Pak. J. Statist.
2017 Vol. 33(5), 355-367

GENERALIZED DIFFERENCE-CUM-EXPONENTIAL ESTIMATOR IN ADAPTIVE CLUSTER SAMPLING

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ABSTRACT

In this paper, a generalized difference-cum-exponential form estimators have been proposed with two auxiliary variables in adaptive cluster sampling design. The suggested estimator is based on the networks averages intersected with the first sample selected by simple random sampling without replacement. It is observed that several estimators are special cases of proposed estimator in adaptive cluster sampling. The expressions for the mean square error and bias of the proposed estimator has been derived. The proposed class of estimators can be used for estimation of the population mean. To reveal and evaluate the efficiencies of the proposed estimator the simulation studies have been carried out.

KEY WORDS

Simulated Population, Expected Final Sample Size, Within Network Variance, Estimated Mean Squared Error, Comparative Percentage Relative Efficiency.

1. INTRODUCTION

The Adaptive Cluster Sampling (ACS) is suitable and efficient for the rare and clustered population. The initial sample in adaptive cluster sampling is selected with usual sampling scheme like a simple random sampling then the neighbourhood of each unit selected is considered. If the sampled unit meet a predefined condition C (usually, $y > 0$) then the neighbouring units are examined and included in the sample. This process stops when the neighbouring unit do not meet the condition. The final sample comprises all the units studied and the initial sample. A network consists of those units that meet the predefined condition. The quadrats that do not meet the pre-condition are identified as edge units. The mixture of network and edge units forms a cluster. The social and institutional associations among units can be used to define the neighbourhood. The sampling unit itself and its four neighbouring units: above, below, right, and left known as the first-order neighbourhood. The first-order neighbouring units and the diagonal quadrats together known as the second-order neighbourhood.

Thompson (1990) introduced the adaptive cluster sampling design to estimate the rare and clustered population and proposed four unbiased estimators in ACS. Smith *et al.*, (1995) estimate the density of wintering water fowl and found that the efficiency is highest as compare to simple random sampling design when the within-network variance is close to population variance. Dryver (2003) found that ACS performs well in a univariate setting. The simulation study for the blue-winged teals and red-winged teals showed that Horvitz-Thompson type estimator has the maximum efficiency by means of the condition of a kind of duck to estimate that kind of duck. The estimators in adaptive cluster sampling perform well to estimate the parameter of interest for highly correlated variables. Chao (2004) suggested a ratio type estimator in adaptive cluster sampling and proved that it generates improved estimation results. Dryver and Chao (2007) suggested the two ratio estimators in ACS. Chutiman and Kumphon (2008) proposed a ratio estimator in ACS utilizing two auxiliary variables. Chutiman (2013) proposed ratio estimators by means of population coefficient of variation and coefficient of kurtosis, regression and difference estimators by using single auxiliary variable, and showed that difference estimator has minimum mean squared error.

2. SOME ESTIMATORS IN SIMPLE RANDOM SAMPLING

Let a sample of size n is selected using simple random sampling without replacement (*srswor*) from the total number of units in the population N. The variable of interest y and auxiliary variable x has $\bar{Y}$ and $\bar{X}$ as the population means, S_y and S_x represents population standard deviations, C_y and C_x represents coefficients of variation respectively. Also let ρ_{xy} denotes population correlation coefficient between study and auxiliary variables, $\theta = \frac{1}{n} - \frac{1}{N}$ and $H_{jk} = \rho_{jk}\frac{C_j}{C_k}$, $j \neq k$.

The mean per unit estimator for a sample of size n drawn from a population of size N is defined as $t_0 = \bar{y}$ with the variance: $VAR(t_0) = \theta\bar{Y}^2 C_y^2$. Cochran (1940) proposed the following classical ratio estimator:

$$t_1 = \bar{y}\left[\frac{\bar{X}}{\bar{x}}\right], \tag{2.1}$$

Mean square error (MSE) of the estimator (2.1) is:

$$MSE(t_1) \approx \theta\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{xy}C_x C_y\right]. \tag{2.2}$$

Hansen *et al.*, (1942) proposed the classical difference estimators for the finite population mean,

$$t_2 = \bar{y} + G\left(\bar{X} - \bar{x}\right). \tag{2.3}$$

where G is an unknown constant.

