Pak. J. Statist.
2017 Vol. 33(6), 491-498

TRANSMUTED WEIGHTED POWER FUNCTION DISTRIBUTIONS: PROPERTIES AND APPLICATIONS

Al Mutairi Alya O.
Department of Mathematics, Faculty of Science, Taibah University, Saudi Arabia
Email: amutairi@taibahu.edu.sa

ABSTRACT

In this paper, the transmuted weighted power (TWP) function distribution which is an extension of the power distribution, is introduced and its properties are explored. The transmuted generalized Power function distribution is exemplified by a real data set in respect of the reliability of semiconductor devices.

KEYWORDS

Transmuted weighted power function distribution; power probability density function; cumulative distribution function; moments; quantile function; random numbers generation; reliability function.

Mathematics Subject Classification: Primary 62F15. Secondary 65.

1. INTRODUCTION

The Power function distribution is a simple life time distribution model. It is a special case of Pareto distribution. The Power function distribution is one of the distributions to study the reliability of electrical devices (Meniconi, B., 1995). The use of simple models are preferred by most of the engineers to find failure rates and reliability info over intricate distributions. In many engineering sciences, modeling and analyzing lifetime data are still problematic with engineers where the real data is not fitted well by any of the classical or standard probability models. The TWP distribution is an extended model to analyze more complex data. Many authors have studied various aspects of the power function. (See Meniconi, 1995; Rider, 1964; Malik, 1967; Lwin (1972) and Arnold and Press (1983). Lutful Kabir and Ahsanullah (1974) obtained estimation for location and scale parameters for power function distribution. Samia and Mohamed (1993) studied five modifications of moment to estimate the parameters of Pareto distribution.

2. POWER PROBABILITY DENSITY FUNCTION

The power probability density function is given by

$$u(x) = \frac{px^{p-1}}{b^p}, \qquad 0 < x < b,\ p > 0. \tag{1}$$

where p is the shape parameter and b is the scale parameter.

The cumulative distribution function is

$$u(x) = \frac{x^p}{b^p}, \qquad 0 < x < b,\ p > 0. \tag{2}$$

The mean and variance of X are $\frac{bp}{p+1}$ and $\frac{b^2 p}{(p+1)^2 (p+2)}$ respectively.

The survival function is

$$S(x) = 1 - \frac{x^p}{b^p}, \qquad 0 < x < b,\ p > 0. \tag{3}$$

The failure rate function is

$$h(x) = \frac{p x^{p-1}}{(b^p - x^p)}, \qquad 0 < x < b,\ p > 0. \tag{4}$$

3. WEIGHTED POWER FUNCTION

A random variable X is said to have weighted power function if its density function is given by

$$f(x) = \frac{w(x)u(x)}{E(w(x))}$$

$$f(x) = \frac{(p+r)\, x^{w+p-1}}{b^{p+w}}$$

$$F(x) = \frac{x^{w+p}}{b^{p+w}} \tag{5}$$

where $w(x)$ is the weight function on R^+. If $w(x) = x^w$, x^2 and x then it is weighted, area-biased and size biased power function distribution respectively.

4. TRANSMUTED WEIGHTED POWER FUNCTION DISTRIBUTION

A random variable X is said to have transmuted distribution if its cumulative distribution function (cdf) is given by

$$G(x) = (1+\theta)F(x) - \theta F^2(x), \quad |\theta| \le 1.$$

where $F(x)$ is the cdf of the base distribution.

If $\theta = 0, G(x) = F(x)$, the base distribution.

The CDF of Transmuted Weighted Power Function Distribution is

$$G(x) = (1+\theta)\left(\frac{x}{b}\right)^{w+p} - \theta\left(\frac{x}{b}\right)^{2(w+p)} \tag{6}$$

$$g(x) = \frac{(w+p)}{b}\left(\frac{x}{b}\right)^{w+p-1}\left[(1+\theta) - 2\theta\left(\frac{x}{b}\right)^{w+p}\right] \tag{7}$$

Ahsan-ul-Haq *et al.*, (2016) studied the transmuted power function distribution and applied it to model lifetime data. In the present study we derive the transmuted weighted power function distribution and obtain some of its properties.

