4}$$

where $\alpha_i = \left[(1-\rho)\varphi_i + \rho\delta_i\right]$ and $\beta_i = (1-\rho)\delta_i$.

In this paper, our main objective is to develop posterior odds ratio for testing the presence of unit root model against the alternative of stationary model. For this purpose we test the null hypothesis $H_0 : \rho = 1$ against the alternative $H_1 : \rho \in S$ with $S = \{a < \rho < 1; a > -1\}$. Under unit root hypothesis the model (3) reduces to

$$\Delta y_{it} = \delta_i + w_t + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{it-j} + \varepsilon_{it} \tag{5}$$

where Δ is the difference operator. We can write the model (4) and model (5) in matrix notation as

Under H_0: $\Delta y = \left(I_n \otimes l_T\right)\delta + l_T' w + X\theta + \varepsilon$ (6)

Under H_1: $y = \rho y_{-1} + Z\gamma + l_T' w + X\theta + \varepsilon$ (7)

where l_T be a $T \times 1$ vector with all element 1 and $l_n = (1, 2, ...T)'$. Further

$$\Delta y_i = (\Delta y_{i1}, \Delta y_{i2}, ..., \Delta y_{it})', \quad \Delta y = \left(\Delta y_1', \Delta y_2', ..., \Delta y_n'\right)',$$

$$\delta = \left(\delta_1, ..., \delta_n\right)', \qquad w = \left(w_1, w_2 \cdots w_T\right)',$$

$$\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)', \qquad \beta = \left(\beta_1, \beta_2, ..., \beta_n\right)',$$

$$\gamma = \left[\alpha \quad \beta\right]', \qquad Z = \left[\left(I_n \otimes l_t\right) \quad \left(I_n \otimes l_n\right)\right],$$

$$\theta_i = \begin{pmatrix} \theta_{i1} \\ \theta_{i2} \\ \vdots \\ \theta_{ik_i} \end{pmatrix} \qquad X_i = \begin{pmatrix} \Delta y_{i\,0} & \Delta y_{i\,-1} & \cdots & \Delta y_{i\,1-k_i} \\ \Delta y_{i\,-1} & \Delta y_{i\,0} & \cdots & \Delta y_{i\,2-k_i} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta y_{i\,T-1} & \Delta y_{i\,T-2} & & \Delta y_{i\,T-k_i} \end{pmatrix}$$

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix} \qquad X = \begin{pmatrix} X_1' & 0 & \cdots & 0 \\ 0 & X_2' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_n' \end{pmatrix}.$$

3. PRIOR DISTRIBUTION AND POSTERIOR ODDS RATIO

For obtaining the posterior probability for the proposed model in Bayesian framework, one may assume some additional information about the parameter known as prior information. We consider the following priors from Schotman and van Dijk (1991 a,b):

$$\delta \sim N\left(0, \frac{1}{\vartheta\tau} I_n\right)$$

$$w \sim N\left(0, \frac{1}{\omega\tau} I_T\right)$$

$$\alpha \sim N\left((1-\rho)y_0, \quad \frac{(1+\rho)}{(1-\rho)\tau} I_n\right)$$

$$\beta \sim N\left(0, \frac{(1-\rho)^2}{\vartheta\tau} I_n\right)$$

$$p(\tau) \propto \frac{1}{\tau};\ 0 < \tau < \infty$$

$$p(\rho) = \frac{1}{1-a};\ a < \rho < 1$$

$$p(\theta) \propto 1;\ \theta \epsilon R^k$$

Then, joint prior for γ is

$$\gamma \sim N\left((1-\rho)\varphi_0, \frac{1}{\tau} V(\rho)^{-1}\right)$$

where

$$\varphi_0 = \begin{pmatrix} y_0 \\ 0 \end{pmatrix}, \quad V(\rho) = \begin{pmatrix} \frac{(1+\rho)}{(1-\rho)} I_n & 0 \\ 0 & \frac{\vartheta}{(1-\rho)^2} I_n \end{pmatrix}$$

y_0 is the initial observations vector for the generating series.