Mean square error (MSE) of the difference estimator (2.3) is as follows:

$$MSE(t_2) \approx \theta\bar{Y}^2\left[C_y^2 + G^2\frac{\bar{X}^2}{\bar{Y}^2}C_x^2 - 2G\frac{\bar{X}}{\bar{Y}}\rho_{xy}C_x C_y\right]. \tag{2.4}$$

Bahl and Tuteja (1991) suggested the exponential ratio estimator to estimate the finite population mean as follows:

$$t_3 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right], \tag{2.5}$$

Mean square error and bias of the exponential ratio estimator t_3 are as follows:

$$MSE(t_3) \approx \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4} - \rho_{xy}C_xC_y\right]. \tag{2.6}$$

$$Bias(t_3) \approx \theta\bar{Y}\left[\frac{3}{8}C_x^2 - \frac{\rho_{xy}C_xC_y}{2}\right]. \tag{2.7}$$

Upadhyaya *et al.*, (2011) recommended generalized exponential ratio estimators:

$$t_{GERU} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+(a-1)\bar{x}}\right]. \tag{2.8}$$

where a is a real constant. The estimator (2.8) provides several estimators as special case for various values of a. For $a = 2$, a special case of (2.10) is the Bahl and Tuteja (1991) estimator.

Mean square error and bias of t_{GERU} are:

$$MSE(t_{GERU}) \approx \bar{Y}^2\theta\left[C_y^2 + \frac{C_x^2}{a^2}\left(1-2aH_{yx}\right)\right]. \tag{2.9}$$

which is minimum for $a = \frac{1}{H_{yx}}$.

$$Bias(t_{GERU}) \approx \frac{\bar{Y}\theta C_x^2}{2a^2}\left[2a\left(1-H_{yx}\right)-1\right]. \tag{2.10}$$

t_{GERU} will be more efficient than classical ratio estimator if $a > \frac{1}{2H_{yx}}$.

3. SOME ESTIMATORS IN ADAPTIVE CLUSTER SAMPLING

Suppose a population of size N is considered as $1,2,3,\ldots,N$ and a first sample of n units is drawn with a *srswor*. Consider the average values of y, x and z, in the network which includes unit i are w_{yi}, w_{xi} and w_{zi} such that, $w_{yi} = \frac{1}{m_i}\sum_{j\in A_i} y_j$, $w_{xi} = \frac{1}{m_i}\sum_{j\in A_i} x_j$

and $w_{zi} = \frac{1}{m_i} \sum_{j \in A_i} z_j$ respectively. Thompson (2002) suggested that ACS can be measured as *srswor* for the transformed population (that is, when the networks averages are taken). Consider that for the transformed populations the sample means of the study and auxiliary variables are $\bar{w}_y$, $\bar{w}_x$ and $\bar{w}_z$ respectively, such that, $\bar{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{yi}$, $\bar{w}_x = \frac{1}{n}\sum_{i=1}^{n} w_{xi}$ and $\bar{w}_z = \frac{1}{n}\sum_{i=1}^{n} w_{zi}$. Let, for the transformed population the coefficient of variations are C_{wy}, C_{wx} and C_{wz} for the study and auxiliary variables respectively. Let the transformed population correlation coefficients ρ_{wxwy} and ρ_{wzwy} are between w_x and w_y and w_z and w_y respectively. Define the following error terms,

$$\bar{e}_{wy} = \frac{\bar{w}_y - \bar{Y}}{\bar{Y}}, \bar{e}_{wx} = \frac{\bar{w}_x - \bar{X}}{\bar{X}}, \text{ and } \bar{e}_{wz} = \frac{\bar{w}_z - \bar{Z}}{\bar{Z}}. \tag{3.1}$$