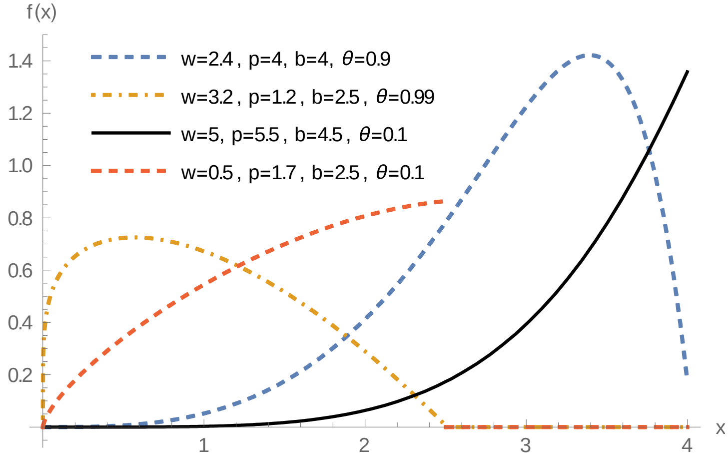


Figure 4.1: Plots of PDF of Transmuted Weighted Power Distribution

Figure 4.1 illustrates some of the possible shapes of the pdf of a transmuted weighted Power function distribution for selected values of the parameters, b, p and θ.

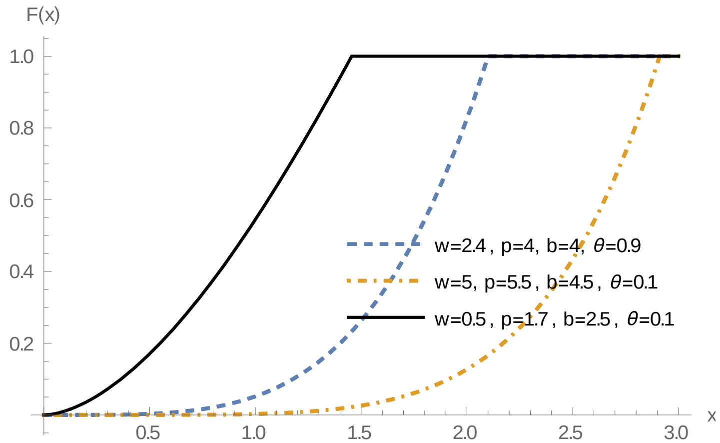


Figure 4.2: Plots of CDF of Transmuted Weighted Power Distribution

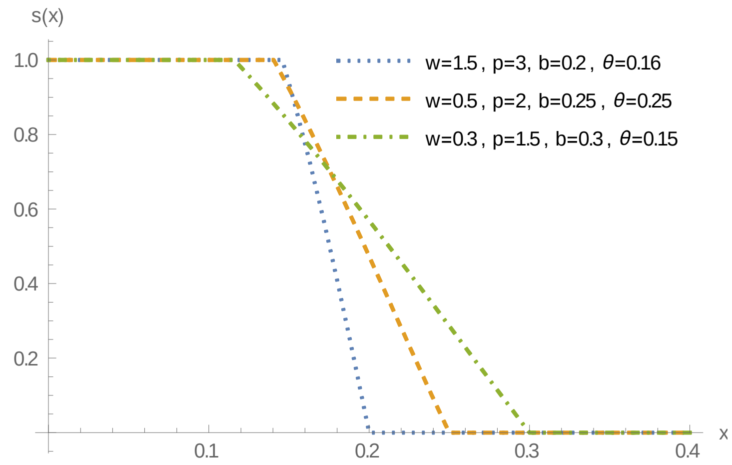


Figure 4.3: Plots of Survival Function of Transmuted Weighted Power Distribution

5. MOMENTS

Theorem 1:

The r^th moment $E\left(X^{r}\right)$ of a transmuted weighted power function distributed random variable X is given as

$$\mu_r' = E\left(x^r\right) = \int_0^b x^r g(x) dx$$

$$= \int_0^b x^r \frac{(w+p)}{b}\left(\frac{x}{b}\right)^{w+p-1}\left[(1+\theta) - 2\theta\left(\frac{x}{b}\right)^{w+p}\right] dx$$

$$\mu_r' = b^{r-1}(w+p)\left(\frac{1+\theta}{r+w+p} - \frac{2\theta}{2(w+p)+r}\right).$$

the central moments transmuted weighted power function are

$$\mu_r = \sum_{i=0}^{r}\left\{\begin{array}{l}\binom{r}{i}(-1)^i\left\{(w+p)\left(\frac{1+\theta}{w+p+1} - \frac{2\theta}{2(w+p)+1}\right)\right\}^i \\ \left\{b^{r-i-1}(w+p)\left(\frac{1+\theta}{r-i+w+p} - \frac{2\theta}{2(w+p)+r-i}\right)\right\}\end{array}\right.$$

