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NEWLY PROPOSED ESTIMATOR FOR RIDGE PARAMETER: AN APPLICATION TO THE NIGERIAN ECONOMY

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ABSTRACT

Different methods have been adopted in the estimation of ridge parameter in ordinary ridge regression estimator. In this study new ridge parameter was introduced and evaluated via simulation study and application to real life data. The proposed parameter is a function of the standard error of regression and independent of the regression coefficients. Results show that the proposed estimators when applied to ridge regression estimator have minimum mean square error (MSE) as compared to other estimators of ridge parameter and the conventional ordinary least square (OLS) estimator.

KEYWORDS

Ridge regression estimator, Simulation Study, Real life data, Standard error.

MSC: 62J07

1. INTRODUCTION

The Classical Linear Regression Model (CLRM) can be written in matrix form as follows:

$$Y = X\beta + e \tag{1}$$

where Y is the vector of response variable ($n \times 1$ dimension), X is design matrix of independent variables ($n \times p$ dimension). β is the unknown regression parameter ($p \times 1$ dimension) and e is vector of uncorrelated error terms with zero mean and constant variance σ^2 ($n \times 1$ dimension). The Ordinary Least Square (OLS) estimator of β is defined as follows provided the determinant of $X'X$ matrix exist:

$$\hat{\beta} = (X'X)^{-1}X'y \tag{2}$$

The OLS estimates is efficient provided the $X'X$ is orthogonal. However, the efficiency is reduced if the $X'X$ matrix is correlated. If these happen, the $X'X$ matrix is no longer orthogonal, this makes the invertibilty difficult and consequently, the OLS estimate and standard error in (2) are inflated (Khalaf and Iguernane, 2016). Moreover, few regression coefficients might become insignificant possessing wrong coefficient sign and making it practically impossible to obtain meaningful statistical inference for practitioners (Dorugade, 2016).

Some biased estimators exists in literature to circumvent this problem. Popularly known among them is the Ridge regression (RR) estimator suggested by Hoerl and Kennard (1970). A basic disadvantage of RR estimator is that it's a nonlinear function of the ridge parameter (or biasing constant) k. Another concern involves the problem of estimating or selecting ridge parameter, k. Researchers have suggested several methods for obtaining ridge parameter e.g. Hoerl and Kennard (1970), McDonald and Galarneau (1975), Lawless and Wang (1976), Hocking et al. (1976), Wichern and Churchill (1978); Gibbons (1981), Nordberg (1982), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi et al. (2006), Muniz and Kibria (2009), Mansson *et al.* (2010) and recently, Khalaf and Iguernane (2016), Dorugade (2016). This paper aims to overcome the problem of multicollinearity by suggesting new estimator for ridge parameter.

2. MODEL AND ESTIMATORS

Assume that the response variable y in equation (1) is centered and the regressors X's are standardized. Let Λ and T be the matrices of eigen values and eigen vectors of $X'X$ respectively such that $T'X'XT = \Lambda$ = diagonal $\left(\lambda_1, \lambda_2, \ldots\lambda_p\right)$, where λ_1 represents the ith eigenvalue of $X'X$ and $T'T = TT' = I_p$. The equivalent model for equation (1) is

$$y = Z\alpha + e \tag{3}$$

where $Z = XT$ such that $Z'Z = \Lambda$ and $\alpha = T'\beta$.

The OLS estimator of α is defined as:

$$\hat{\alpha}_{OLS} = (Z'Z)^{-1} Z'Y = \Lambda^{-1} Z'Y \tag{4}$$

The relationship between the OLS estimator of β and α is given by $\hat{\beta} = T\,\hat{\alpha}_{OLS}$

2.1 Ridge Regression and Newly Proposed Ridge Parameter

Hoerl and Kennard (1970) added ridge parameter (k) to the diagonal elements of the least square estimator. It is given as:

$$\hat{\alpha}_{RR} = (I - k\left(\Lambda + kI_p\right)^{-1})\hat{\alpha}_{OLS} \tag{5}$$

where $k \geq 0$. Therefore, ridge estimator for β is given as $\hat{\beta}_{RR} = T\hat{\alpha}_{RR}$. The mean square error of $\hat{\alpha}_{RR}$ is

$$\text{MSE}(\hat{\alpha}_{RR}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{(\lambda_i + k)^2} \tag{6}$$

When k reduces to zero, MSE of $\hat{\alpha}_{RR}$ becomes MSE of OLS. Hence,

$$\text{MSE}(\hat{\alpha}_{LS}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \tag{7}$$

Hoerl et al. (1975) suggested that k should be chosen small enough so that its produce a minimum mean squared error (MSE) when compared to MSE of Least Square estimator. Hoerl and Kennard (1970) suggested estimating ridge parameter by taking the maximum of α_i^2 such that the estimator of K is:

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{Max(\hat{\alpha}_i{}^2)} \tag{8}$$

Hoerl Kennard and Baldwin (1975) estimated the value of k by taking the harmonic mean of the ridge parameterK_{HK_i}. This estimator is given as:

$$\hat{k}_{HKB} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\alpha}_i{}^2} \tag{9}$$

Dorugade (2016) suggested an estimator of k that depends on the variance of the regression model, $\hat{\sigma}^2$.

$$\hat{k}_D=\hat{\sigma} \tag{10}$$

where $\hat{\sigma} = \sqrt{\dfrac{Y'Y - \alpha'_{LS}Z'Y}{n-p}}$

Dorugade (2016) defined the MSE of ridge estimator in (10) as:

$$\text{MSE } (\hat{\alpha}_{RR}^{D}) = \hat{\sigma}\sum_{i=1}^{p}\frac{\left(\lambda_i + \hat{\alpha}_i^2\right)}{\left(\lambda_i + \hat{\sigma}\right)^2} \tag{11}$$

In the light of equation (10), the following k was proposed.

$$\hat{k}_{AL1}= p\hat{\sigma} \tag{12}$$

3. SIMULATION STUDY

The model is specified as follows:

$$\mathrm{Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_p X_{tp} + e_t} \tag{13}$$

$\mathrm{t = 1,2, \ldots, n \,;\, p = 3,7}$ where $\mathrm{e_t \sim N(0, \sigma^2)}$

Following Kibria (2003), independent variables are generated by

$$\mathrm{X_{ti} = (1 - \rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{tp}} \tag{14}$$

$t = 1, 2, 3, \ldots, n.\ i = 1,\ 2, \ldots p.$

where $\mathrm{Z_{ti}}$ is independent standard normal distribution with mean zero and unit variance. ρ is the correlation between the X’s while p denotes the number of regressors. ρ were taken as 0.85, 0.9, 0.95 and 0.99. In this study, the number of regressors (p) was taken to be three (3) and seven (7). β_0 was taken to be identically zero. When $p = 3$, the values of β were chosen to be: β_1= 0.8, β_2= 0.1, β_3= 0.6. When $p = 7$, the values of β were chosen to be: β_1= 0.4, β_2= 0.1, β_3= 0.6, β_4= 0.2, β_5= 0.25, β_6= 0.3, β_7= 0.53. Sample sizes were varied between 25, 60 and 100. Four different values of σ^2: 1, 5, 9 and 25 are also used. The average MSE of the estimators over 1500 replication was computed using the following equation:

$$AMSE(\hat{\beta}) = \frac{1}{1500}\sum_{j=1}^{1500}\sum_{i=1}^{p}\left(\hat{\beta}_{ij} - \beta_i\right)^2 \tag{15}$$

where $\hat{\beta}_{ij}$ is ith element of the estimator βin the jth replication which gives the estimate of β_i. β_i are the true value of the parameter previously mentioned. Estimators with the minimum AMSE are considered best.

4. RESULT

The AMSE of the existing estimators and the proposed ones are provided in Table 1.

Table 1
AMSE of OLS and Some Ridge Estimators

ρ	Estimator	n=25							
		p=3				p=7			
		σ^2=1	5	9	25	σ^2=1	5	9	25
0.85	**OLS**	1.3503	3.6143	5.8904	15.0221	2.3952	8.3625	14.3278	38.1841
	$\mathbf{k_{HK}}$	0.3624	1.5620	2.7559	7.5224	0.9778	4.4445	7.9543	22.0124
	$\mathbf{k_{HKB}}$	0.2961	1.0519	1.7596	4.5425	0.5778	2.2078	3.8068	10.1851
	$\mathbf{k_D}$	0.3313	1.0368	1.4935	2.6463	0.8573	2.7726	4.0563	7.3472
	$\mathbf{k_{AL1}}$	**0.1865**	**0.4529**	**0.6001**	**0.9421**	**0.2723**	**0.5193**	**0.6301**	**0.8557**
0.9	**OLS**	1.6692	5.1351	8.6156	22.5704	3.1969	12.2988	21.4101	57.8766
	$\mathbf{k_{HK}}$	0.5162	2.3294	4.1391	11.3725	1.4206	6.7333	12.0839	33.4990
	$\mathbf{k_{HKB}}$	0.3985	1.4876	2.5364	6.6973	0.7880	3.2097	5.6093	15.1978
	$\mathbf{k_D}$	0.3998	1.1104	1.5328	2.5359	1.0451	3.0567	4.3009	7.2588
	$\mathbf{k_{AL1}}$	**0.2042**	**0.4465**	**0.5720**	**0.8582**	**0.2777**	**0.4723**	**0.5553**	**0.7286**
0.95	**OLS**	2.6394	9.8912	17.1635	46.2985	5.6426	24.70855	43.7882	120.1375
	$\mathbf{k_{HK}}$	0.9993	4.7468	8.4947	23.4844	2.8354	13.9489	25.0831	69.6367
	$\mathbf{k_{HKB}}$	0.6917	2.8431	4.9681	13.4505	1.4239	6.3322	11.2291	30.8134
	$\mathbf{k_D}$	0.4851	1.1094	1.4341	2.1394	1.3209	3.2272	4.2285	6.2895
	$\mathbf{k_{AL1}}$	**0.2287**	**0.4204**	**0.5117**	**0.7262**	**0.2699**	**0.3882**	**0.4395**	**0.5607**
0.99	**OLS**	10.8487	50.8773	90.9515	251.3505	27.1691	132.6550	238.1725	660.3137
	$\mathbf{k_{HK}}$	5.1718	25.6261	46.0797	127.8975	15.2618	76.2232	137.1939	381.1007
	$\mathbf{k_{HKB}}$	3.0337	14.4243	25.8099	71.3530	6.7294	32.8121	58.8943	163.2263
	$\mathbf{k_D}$	0.4767	0.7366	0.8469	1.1080	1.3094	2.0053	2.2453	2.6562
	$\mathbf{k_{AL1}}$	**0.2640**	**0.3537**	**0.4047**	**0.5578**	**0.2239**	**0.2676**	**0.2966**	**0.3866**