Here $\bar{e}_{wy}$, $\bar{e}_{wx}$ and $\bar{e}_{wz}$ are the sampling errors for the variable of interest and for the auxiliary variables respectively, such that:

$$E\left(\bar{e}_{wy}\right) = E\left(\bar{e}_{wx}\right) = E\left(\bar{e}_{wz}\right) = 0 \tag{3.2}$$

$$E\left(\bar{e}_{wx}\bar{e}_{wy}\right) = \theta\rho_{wxwy}C_{wx}C_{wy} \text{ and } E\left(\bar{e}_{wz}\bar{e}_{wy}\right) = \theta\rho_{wzwy}C_{wz}C_{wy} \tag{3.3}$$

$$E\left(\bar{e}_{wy}^2\right) = \theta C_{wy}^2, E\left(\bar{e}_{wx}^2\right) = \theta C_{wx}^2 \text{ and } E\left(\bar{e}_{wz}^2\right) = \theta C_{wz}^2 \tag{3.4}$$

$$C_{wy} = \frac{S_{wi}}{\bar{i}}, \ \rho_{wij} = \frac{S_{wij}}{S_{wi}S_{wj}}, \ H_{wij} = \rho_{wij}\frac{C_{wi}}{C_{wj}}. \quad \begin{matrix} i = x, y, z \\ j = x, y, z \quad i \neq j \end{matrix} \tag{3.5}$$

Thompson (1990) proposed an unbiased estimator to estimate the finite population mean in ACS using the transformed population based upon the adaptation of the Hansen-Hurwitz estimator:

$$t_4 = \frac{1}{n}\sum_{i=1}^{n} w_{yi} = \bar{w}_y \tag{3.6}$$

$$Var(t_4) = \frac{\theta}{N-1}\sum_{i=1}^{N}\left(w_{yi} - \bar{Y}\right)^2. \tag{3.7}$$

Dryver and Chao (2007) modified the conventional ratio estimator for the population mean in ACS using the transformed population:

$$t_5 = \left[\frac{\sum_{i \in s_0} w_{yi}}{\sum_{i \in s_0} w_{xi}} \right] \bar{X} = \hat{R}\bar{X} \tag{3.8}$$

$$MSE(t_5) = \frac{\theta}{N-1} \sum_{i=1}^{N} \left(w_{yi} - R w_{xi} \right)^2 . \tag{3.9}$$

Chutiman (2013) proposed a modified difference estimator in adaptive cluster sampling as follows:

$$t_6 = \bar{w}_y + (\bar{X} - \bar{w}_x) . \tag{3.10}$$

The approximate mean squared error of t_6 is given by:

$$MSE(t_6) \approx \theta \bar{Y} (C_{wy}^2 + C_{wx}^2 - 2C_{wxwy}) . \tag{3.11}$$

4. PROPOSED GENERALIZED DIFFERENCE-CUM-EXPONENTIAL ESTIMATOR IN ADAPTIVE CLUSTER SAMPLING

Following the concept of Chutiman (2013) and Chaudhry and Hanif (2015) the proposed generalized exponential difference estimator in adaptive cluster sampling with two auxiliary variables using average values of networks in ACS are:

$$t_{GRE1} = \left\{ \bar{w}_y + K_1 \left(\bar{Z} - \bar{w}_z \right) \right\} \exp \left[\alpha_1 \left\{ \frac{\bar{X} - \bar{w}_x}{\bar{X} + (a-1)\bar{w}_x} \right\} \right] \tag{4.1}$$

where K_1, α_1 and a are real constants. Several non-exponential and exponential type estimators as new family of estimators can be obtained for various values of K_1, α_1 and a.