the skewness of transmuted weighted power function are

$$\sqrt{\beta_1}=\frac{\sum_{i=0}^{3}\left\{\begin{array}{l}\binom{3}{i}(-1)^i\left\{(w+p)\left(\frac{1+\theta}{w+p+1}-\frac{2\theta}{2(w+p)+1}\right)\right\}^i\\ \left\{b^{r-i-1}(w+p)\left(\frac{1+\theta}{3-i+w+p}-\frac{2\theta}{2(w+p)+3-i}\right)\right\}\end{array}\right\}}{\left[\sum_{i=0}^{2}\left\{\begin{array}{l}\binom{2}{i}(-1)^i\left\{(w+p)\left(\frac{1+\theta}{w+p+1}-\frac{2\theta}{2(w+p)+1}\right)\right\}^i\\ \left\{b^{r-i-1}(w+p)\left(\frac{1+\theta}{2-i+w+p}-\frac{2\theta}{2(w+p)+2-i}\right)\right\}\end{array}\right\}\right]^{\frac{3}{2}}}$$

the kurtosis of transmuted weighted power function are

$$\beta_2=\frac{\sum_{i=0}^{4}\left\{\begin{array}{l}\binom{4}{i}(-1)^i\left\{(w+p)\left(\frac{1+\theta}{w+p+1}-\frac{2\theta}{2(w+p)+1}\right)\right\}^i\\ \left\{b^{r-i-1}(w+p)\left(\frac{1+\theta}{4-i+w+p}-\frac{2\theta}{2(w+p)+4-i}\right)\right\}\end{array}\right\}}{\left[\sum_{i=0}^{2}\left\{\begin{array}{l}\binom{2}{i}(-1)^i\left\{(w+p)\left(\frac{1+\theta}{w+p+1}-\frac{2\theta}{2(w+p)+1}\right)\right\}^i\\ \left\{b^{r-i-1}(w+p)\left(\frac{1+\theta}{2-i+w+p}-\frac{2\theta}{2(w+p)+2-i}\right)\right\}\end{array}\right\}\right]^{2}}$$

6. QUANTILE FUNCTION AND RANDOM NUMBER GENERATION

Using the method of inversion we can generate random numbers from the transmuted weighted Power function distribution as

From equation (6) let

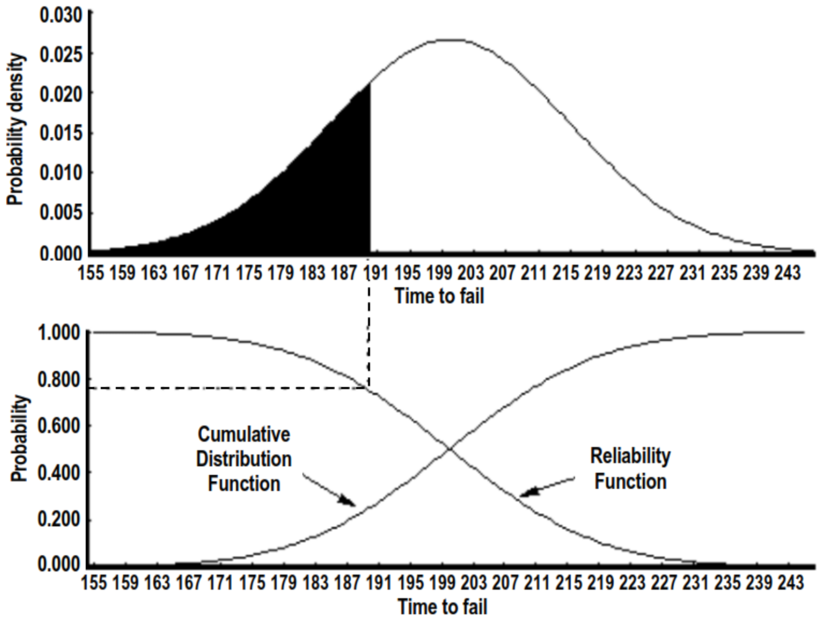
$$u=(1+\theta)\left(\frac{x}{b}\right)^{w+p}-\theta\left(\frac{x}{b}\right)^{2(w+p)}, \qquad 0<x<b,\ p>0$$

$$x_q=b\left[\sqrt{\left(\frac{1+\theta}{2p}\right)^2-\frac{u}{p}}+\frac{1+\theta}{2p}\right]^{\frac{1}{w+p}}, \qquad |\theta|\le 1,\ p>0$$

is the quantile function, for q=0.5 we get the median of the proposed distribution. If u is taken as uniform random variable then we can generate random numbers follows proposed distribution.

7. RELIABILITY ANALYSIS

The reliability function is the survival function. Thus, the cumulative distribution function increases from zero to one as the value of *x* increases, and the reliability function decreases from one to zero as the value of x increases. The reliability function is $R(x) = \int_x^\infty \mathrm{f(t)dt}$ and is shown in the Figure 7.1:



Source: (www.engineeredsoftware.com/nasa/reliabil.htm)

Figure 7.1: The Reliability Function

The Figure 7.1 shows the probability that the time to fail is larger than 190 is 0.7475. It shows that reliability at time = 190. The probability that the time to fail is less than 190 is 1 - 0.7475 = 0.2525. (www.engineeredsoftware.com/nasa/reliabil.htm).