Now we define

$$\Sigma = I_{nT} - X\left(X'X\right)^{-1} X'$$

$$D = l_T \Sigma l_T^{'} + \omega I_T$$

$$H = \Sigma - \Sigma l_T^{'} D^{-1} l_T \Sigma$$

$$K = \left(I_n \otimes l_T\right)' H \left(I_n \otimes l_T\right) + \vartheta I_n$$

$$K(\rho) = Z' HZ + V(\rho)$$

$$\Phi = \Delta y' \left(H - H'\left(I_n \otimes l_T\right) K^{-1} \left(I_n \otimes l_T\right)' H \right) \Delta y$$

$$\Phi(\rho) = \left(y - \rho y_{-1}\right)' H \left(y - \rho y_{-1}\right) + \left(1 - \rho^2\right) \varphi_0^{'} \varphi_0$$
$$- \left(Z'H'\left(y - \rho y_{-1}\right) + (1+\rho)\varphi_0\right)' K^{-1}(\rho) \left(Z'H'\left(y - \rho y_{-1}\right) + (1+\rho)\varphi_0\right)$$

Theorem:

For observing the presence of unit root in panel AR(1) time series model with time effect, the posterior odds ratio for testing difference stationarity against the alternative of trend stationarity with prior odds ratio $\frac{p_0}{1-p_0}$ is

$$\beta_{01} = \frac{p_0}{1-p_0} \frac{1-a}{|K|^{\frac{1}{2}} \left[\Phi\right]^{\frac{nT-k}{2}}} \left[\int_a^1 \frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}} |K(\rho)|^{\frac{1}{2}} \left[\Phi(\rho)\right]^{\frac{nT-k}{2}}} d\rho \right]^{-1} \quad (8)$$

Proof:

Under unit root hypothesis, likelihood function is written as

$$p\left(y \middle| \delta, \theta, w, \tau\right) = \frac{\tau^{\frac{nT}{2}}}{(2\pi)^{\frac{nT}{2}}} \exp\left[-\frac{\tau}{2} \left\{ \left(\Delta y - (I_n \otimes l_T)\delta - l_T^{'} w - X\theta\right)' \right.\right.$$
$$\left.\left. \left(\Delta y - (I_n \otimes l_T)\delta - l_T^{'} w - X\theta\right) \right\} \right]$$

Combining the likelihood function with the prior distributions lead to

$$p\left(y\middle|H_0\right)=\int_0^\infty\int_{R^n}\int_{R^T}\int_{R^k}\frac{\tau^{\frac{nT+T+n}{2}-1}\omega^{\frac{T}{2}}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+T+n}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta y-(I_n\otimes l_T)\delta-l_T^{'}w-X\theta\right)'\right.\right.$$

$$\left.\left.\left(\Delta y-(I_n\otimes l_T)\delta-l_T^{'}w-X\theta\right)+\vartheta\delta'\delta+w'\omega I_T w\right\}\right]d\theta dw d\delta d\tau$$

Let

$$\tilde{\theta}=\left(X'X\right)^{-1}X'\left(\Delta y-\left(I_n\otimes l_T\right)\delta-\left(l_n\otimes I_T\right)w\right)$$

$$=\int_0^\infty\int_{R^n}\int_{R^T}\int_{R^k}\frac{\tau^{\frac{nT+T+n}{2}-1}\omega^{\frac{T}{2}}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+T+n}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta y-(I_n\otimes l_T)\delta-l_T^{'}w\right)'\right.\right.$$

$$\sum\left(\Delta y-(I_n\otimes l_T)\delta-l_T^{'}w\right)+\left(\theta-\tilde{\theta}\right)'X'X\left(\theta-\tilde{\theta}\right)$$

$$\left.\left.+\vartheta\delta'\delta+w'\omega I_T w\right\}\right]d\theta dw d\delta d\tau$$

$$=\int_0^\infty\int_{R^n}\int_{R^T}\frac{\tau^{\frac{nT+T+n-k}{2}-1}\vartheta^{\frac{n}{2}}\omega^{\frac{T}{2}}}{(2\pi)^{\frac{nT+T+n-k}{2}}\left|X'X\right|^{\frac{1}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta y-(I_n\otimes l_T)\delta\right)'\right.\right.$$

$$\sum\left(\Delta y-(I_n\otimes l_T)\delta\right)+w'\left(l_T\sum l_T^{'}+\omega I_T\right)w$$

$$\left.\left.-2w'l_T^{'}\sum\left(\Delta y-(I_n\otimes l_T)\delta\right)+\vartheta\delta'\delta\right\}\right]dw d\delta d\tau$$