5. THE BIAS AND MEAN SQUARE ERROR OF THE PROPOSED ESTIMATOR

The notations (3.1) can be used to rewrite the estimator (4.1) as follows:

$$\text{or} \quad t_{GRE1} = \left[\bar{Y} \left(1 + \bar{e}_{wy} \right) - K_1 \bar{Z} \bar{e}_{wz} \right] \exp \left[\frac{-\alpha_1 \bar{e}_{wx}}{a} \left\{ 1 + \left(\frac{a-1}{a} \right) \bar{e}_{wx} \right\}^{-1} \right] . \tag{5.1}$$

Opening the series in (5.1) up to the first order:

$$\text{or} \quad t_{GRE1} \approx \left[\bar{Y} \left(1 + \bar{e}_{wy} \right) - K_1 \bar{Z} \bar{e}_{wz} \right] \exp \left[\frac{-\alpha_1 \bar{e}_{wx}}{a} \left\{ 1 - \bar{e}_{wx} + \frac{\bar{e}_{wx}}{a} \right\} \right], \tag{5.2}$$

Opening the exponential term in (5.2) up-to the second degree, and simplifying by discarding the expressions with degree three or more:

$$\text{or} \quad t_{GRE1} - \bar{Y} \approx \bar{Y}\bar{e}_{wy} - \frac{\alpha_1 \bar{Y}\bar{e}_{wx}}{a} + \frac{\alpha_1 \bar{Y}\bar{e}_{wx}^2}{a} - \frac{\alpha_1 \bar{Y}\bar{e}_{wx}^2}{a^2} + \frac{\alpha_1^2 \bar{Y}\bar{e}_{wx}^2}{2a^2}$$

$$- \frac{\alpha_1 \bar{Y}\bar{e}_{wx}\bar{e}_{wy}}{a} - K_1 \bar{Z}\bar{e}_{wz} + \frac{\alpha_1 K_1 \bar{Z}\bar{e}_{wx}\bar{e}_{wz}}{a}. \tag{5.3}$$

Expectations are applying on both sides of (5.3), using the notations (3.4) as:

$$\text{or} \quad Bias(t_{GRE1}) \approx \frac{\alpha_1 \theta \bar{Y} C_{wx}^2}{2a^2}\left[\alpha_1 + 2(a-1) - 2aH_{wyx}\right] + \frac{\alpha_1 \theta K_1 \bar{Z} H_{wxz} C_{wz}^2}{a}. \tag{5.4}$$

The mean square error of t_{GRE1} is derived using (5.2). Discarding the expressions with power two or more we get,

$$t_{GRE1} \approx \left[\bar{Y} + \bar{Y}\bar{e}_{wy} - K_1 \bar{Z}\bar{e}_{wz}\right] \exp\left[\frac{-\alpha_1 \bar{e}_{wx}}{a}\right]. \tag{5.5}$$

Opening the exponential expression up-to the first degree, we get (5.6) as:

$$\text{or} \quad t_{GRE1} - \bar{Y} \approx \bar{Y}\bar{e}_{wy} - K_1 \bar{Z}\bar{e}_{wz} - \frac{\alpha_1 \bar{Y}\bar{e}_{wx}}{a} - \frac{\alpha_1 \bar{Y}\bar{e}_{wx}\bar{e}_{wy}}{a} + \frac{\alpha_1 K_1 \bar{Z}\bar{e}_{wx}\bar{e}_{wz}}{a}. \tag{5.6}$$

Applying the square and expectations on the both sides of (5.6) and using the notations (3.3), we get (5.7) as

$$MSE(t_{GRE1}) \approx \Bigg[\theta \bar{Y}^2 C_{wy}^2 + \theta K_1^2 \bar{Z}^2 C_{wz}^2 - 2\theta K_1 \overline{YZ} \rho_{wywz} C_{wy} C_{wz}$$

$$+ \frac{\theta \alpha_1^2 \bar{Y}^2 C_{wx}^2}{a^2} - \frac{2\theta \alpha_1 \bar{Y}^2 \rho_{wxwy} C_{wx} C_{wy}}{a} + \frac{2\theta \alpha_1 K_1 \overline{YZ} \rho_{wxwz} C_{wx} C_{wz}}{a}\Bigg]. \tag{5.7}$$