The reliability function of a transmuted size-biased power function distribution is given by

$$R(t) = 1 - (1+\theta)\left(\frac{t}{b}\right)^{w+p} + \theta\left(\frac{t}{b}\right)^{2(w+p)}, \qquad |\theta| \le 1,\ p > 0 \qquad (8)$$

which is an important quantity characterizing life phenomenon. It can be loosely interpreted as the conditional probability of failure, given it has survived to time *t*. The hazard rate function for a transmuted generalized power function random variable is given by

$$h(t)=\frac{\frac{(w+p)}{b}\left(\frac{t}{b}\right)^{w+p-1}\left[(1+\theta)-2\theta\left(\frac{t}{b}\right)^{w+p}\right]}{1-(1+\theta)\left(\frac{t}{b}\right)^{w+p}+\theta\left(\frac{t}{b}\right)^{2(w+p)}},\qquad |\theta|\le 1,\ p>0 \tag{9}$$

8. APPLICATION

In this section, we use a real data set taken from Smith and Naylor (1987) to show that the transmuted generalized Power function distribution can be a better model than the generalized power function, power function and transmuted power function distribution.

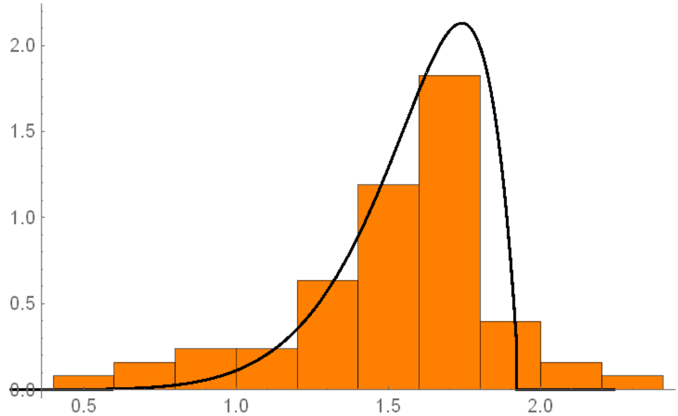


Figure 8.1: Empirical Histogram and Fitted Distributions for Data

Table 1
Descriptive statistics for data

Observations	Min.	Max.	Mean	SD	Skewness	Kurtosis
63	0.55	2.24	1.507	0.324	-0.90	3.92

Table 2
Parameters estimates

Parameter	$\hat{w}$	$\hat{p}$	$\hat{b}$	$\hat{\theta}$
Estimated values	2.920	4.520	1.920	0.931

CONCLUSION

In this paper, the transmuted weighted power (TWP) function distribution is discussed and the properties are explored. We concluded from the current study that, the transmuted generalized power function distribution can be a better model than the generalized power function, power function and transmuted power function distribution.

ACKNOWLEDGEMENTS

This research was supported financially by Taibah University, Medina, Saudi Arabia. Special thanks to Professor Dr. Munir Ahmad (Professor of Statistics and Rector National College of Business Administration and Economics) and Professor Syed Azeem Ali for their positive suggestions in improving the manuscript.

REFERENCES

1. Lutful Kabir, A.B.M. and Ahsanullah, M. (1974). Estimation of the location and scale parameters of a power-function distribution by linear functions of order statistics. *Communications in Statistics-Theory and Methods*, 3(5), 463-467.
2. Arnold, B.C. and Press, S.J. (1983). Bayesian inference for Pareto populations. *Journal of Econometrics*, 21(3), 287-306.
3. Lwin, T. (1972). Estimation of the tail of the Paretian law. *Scandinavian Actuarial Journal*, 1972(2), 170-178.
4. Malik, H.J. (1967). Exact moments of order statistics from a power-function distribution. *Scandinavian Actuarial Journal*, 1967(1-2), 64-69.
5. Meniconi, B. (1995). The power function distribution: A useful and simple distribution to assess electrical component reliability. *Microelectronics Reliability*, 36(9), 1207-1212.
6. Rider, P.R. (1964). Distribution of the product, quotient of maximum values in samples from a power function population. *Journal of the American Statistical Association*, 59, 877-880.
7. Samia, A.S. and Mohammed M.M. (1993). Modified Moment Estimators for three parameters Pareto distribution. *ISSR, Cairo University*, 28(2).
8. Ahsan-ul-Haq, M., Butt, N.S., Usman, R.M. and Fattah, A.A. (2016). Transmuted Power Function Distribution. *Gazi University Journal of Science*, 29(1), 177-185.