ρ	Estimator	n=60							
		p=3				p=7			
		σ^2=1	5	9	25	σ^2=1	5	9	25
0.85	**OLS**	0.7192	1.4718	2.2223	5.2196	1.9214	4.2821	6.6459	16.1082
	k_{HK}	0.1502	0.5637	0.9499	2.4876	0.4473	1.7844	3.1380	8.6295
	k_{HKB}	0.1330	0.4402	0.6943	1.6427	0.2754	0.9191	1.5209	3.9883
	k_D	0.1549	0.6306	1.0132	2.1692	0.4484	1.7312	2.7325	5.7217
	k_{AL1}	**0.1170**	0.3740	0.5406	0.9558	0.1899	0.4760	0.6287	0.9498
0.9	**OLS**	0.8633	2.0058	3.1456	7.6993	2.2008	5.8559	9.5149	24.1595
	k_{HK}	0.2083	0.8033	1.3828	3.7019	0.6313	2.6934	4.7999	13.2741
	k_{HKB}	0.1793	0.5920	0.9526	2.3395	0.3659	1.3054	2.2106	5.8104
	k_D	0.2135	0.7962	1.2275	2.4246	0.5997	2.1245	3.2413	6.3813
	k_{AL1}	**0.1439**	0.4058	0.5595	0.9159	0.2075	0.4657	0.5898	0.8322
0.95	**OLS**	1.2356	3.6045	5.9697	15.4216	3.2481	11.0434	18.8324	49.9740
	k_{HK}	0.3685	1.5648	2.7605	7.5468	1.1940	5.6140	10.0792	27.9526
	k_{HKB}	0.2985	1.0459	1.7496	4.5285	0.6254	2.5075	4.3674	11.7958
	k_D	0.3370	1.0351	1.4712	2.5161	0.8895	2.7159	3.9086	6.8805
	k_{AL1}	**0.1828**	0.4159	0.5326	0.7800	0.2261	0.4133	0.4882	0.6236
0.99	**OLS**	3.0906	16.9612	29.9219	81.7454	12.2260	56.0733	99.9068	275.2102
	k_{HK}	1.2430	8.2213	14.7459	40.8406	6.2590	31.1687	56.0814	155.7302
	k_{HKB}	0.8398	4.8308	8.5398	23.3660	2.6941	12.7328	22.7661	62.8979
	k_D	0.4344	0.9527	1.1397	1.4901	1.3731	2.7517	3.3316	4.3264
	k_{AL1}	**0.2049**	**0.3379**	**0.3836**	**0.4916**	0.2122	0.2649	0.2841	0.3283
		n=100							
ρ		p=3				p=7			
0.85	**OLS**	0.6534	1.0930	1.5295	3.2679	1.3112	2.4781	3.6487	8.3399
	k_{HK}	0.0941	0.3511	0.5767	1.4567	0.2527	0.9571	1.6101	4.3062
	k_{HKB}	0.0857	0.2902	0.4464	1.0010	0.1762	0.5473	0.8634	2.0788
	k_D	0.0965	0.4238	0.7078	1.6434	0.2572	1.1131	1.8495	4.2682
	k_{AL1}	0.0805	0.2938	0.4480	0.8663	0.1497	0.4432	0.6219	1.0346
0.9	**OLS**	0.7618	1.4304	2.0946	4.7420	1.43775	3.2351	5.0368	12.2536
	k_{HK}	0.1335	0.4928	0.8272	2.1575	0.3599	1.3865	2.4042	6.5694
	k_{HKB}	0.1185	0.3864	0.6023	1.4047	0.2339	0.7477	1.2119	3.0320
	k_D	0.1383	0.5707	0.9228	1.9993	0.3657	1.4819	2.3875	5.1857
	k_{AL1}	0.1059	0.3450	0.5014	0.8894	0.1783	0.4691	0.6284	0.9647
0.95	**OLS**	1.0134	2.4018	3.7832	9.2929	1.8967	5.6873	9.4936	24.6822
	k_{HK}	0.2362	0.9352	1.6243	4.3859	0.6519	2.7824	4.9616	13.7138
	k_{HKB}	0.2009	0.6611	1.0712	2.6688	0.3835	1.3535	2.2873	6.0014
	k_D	0.2406	0.8492	1.2764	2.4008	0.6183	2.1749	3.3032	6.4217
	k_{AL1}	0.1514	0.3962	0.5299	0.8189	0.2201	0.4647	0.5739	0.7735
0.99	**OLS**	2.6867	10.2919	17.8761	48.1735	6.1798	27.3298	48.4918	133.1665
	k_{HK}	1.0167	4.8116	8.6085	23.7897	3.0640	15.1202	27.1855	75.4436
	k_{HKB}	0.7009	2.8651	5.0053	13.5533	1.4336	6.4022	11.3604	31.1891
	k_D	0.4835	1.0398	1.2965	1.7805	1.2737	3.0057	3.8604	5.4898
	k_{AL1}	0.2149	0.3507	0.4019	0.5005	0.2367	0.3129	0.3373	0.3821