Partially differentiate (5.7) with respect to (w.r.t) " K_1 ", and set equal to zero:

$$\left[2K_1 \theta \bar{Z}^2 C_{wz}^2 - 2\theta \overline{YZ} \rho_{wywz} C_{wy} C_{wz} + \frac{2\theta \alpha_1 \overline{YZ} \rho_{wxwz} C_{wx} C_{wz}}{a}\right] = 0 \tag{5.8}$$

After simplification, we get optimum value of K_1,

$$K_1 = \frac{\bar{Y}}{\bar{Z}}\left[H_{wyz} - \frac{\alpha_1 H_{wxz}}{a}\right]. \tag{5.9}$$

Partially differentiate (5.7) w.r.t" a ", and set equal to zero:

$$\left[\frac{-2\theta \alpha_1^2 \bar{Y}^2 C_{wx}^2}{a^3} - \frac{2\theta \alpha_1 K_1 \overline{YZ} \rho_{wxwz} C_{wx} C_{wz}}{a^2} + \frac{2\theta \alpha_1 \bar{Y}^2 \rho_{wxwy} C_{wx} C_{wy}}{a^2}\right] = 0 \tag{5.10}$$

After some simplification, we get optimum value of a as follows:

$$a = \frac{\alpha_1\left(1-\rho^2_{wxwz}\right)}{H_{wyx} - H_{wxz}H_{wyz}\dfrac{C^2_{wz}}{C^2_{wx}}}. \tag{5.11}$$

Put the value of "a" (5.11) in equation (5.9) we get:

$$K_1 = \frac{\bar{Y}}{\bar{Z}}\left[\frac{H_{wyz} - H_{wyx}H_{wxz}}{1-\rho^2_{wxwz}}\right]. \tag{5.12}$$

To get minimized mean square error substitute the value of "K_1" (5.12) and "a" (5.11) in equation (5.7), after some simplifications, we get:

$$MSE\left(t_{GRE1}\right)_{\min} \approx \frac{\theta\bar{Y}^2C^2_{wy}}{\left(1-\rho^2_{wxwz}\right)^2}\Big[(1-\rho^2_{wxwz})^2 - \rho^2_{wywz} + \rho^2_{wxwz}\rho^2_{wxwy} - \rho^2_{wxwy}$$

$$-\rho^2_{wxwz}\rho^2_{wxwz} - 2\rho_{wywz}\rho_{wxwy}\rho^3_{wxwz} + 2\rho_{wywz}\rho_{wxwy}\rho_{wxwz}\Big]. \tag{5.13}$$

5.1 Special Cases of the Proposed Generalized Estimators

Several non-exponential and exponential type estimators as new family of estimators can be obtained for various values of K_1, α_1 and a with proposed generalized estimator t_{GRE1}:

Table 5.1
Special Cases from t_{GRE1}

K_1	α_1	a	Estimator
0	0	a	t_4
0	1	1	$t_7 = \bar{w}_y \exp\left[\frac{\bar{X} - \bar{w}_x}{\bar{X}}\right]$
0	1	2	$t_8 = \bar{w}_y \exp\left[\frac{\bar{X} - \bar{w}_x}{\bar{X} + \bar{w}_x}\right]$
0	-1	1	$t_9 = \bar{w}_y \exp\left[\frac{\bar{w}_x - \bar{X}}{\bar{X}}\right]$
0	-1	2	$t_{10} = \bar{w}_y \exp\left[\frac{\bar{w}_x - \bar{X}}{\bar{w}_x + \bar{X}}\right]$
1	0	a	$t_{11} = \bar{w}_y + \left(\bar{Z} - \bar{w}_z\right)$
1	1	1	$t_{12} = \left\{\bar{w}_y + \left(\bar{Z} - \bar{w}_z\right)\right\} \exp\left[\frac{\bar{X} - \bar{w}_x}{\bar{X}}\right]$
1	1	2	$t_{13} = \left\{\bar{w}_y + \left(\bar{Z} - \bar{w}_z\right)\right\} \exp\left[\frac{\bar{X} - \bar{w}_x}{\bar{X} + \bar{w}_x}\right]$
1	-1	1	$t_{14} = \left\{\bar{w}_y + \left(\bar{Z} - \bar{w}_z\right)\right\} \exp\left[\frac{\bar{w}_x - \bar{X}}{\bar{X}}\right]$
1	-1	2	$t_{15} = \left\{\bar{w}_y + \left(\bar{Z} - \bar{w}_z\right)\right\} \exp\left[\frac{\bar{w}_x - \bar{X}}{\bar{X} + \bar{w}_x}\right]$