Further, we observe that

$$w'\left(l_T\sum l_T^{'}+\omega I_T\right)w-2w'l_T\sum\left(\Delta y-(I_n\otimes l_T)\delta\right)$$

$$=\left(w-\tilde{w}\right)^{'}D\left(w-\tilde{w}\right)-\left(\Delta y-(I_n\otimes l_T)\delta\right)'\sum l_T^{'}D^{-1}l_T\sum\left(\Delta y-(I_n\otimes l_T)\delta\right)$$

where

$$\tilde{w}=D^{-1}l_T\sum\left(\Delta y-(I_n\otimes l_T)\delta\right)$$

Then, we have

$$p\left(y\middle|H_0\right)=\int_0^\infty\int_{R^n}\frac{\tau^{\frac{nT+n-k}{2}-1}\omega^{\frac{T}{2}}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n-k}{2}}\left|X'X\right|^{\frac{1}{2}}\left|D\right|^{\frac{1}{2}}}\exp\left[-\frac{\tau}{2}\left\{\Delta y^{'}H\Delta y\right.\right.$$

$$\left.\left.+\delta^{'}\left((I_n\otimes l_T)'H(I_n\otimes l_T)+\vartheta I_n\right)\delta-2\delta'(I_n\otimes l_T)'H\Delta y\right\}\right]d\delta d\tau$$

Let us consider $\tilde{\delta} = K^{-1}(I_n \otimes l_T)' H \Delta y$, then

$$\delta' \left((I_n \otimes l_T)' H (I_n \otimes l_T) + \vartheta I_n \right) \delta - 2\delta' (I_n \otimes l_T)' H \Delta y = \left(\delta - \tilde{\delta}\right) K \left(\delta - \tilde{\delta}\right)$$
$$- \Delta y' H' (I_n \otimes l_T) K^{-1} (I_n \otimes l_T)' H \Delta y$$

Then, we get the reduce form

$$P\left(y \middle| H_0\right) = \frac{\omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT-k}{2}} |D|^{\frac{1}{2}} |K|^{\frac{1}{2}} |X'X|^{\frac{1}{2}}} \int_0^\infty \tau^{\frac{nT-k}{2}-1} \exp\left[-\frac{\tau}{2} \{\Phi\} \right]$$
$$\int_{R^n} \frac{\tau^{\frac{n}{2}} |K|^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}} \exp\left[-\frac{\tau}{2} \left\{ \left(\delta - \tilde{\delta}\right)' K \left(\delta - \tilde{\delta}\right) \right\} \right] d\delta d\tau$$
$$= \frac{\omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT-k}{2}} |D|^{\frac{1}{2}} |K|^{\frac{1}{2}} |X'X|^{\frac{1}{2}}} \int_0^\infty \tau^{\frac{nT-k}{2}-1} \exp\left[-\frac{\tau}{2} \{\Phi\} \right] d\tau$$
$$= \frac{\omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} \Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}} |X'X|^{\frac{1}{2}} |D|^{\frac{1}{2}} |K|^{\frac{1}{2}} [\Phi]^{\frac{nT-k}{2}}} \tag{9}$$

Further, under H_1, the likelihood function is given by

$$p\left(y \middle| y_0, \gamma, w, \theta, \tau, \rho\right) = \exp\left[-\frac{\tau}{2} \left\{ \left(y - \rho y_{-1} - Z\gamma - l_T' w - X\theta \right)' \right.\right.$$
$$\left.\left. \left(y - \rho y_{-1} - Z\gamma - l_T' w - X\theta \right) \right\} \right]$$

Combining the likelihood function with the prior distributions lead to

$$p(y|H_1) = \int_a^1 \int_0^\infty \int_{R^{2n}} \int_{R^{Tk}} \int_{R^k} \frac{\tau^{\frac{nT+T}{2}+n-1} \omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} |V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT+T}{2}+n} (1-a)}$$
$$\exp\left[-\frac{\tau}{2} \left\{ \left(y - \rho y_{-1} - Z\gamma - l_T' w - X\theta \right)' \right.\right.$$
$$\left(y - \rho y_{-1} - Z\gamma - l_T' w - X\theta \right) + w' \omega I_T w$$
$$\left.\left. + \left(\gamma - (1-\rho)\varphi_0 \right)' V(\rho) (\gamma - (1-\rho)\varphi_0) \right\} \right] d\theta d\gamma d\tau d\rho dw$$

