Pak. J. Statist.
2018 Vol. 34(5), 377-380

REMARK ON THE EIWG AND BGIWG DISTRIBUTIONS AND THEIR CHARACTERIZATIONS

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ABSTRACT

The distribution proposed by Chung et al. (2017) is the same as the one proposed by Elbatal et al. (2017), which has been, we believe independently, the subject of the investigation of two groups of authors. We mention this distribution here and present certain characterizations of it.

1. INTRODUCTION

The Beta Generalized Inverse Weibull Geometric (BGIWG) distribution of Elbatal et al. (2017) has the cdf (cumulative distribution function) and pdf (probability density function) given, respectively, by

$$F(x;\alpha,\theta,\gamma,p)=\frac{e^{-\gamma(\alpha x)^{-\theta}}}{1-p\left[1-e^{-\gamma(\alpha x)^{-\theta}}\right]}, x\geq 0, \tag{1}$$

and

$$f(x;\alpha,\theta,\gamma,p)=\frac{(1-p)\alpha\theta\gamma(\alpha x)^{-\theta-1}e^{-\gamma(\alpha x)^{-\theta}}}{\left\{1-p\left[1-e^{-\gamma(\alpha x)^{-\theta}}\right]\right\}^{2}}, x>0, \tag{2}$$

where α,θ,γ positive and $p\in(0,1)$ are parameters.

The Exponentiated Inverse Weibull Geometric (BGIWG) distribution of Chung et al. (2017) has the cdf and pdf given, respectively, by

$$F(x;\alpha,\gamma,q)=\frac{e^{-\gamma x^{-\alpha}}}{1-q[1-e^{-\gamma x^{-\alpha}}]}, x\geq 0, \tag{3}$$

and

$$f(x;\alpha,\gamma,q)=\frac{(1-q)\alpha\gamma x^{-(\alpha+1)}e^{-\gamma x^{-\alpha}}}{\{1-q[1-e^{-\gamma x^{-\alpha}}]\}^{2}}, x>0, \tag{4}$$

where α,γ positive and $q\in(0,1)$ are parameters.

Remark 1:

Observe that the extra parameter in BGIWG does not have a significant role, so for all the theoretical as well as the practical purposes, (1) and (3) are the same. In characterization results, the preference is to have less number of parameters to make the characterizations more elegant. So, we will present certain characterizations of (3) in the following section.

2. CHARACTERIZATIONS OF EIWG DISTRIBUTION

This section deals with the characterizations of EIWG distribution based on: (i) the ration of two truncated moments; (ii) the reverse (reversed) hazard function. These characterizations will be presented in two subsections.

2.1 Characterization based on the Ratio of Two Truncated Moments

Our first characterization employs a theorem due to Glänzel (1987), see Theorem 1 of Appendix A. The result, however, holds also when the interval H is not closed, since the condition of the Theorem is on the interior of H.

Proposition 2.1:

Let $X: \Omega \to (0, \infty)$ be a continuous random variable and let $h(x) = \left\{1 - q\left[1 - e^{-\gamma x^{-\alpha}}\right]\right\}^2$ and $g(x) = h(x) e^{-\gamma x^{-\alpha}}$ for $x > 0$. Then, the random variable X has pdf (4) if and only if the function ξ defined in Theorem 1 is of the form

$$\xi(x) = \frac{1}{2}\left\{1 + e^{-\gamma x^{-\alpha}}\right\}, x > 0.$$

Proof:

Suppose the random variable X has pdf (4), then

$$\left(1 - F(x)\right) E[h(X)|X \geq x] = (1 - q)\left(1 - e^{-\gamma x^{-\alpha}}\right), x > 0,$$

and

$$\left(1 - F(x)\right) E[g(X)|X \geq x] = \frac{(1 - q)}{2}\left(1 - e^{-2\gamma x^{-\alpha}}\right), x > 0.$$

Further,

$$\xi(x) h(x) - g(x) = \frac{1}{2} h(x)\left(1 - e^{-\gamma x^{-\alpha}}\right) > 0 \; for \; x > 0.$$

Conversely, if ξ is of the above form, then

$$s'(x) = \frac{\xi'(x) h(x)}{\xi(x) h(x) - g(x)} = \frac{\alpha \gamma x^{-(\alpha+1)} e^{-\gamma x^{-\alpha}}}{1 - e^{-\gamma x^{-\alpha}}}, x > 0,$$

and consequently

$$s(x) = -\log\left\{\left(1 - e^{-\gamma x^{-\alpha}}\right)\right\}, x > 0.$$

Now, according to Theorem 1, X has density (4).