Results provided in Table 1 shows that the proposed estimator perform consistently well than OLS and existing ridge estimation techniques. Particularly, proposed k_{AL1} consistently gives minimum AMSE values as compared to others.

5. APPLICATION

Consider an example that focuses on the impact of Government Expenditure and revenue on Nigerian Economic growth. Dataset extracted from CBN Statistical bulletin cover the period 1970 to 2013. The following regression model is considered:

$$Y_t = \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_8 X_{t8} + U_t \quad (17)$$

$t = 1,2,\ldots,34$

where

Y_t is the gross domestic product

X_{t1} represent Recurrent Expenditure on Economic Services

X_{t2} represent Recurrent Expenditure on Social and Community Services

X_{t3} represent Recurrent Expenditure on Transfers

X_{t4} represent Capital Expenditure on Economic Services

X_{t5} represent Capital Expenditure on Social and Community Services

X_{t6} represent Capital Expenditure on Transfers

X_{t7} represent Oil Revenue

X_{t8} represent Non-oil Revenue

Table 2
Variance Inflation Factors among the Explanatory Variables

	X_{t1}	X_{t2}	X_{t3}	X_{t4}	X_{t5}	X_{t6}	X_{t7}	X_{t8}
VIF	1.786	49.547	27.022	4.598	14.606	10.632	14.427	100.959

From Table 2, it was observed that the model suffers multicollinearity since the VIF of some of the variables are greater than 10. Moreover, a condition index of 1581 shows that there is multicollinearity. Existing ridge estimators and proposed ridge estimator are applied to the dataset. The result is presented in Table 3.

Table 3
Mean Squared Error and Regression Coefficients of the Existing and Proposed Estimators

Estimators	MSE	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$
OLS	6753545.9	0.11	89.69	15.44	-18.49	50.92	17.93	-1.34	-4.08
$\mathbf{k_{HK}}$	5584429.6	0.11	87.17	15.08	-18.79	48.09	18.14	-1.28	-3.18
$\mathbf{k_{HKB}}$	2789084.8	0.11	78.10	13.75	-19.85	38.23	18.63	-1.04	0.08
$\mathbf{k_{D}}$	4077269.6	0.11	82.68	14.43	-19.32	43.14	18.43	-1.16	-1.57
$\mathbf{k_{AL1}}$	463.5	0.15	57.49	10.43	-21.87	18.49	17.93	-0.52	7.53

The result obtained in Table 3 agrees with the result of the simulation study. It is evident that k_{AL1} gives the minimum MSE. From the regression coefficients with the use of the proposed ridge estimator, $\mathbf{k_{AL1}}$, the following factors are responsible for the increase in Nigerian Economy:

Recurrent Expenditure on Economic Services, Recurrent Expenditure on Social and Community Services, Recurrent Expenditure on Transfers, Capital Expenditure on Social and Community Services, Capital Expenditure on Transfers and Non-oil Revenue.

6. CONCLUSION

A new ridge parameter that is dependent on the standard error but not a function of the regression coefficient was proposed in this study. Results from simulation study and its application to real life data shows that the proposed method consistently performs better than the conventional OLS and the existing ridge parameters.

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