Further writing

$$\hat{\theta} = (X'X)^{-1} X'\left(y - \rho y_{-1} - Z\gamma - l_T' w\right)$$
$$\hat{w} = D^{-1}(l_n \otimes I_T)'\Sigma(y - \rho y_{-1} - Z\gamma)$$
$$\hat{\gamma} = K^{-1}(\rho)\left(Z'H(y - \rho y_{-1}) + (1+\rho)\varphi_0\right)$$

and integrating with respect to θ, *w*, and $\gamma, \rho(y \mid H_1)$ reduces as

$$p(y|H_1) = \int_a^1 \int_0^\infty \int_{R^{2n}} \int_{R^T} \int_{R^k} \frac{\tau^{\frac{nT+T}{2}+n-1} \omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} |V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT+T}{2}+n}(1-a)} \exp\left[-\frac{\tau}{2}\left\{\left(y - \rho y_{-1} - Z\gamma - l_T' w\right)'\right.\right.$$
$$\Sigma\left(y - \rho y_{-1} - Z\gamma - l_T' w\right) + \left(\theta - \hat{\theta}\right)' X'X\left(\theta - \hat{\theta}\right) + w'\omega I_T w$$
$$\left.\left. + \left(\gamma - (1-\rho)\varphi_0\right)' V(\rho)\left(\gamma - (1-\rho)\varphi_0\right)\right\}\right] d\theta\, dw\, d\gamma\, d\tau\, d\rho$$

$$= \int_a^1 \int_0^\infty \int_{R^{2n}} \int_{R^T} \frac{\tau^{\frac{nT+T-k}{2}+n-1} \omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} (1+\rho)^{\frac{n}{2}}}{(2\pi)^{\frac{nT+T-k}{2}+n} (1-\rho)^{\frac{3n}{2}} (1-a)|X'X|^{\frac{1}{2}}} \exp\left[-\frac{\tau}{2}\left\{\left(y - \rho y_{-1} - Z\gamma\right)'\right.\right.$$
$$H(y - \rho y_{-1} - Z\gamma) + \left(\gamma - (1-\rho)\varphi_0\right)' V(\rho)\left(\gamma - (1-\rho)\varphi_0\right)$$
$$\left.\left. + (w - \hat{w})' D (w - \hat{w}) \right\}\right] dw\, d\gamma\, d\tau\, d\rho$$

$$= \int_a^1 \int_0^\infty \int_{R^{2n}} \frac{\tau^{\frac{nT-k}{2}+n-1} \omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} |V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT-k}{2}+n} (1-a)|X'X|^{\frac{1}{2}} |D|^{\frac{1}{2}}}$$
$$\exp\left[-\frac{\tau}{2}\left\{\varsigma(\rho) + (\gamma - \tilde{\gamma})' K(\rho)(\gamma - \tilde{\gamma})\right\}\right] d\gamma\, d\tau\, d\rho$$

$$= \int_a^1 \int_0^\infty \frac{\tau^{\frac{nT-k}{2}-1} \omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} (1+\rho)^{\frac{n}{2}}}{(2\pi)^{\frac{nT-k}{2}} (1-\rho)^{\frac{3n}{2}} (1-a)|X'X|^{\frac{1}{2}} |D|^{\frac{1}{2}} |K(\rho)|^{\frac{1}{2}}} \exp\left[-\frac{\tau}{2}\Phi(\rho)\right] d\tau\, d\rho$$

$$= \frac{\omega^{\frac{T}{2}} \vartheta^{\frac{n}{2}} \Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}} (1-a)|X'X|^{\frac{1}{2}} |D|^{\frac{1}{2}}} \int_a^1 \frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}} |K(\rho)|^{\frac{1}{2}} \left[\Phi(\rho)\right]^{\frac{nT-k}{2}}} d\rho \qquad (10)$$

Utilizing (9) and (10), we get the required expression for posterior odds ratio.

4. NUMERICAL ILLUSTRATION

Understanding the need of establishment of proposed procedure is very important before actually implementing it for any real situation. For defending the worthiness of findings, empirical as well as simulation studies are frequently used methodologies. Therefore an empirical analysis for the model under study is explored and then a simulation study is also carried out.