Corollary 2.1:

Let $X:\Omega \rightarrow (0,\infty)$ be a continuous random variable and let $h(x)$ be as in Proposition 2.1. The random variable X has pdf (4) if and only if there exist functions g and ξ defined in Theorem 1 satisfying the following differential equation

$$\frac{\xi'(x)h(x)}{\xi(x)h(x)-g(x)} = \frac{\alpha\gamma x^{-\alpha-1}e^{-\gamma x^{-\alpha}}}{1-e^{-\gamma x^{-\alpha}}}, x>0.$$

The general solution of the differential equation in Corollary 2.1 is

$$\xi(x) = \left(1-e^{-\gamma x^{-\alpha}}\right)^{-1}\left[-\int \alpha\gamma x^{-(\alpha+1)}e^{-\gamma x^{-\alpha}}\left(h(x)\right)^{-1}g(x)dx + D\right],$$

where D is a constant. We like to point out that one set of functions satisfying the above differential equation is given in Proposition 2.1 with $D=\frac{1}{2}$. Clearly, there are other triplets (h,g,ξ) which satisfy conditions of Theorem1.

2.2 Characterization in Terms of the Reverse Hazard Function

The reverse hazard function, r_F, of a twice differentiable distribution function, F, is defined as

$$r_F(x) = \frac{f(x)}{F(x)}, x \in support\ of\ F.$$

Proposition 2.2:

Let $X:\Omega \rightarrow (0,\infty)$ be a continuous random variable. The random variable X has pdf (4) if and only if its reverse hazard function $r_F(x)$ satisfies the following differential equation

$$r_F'(x) + \frac{\alpha+1}{x}r_F(x) = \frac{q(1-q)\alpha^2\gamma^2x^{-2(\alpha+1)}e^{-\gamma x^{-\alpha}}}{\{1-q[1-e^{-\gamma x^{-\alpha}}]\}^2}, x>0. \tag{5}$$

Proof:

If X has pdf (4), then clearly (5) holds. Now, if (5) holds, then

$$\frac{d}{dx}\left\{x^{(\alpha+1)}r_F(x)\right\} = \frac{q(1-q)\alpha^2\gamma^2x^{-(\alpha+1)}e^{-\gamma x^{-\alpha}}}{\{1-q[1-e^{-\gamma x^{-\alpha}}]\}^2}$$

$$= \alpha\gamma(1-q)\frac{d}{dx}\left\{\left[1-q\left[1-e^{-\gamma x^{-\alpha}}\right]\right]^{-1}\right\},$$

from which we arrive at the reverse hazard function of the random variable X with pdf (4).

REFERENCES

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APPENDIX A

Theorem 1:

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a given probability space and let $H = [d, e]$ be an interval for some $d < e$ $(d = -\infty, e = \infty$ mightaswellbeallowed$)$. Let $X: \Omega \rightarrow H$ be a continuous random variable with the distribution function F and let h and g be two real functions defined on H such that

$$\frac{\mathbf{E}[g(X)|X \geq x]}{\mathbf{E}[h(X)|X \geq x]} = \xi(x), x \in H,$$

is defined with some real function ξ. Assume that $h, g \in C^1(H)$, $\xi \in C^2(H)$ and F is twice continuously differentiable and strictly monotone function on the set H. Finally, assume that the equation $\xi h = g$ has no real solution in the interior of H. Then F is uniquely determined by the functions h, g and ξ, particularly

$$F(x) = \int_a^x C \left| \frac{\xi'(u)}{\xi(u)h(u) - g(u)} \right| \exp(-s(u)) du,$$

where the function s is a solution of the differential equation $s' = \frac{\xi' h}{\xi h - g}$ and C is the normalization constant, such that $\int_H dF = 1$.