4.1 Empirical Analysis

National Pension Scheme (NPS) was started by government of India as a new yardstick to begin with system which will facilitate all citizens of India to secure their future when they are not in job or not in position to work due to old age. During the reformation of the pensions, this was mainly designed to extend the coverage, shape a pension plan for overall population and also provide more opportunities to the employee and benefits after retirement. This was first implemented for central government employees in 2004 and later on accepted by other state governments. Initially, NPS was introduced for the new government recruits (except armed forces) with effect from 1st May, 2009. Since 2010 the contributed pension scheme was extended to unorganized sector under *Swawlamban* national pension scheme. NPS funds are managed by multiple banks through two mechanisms; first which does not allow withdrawal before retirement named Tier-I and second which allows withdrawal named Tier-II. Both funds are invested in equity market (E), fixed return (C) and government bonds (G). NPS is a voluntary retirement savings scheme and planned to enable systematic savings during the subscriber's working life. It is an attempt toward providing adequate retirement income to every citizen of India. In India, there is a big young population at working position. If this young population is placed into a modern pension system, they will be able to accumulate pension assets for their old age, and India would then be able to tackle the crises related with an ageing population. The pension reforms effort that has spread-out in India from 1998 to 2005. Shah (2005) summarized the pension reform and its goal with detailed comparison of new and old pension schemes. For the literature on the pension funds discussing the performance, challenges, issues and required reforms for pension funds, one may refer to Black (1989), Brown *et al.* (2011), Dushi *et al.* (2010) and Franzen (2010). Recently Sane and Thomas (2013) addressed all associate issues with NPS and explore the possibilities and benefits of NPS on the market.

We have recorded the time series of daily NAV and considered the Schemes as panel in respect to bank and tier. We focus on testing whether the series is difference stationary or trend stationary or equivalently in testing the unit root hypothesis H_0:ρ=1 against the alternative H_1:$\rho\in S$ with $S=\{a<\rho<1;\ a>-1\}$.Let $\{y(i,j,k)_t\}$ denotes the recorded NAV of i^{th} Bank, j^{th} Tier and k^{th} Scheme, i=1(ICICI), 2(SBI) and 3(KM) stands for bank; j=1(Tier-1), 2(Tier-2) stands for tier and k=1(E), 2(C), 3(G) stands for scheme. We have considered the model for the analysis:

$$y(i,j,k)_t = \alpha(i,j,k) + \beta\,(i,j,k)*t + \rho(i,j,k)\,y(i,j,k)_{t-1} + w(i,j,k)_t + \varepsilon(i,j,k)_t$$

We have taken the time series of recorded daily NAV for the period February 01, 2010 to December 31, 2015. For analysis purpose, series is converted into monthly average and test the unit root hypothesis using the derived posterior odds ratio considering panel in respect to Schemes.

The model under study is:

$$NAV(i,j)_{k,t} = \text{intercept}(i,j)_k + \text{trend}(i,j)_k t + \rho(i,j) NAV(i,j)_{k,t-1} + w(i,j)_t + \varepsilon(i,j)_k$$

The maximum likelihood estimates of autoregressive coefficient, intercept (i, j), trend (i, j) and $\hat{\rho}(i, j)$ are given in appendix A-1 to A-3 along with the variance covariance matrix of regression coefficients. The posterior odds ratio, estimated value of $\hat{\rho}$ with $SE(\hat{\rho})$ and error variance ($\hat{\sigma}^2$) are recorded in Table 1.

Table 1
POR Values with Parameter Estimates

Banks	Tier I				Tier II			
	β_{01}	$\hat{\rho}$	$SE(\hat{\rho})$	$\hat{\sigma}^2$	β_{01}	$\hat{\rho}$	$SE(\hat{\rho})$	$\hat{\sigma}^2$
ICICI	1.09E-24	0.9399	0.5651	8.6974	1.49E-38	0.9303	0.4356	5.3026
KM	6.3E-113	0.9337	0.5148	6.8313	2.24E-111	0.9361	0.4262	4.9772
SBI	3.42E-33	0.9346	0.4963	6.6698	5.12E-35	0.9309	0.455	5.5286

It is evident from the table 1 that all observed posterior odds ratios for both tiers of the banks under study are less than one leading to the rejection of the unit root hypothesis. From the analysis it can be concluded that all the NAV series of different banks under study are trend stationary.

4.2 Simulation Study

This section carries out simulation using Matlab software with the objective of investigating the performance of derived posterior odds ratio. We have written the code to generate data and validate the situation through a simulated data for the established model under study:

$$y_{it} = \rho y_{it-1} + \left[(1-\rho)\mu_i + \rho\delta_i\right] + (1-\rho)\delta_i t + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{it-j} + w_t + \varepsilon_{it}\,;\ (i=1,\dots,n;\ t=1,\dots,T)$$

We consider three panels with $\varepsilon_{it} \sim N(0,1)$ to generate a panel data time series of size 25. For generating the series the initial observations for three panels are taken as $y_{10} = 1000$, $y_{20} = 1500$, $y_{30} = 2000$, coefficients of augmentation term of order 1 as $\theta_{11} = \theta_{21} = \theta_{31} = 1$, time effect term following $N(0,(\omega\tau)^{-1} I_n)$, intercept term as μ_1=750, μ_2=1000, μ_3=1250 which are fixed while generating series for all the possible 27 combinations of values $\{\delta_1, \delta_2, \delta_3\} = \{1, 1.25, 1.5\}$.

We have tested the presence of unit root using model (3) for $\rho = \{0.92, 0.94, 0.96, 0.98\}$ and consider equal prior probabilities for the null and alternative hypothesis. The posterior odds ratio(POR) and estimated value $\hat{\rho}$ with $SE(\hat{\rho})$ for testing unit root hypothesis for various values of the autoregressive coefficient, are obtained and presented graphically in Figure 1:

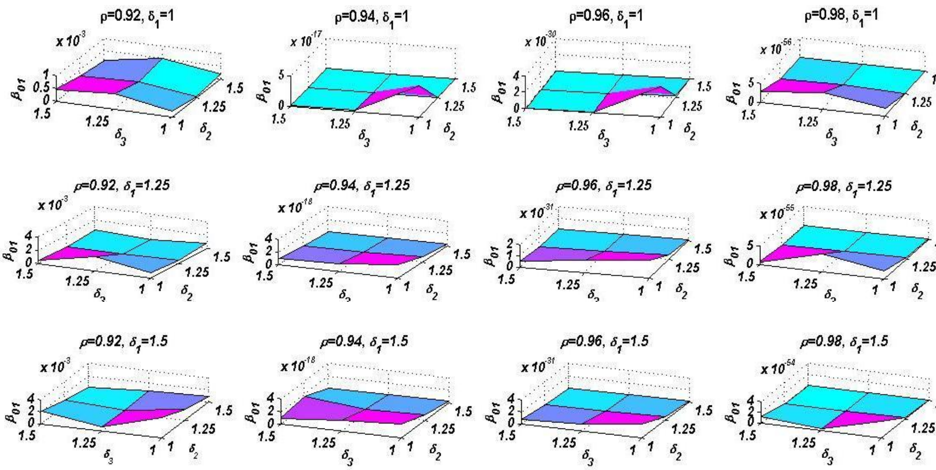

Figure 1

Here, we have plotted POR against the nine different possible combination of coefficient of time trend $\delta = (\delta_1, \delta_2, \delta_3)$ while fixing δ_1 as 1, 1.25 and 1.5 respectively, it is clear from the figure that the values of POR is tending to zero with the increasing value of autoregressive coefficient ρ.

5. CONCLUDING REMARKS

The present paper focuses on developing the POR for testing the unit root hypothesis for PAR (1) model with time effect. A simulation study reveals that POR increases with increase in the value of auto regression coefficient. An empirical study for NPS data is also presented for illustration purpose. The study may be extended for the panel data time series models involving non-normal errors as well as for model with covariate and structural break.

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APPENDIX

Table A-1

ICICI
With Time Effect and Augmentation Term of Order 1

Coefficient	Variance Covariance Matrix $(\hat{\Sigma})$									
					Tier-I					
$\hat{\rho}$ 0.940	0.319	-3.072	-3.003	-2.964	-0.043	-0.041	-0.03	-0.159	-0.215	-0.179
$\hat{\mu}_1$ 0.706	-3.072	33.595	28.888	28.505	0.34	0.39	0.284	1.156	2.07	1.723
$\hat{\mu}_2$ 0.626	-3.003	28.888	32.641	27.872	0.408	0.311	0.278	1.496	-2.953	1.684
$\hat{\mu}_3$ 0.585	-2.964	28.505	27.872	31.544	0.403	0.377	0.198	1.476	1.997	0.627
$\hat{\beta}_1$ 0.008	-0.043	0.34	0.408	0.403	0.008	0.006	0.004	0.02	0.029	0.024
$\hat{\beta}_2$ 0.008	-0.041	0.39	0.311	0.377	0.006	0.007	0.004	0.02	-0.061	0.023
$\hat{\beta}_3$ 0.006	-0.03	0.284	0.278	0.198	0.004	0.004	0.005	0.015	0.02	-0.032
$\hat{\theta}_{11}$ 0.004	-0.159	1.156	1.496	1.476	0.02	0.02	0.015	3.277	0.107	0.089
$\hat{\theta}_{12}$ 0.360	-0.215	2.07	-2.953	1.997	0.029	-0.061	0.02	0.107	64.207	0.121
$\hat{\theta}_{13}$ 0.457	-0.179	1.723	1.684	0.627	0.024	0.023	-0.032	0.089	0.121	30.857
					Tier-II					
$\hat{\rho}$ 0.930	0.1897	-1.6032	-1.823	-1.784	-0.021	-0.0237	-0.0181	-0.0824	-0.11	-0.098
$\hat{\mu}_1$ 0.621	-1.6032	15.241	15.401	15.074	0.1404	0.2002	0.1529	0.6306	0.932	0.8267
$\hat{\mu}_2$ 0.733	-1.8225	15.401	19.381	17.136	0.2016	0.1936	0.1738	0.7915	-1.182	0.9398
$\hat{\mu}_3$ 0.684	-1.7838	15.074	17.136	18.481	0.1973	0.2228	0.1339	0.7746	1.037	0.4389
$\hat{\beta}_1$ 0.009	-0.021	0.1404	0.2016	0.1973	0.0034	0.0026	0.002	0.0057	0.0122	0.0108
$\hat{\beta}_2$ 0.009	-0.0237	0.2002	0.1936	0.2228	0.0026	0.0041	0.0023	0.0103	-0.023	0.0122
$\hat{\beta}_3$ 0.007	-0.0181	0.1529	0.1738	0.1339	0.002	0.0023	0.0028	0.0079	0.0105	-0.012
$\hat{\theta}_{11}$ 0.003	-0.0824	0.6306	0.7915	0.7746	0.0057	0.0103	0.0079	2.0786	0.0479	0.0425
$\hat{\theta}_{12}$ 0.343	-0.1103	0.932	-1.182	1.037	0.0122	-0.0231	0.0105	0.0479	27.654	0.0569
$\hat{\theta}_{13}$ 0.475	-0.0978	0.8267	0.9398	0.4389	0.0108	0.0122	-0.0118	0.0425	0.0569	12.614

Table A-2

KM
With Time Effect and Augmentation Term of Order 1

Coefficient	Variance Covariance Matrix $(\hat{\Sigma})$										
Tier-I											
$\hat{\rho}$	0.934	0.265	-2.535	-2.723	-2.579	-0.034	-0.034	-0.025	-0.113	-0.139	-0.153
$\hat{\mu}_1$	0.729	-2.535	26.982	26.045	24.676	0.269	0.327	0.237	0.989	1.334	1.462
$\hat{\mu}_2$	0.739	-2.723	26.045	30.917	26.498	0.351	0.298	0.255	1.162	-1.434	1.570
$\hat{\mu}_3$	0.681	-2.579	24.676	26.498	27.861	0.333	0.333	0.184	1.100	1.358	0.729
$\hat{\beta}_1$	0.009	-0.034	0.269	0.351	0.333	0.006	0.004	0.003	0.010	0.018	0.020
$\hat{\beta}_2$	0.009	-0.034	0.327	0.298	0.333	0.004	0.006	0.003	0.015	-0.036	0.020
$\hat{\beta}_3$	0.007	-0.025	0.237	0.255	0.184	0.003	0.003	0.004	0.011	0.013	-0.016
$\hat{\theta}_{11}$	-0.012	-0.113	0.989	1.162	1.100	0.01	0.015	0.011	2.570	0.060	0.065
$\hat{\theta}_{12}$	0.422	-0.139	1.334	-1.434	1.358	0.018	-0.036	0.013	0.060	36.847	0.080
$\hat{\theta}_{13}$	0.448	-0.153	1.462	1.570	0.729	0.02	0.02	-0.016	0.065	0.080	18.942
Tier-II											
$\hat{\rho}$	0.936	0.182	-1.536	-1.685	-1.686	-0.021	-0.019	-0.016	-0.080	-0.097	-0.106
$\hat{\mu}_1$	0.625	-1.536	14.433	14.246	14.253	0.146	0.161	0.133	0.591	0.823	0.893
$\hat{\mu}_2$	0.633	-1.685	14.246	17.174	15.640	0.194	0.149	0.146	0.747	-0.813	0.980
$\hat{\mu}_3$	0.621	-1.686	14.253	15.64	17.118	0.194	0.177	0.116	0.747	0.904	0.424
$\hat{\beta}_1$	0.008	-0.021	0.146	0.194	0.194	0.003	0.002	0.002	0.007	0.011	0.012
$\hat{\beta}_2$	0.007	-0.019	0.161	0.149	0.177	0.002	0.003	0.002	0.008	-0.033	0.011
$\hat{\beta}_3$	0.006	-0.016	0.133	0.146	0.116	0.002	0.002	0.002	0.007	0.008	-0.008
$\hat{\theta}_{11}$	0.009	-0.080	0.591	0.747	0.747	0.007	0.008	0.007	1.897	0.043	0.047
$\hat{\theta}_{12}$	0.403	-0.097	0.823	-0.813	0.904	0.011	-0.033	0.008	0.043	30.743	0.057
$\hat{\theta}_{13}$	0.467	-0.106	0.893	0.98	0.424	0.012	0.011	-0.008	0.047	0.057	12.901

Table A-3

SBI
With Time Effect and Augmentation Term of Order 1

Coefficient	Variance Covariance Matrix $(\hat{\Sigma})$										
					Tier-I						
$\hat{\rho}$	0.935	0.246	-2.095	-2.366	-2.417	-0.027	-0.032	-0.025	-0.113	-0.144	-0.143
$\hat{\mu}_1$	0.63	-2.095	20.177	20.123	20.555	0.184	0.27	0.216	0.839	1.224	1.218
$\hat{\mu}_2$	0.689	-2.366	20.123	25.351	23.218	0.259	0.262	0.244	1.088	-1.468	1.376
$\hat{\mu}_3$	0.694	-2.417	20.555	23.218	26.134	0.265	0.311	0.204	1.111	1.412	0.551
$\hat{\beta}_1$	0.008	-0.027	0.184	0.259	0.265	0.004	0.003	0.003	0.009	0.016	0.016
$\hat{\beta}_2$	0.009	-0.032	0.27	0.262	0.311	0.003	0.005	0.003	0.015	-0.013	0.018
$\hat{\beta}_3$	0.007	-0.025	0.216	0.244	0.204	0.003	0.003	0.004	0.012	0.015	0.003
$\hat{\theta}_{11}$	-0.01	-0.113	0.839	1.088	1.111	0.009	0.015	0.012	2.777	0.066	0.066
$\hat{\theta}_{12}$	0.425	-0.144	1.224	-1.468	1.412	0.016	-0.013	0.015	0.066	30.918	0.084
$\hat{\theta}_{13}$	0.43	-0.143	1.218	1.376	0.551	0.016	0.018	0.003	0.066	0.084	11.9
					Tier-II						
$\hat{\rho}$	0.931	0.207	-1.698	-2.038	-2.073	-0.023	-0.024	-0.021	-0.083	-0.128	-0.119
$\hat{\mu}_1$	0.624	-1.698	15.719	16.724	17.006	0.147	0.196	0.17	0.647	1.054	0.973
$\hat{\mu}_2$	0.749	-2.038	16.724	22.076	20.411	0.222	0.199	0.204	0.82	-1.052	1.168
$\hat{\mu}_3$	0.742	-2.073	17.006	20.411	22.583	0.225	0.24	0.17	0.834	1.286	0.563
$\hat{\beta}_1$	0.009	-0.023	0.147	0.222	0.225	0.004	0.003	0.002	0.005	0.014	0.013
$\hat{\beta}_2$	0.008	-0.024	0.196	0.199	0.24	0.003	0.004	0.002	0.01	-0.004	0.014
$\hat{\beta}_3$	0.007	-0.021	0.17	0.204	0.17	0.002	0.002	0.003	0.008	0.013	0.002
$\hat{\theta}_{11}$	-0.01	-0.083	0.647	0.82	0.834	0.005	0.01	0.008	2.304	0.052	0.048
$\hat{\theta}_{12}$	0.42	-0.128	1.054	-1.052	1.286	0.014	-0.004	0.013	0.052	24.547	0.074
$\hat{\theta}_{13}$	0.432	-0.119	0.973	1.168	0.563	0.013	0.014	0.002	0.048	0.074	9.058