The bias and mean square error expressions for special cases may be obtained by using the different values of generalizing and optimization constants in (5.4) and (5.7) for t_{GRE1}.

6. SIMULATION STUDY

The efficiency of suggested estimators is compare with the existing estimators by a simulated population and simulations are performed for the thorough study. The condition $y > 0$ is assumed to include new units in the sample. According to this condition, the values of the study variable y are acquired and these values are averaged for sample network. Similarly, the values of the auxiliary variables x and z are gained and averaged for each sample network. Ten thousands iteration has been performed in the simulation study for all estimators to obtain accuracy estimates with the *srswor*. The preliminary sample sizes of 5, 10, 15, 20 and 25 are used in this simulation study.

In adaptive cluster sampling, the ultimate sample amount is generally bigger than the preliminary sample amount. The expected final sample size in adaptive cluster sampling is the summation of the probabilities of inclusion of every quadrat. Let the expected final sample size denoted by $E(v)$, then:

$$E(v)=\sum_{i=1}^{N}\pi_i \quad . \tag{6.1}$$

The expected final sample size differs from one sample to another sample in ACS. The sample mean from a srswor based on E(v) has variance using the formula:

$$Var(\bar{y}^*)=\frac{S_y^2(N-E(v))}{NE(v)} \quad . \tag{6.2}$$

This can be used for the comparison with the adaptive estimators.

The estimated mean squared error of the estimated mean is:

$$\hat{MSE}(t_*)=\frac{1}{r}\sum_{i=1}^{r}(t_*-\bar{Y})^2 \quad . \tag{6.3}$$

where t_* is the related estimator for the sample i and iterations size are represented with r .

Thus, comparative percentage relative efficiency is defined as:

$$PRE=\frac{Var(\bar{y}^*)}{\hat{MSE}(t_*)}\times 100 \quad . \tag{6.4}$$

Thus, estimated relative bias is defined as:

$$\hat{RBias}(t_*)=\frac{\frac{1}{r}\sum_{i=1}^{r}(t_*)-\bar{Y}}{\bar{Y}} \quad . \tag{6.5}$$

6.1 Simulated Population

In this population, a pair of auxiliary variables is considered. This pair was taken from smith et al. (1995), 5000 km^2 was the whole region for original observations which was divided into 50 100-km^2 quadrats in central Florida. In this pair, blue-winged teal data (Table 6.1) and Green- winged teal data

(Table 6.2) has been used as auxiliary variable x and z respectively.

Table 6.1
Blue-Winged Ducks(x)

0	0	3	5	0	0	0	0	0	0
0	0	0	24	14	0	0	10	103	0
0	0	0	0	2	3	2	0	13639	1
0	0	0	0	0	0	0	0	14	122
0	0	0	0	0	0	2	0	0	177

Table 6.2
Green-Winged Ducks(z)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	35	75	0
0	0	0	0	0	0	0	0	2255	13
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	24

Dryver and Chao (2007) used the following two models to produce the values for the study variable:

$$y_i = 4x_i + \varepsilon_i \qquad \text{where} \qquad \varepsilon_i \sim N(0, x_i) \tag{6.6}$$

$$y_i = 4w_{xi} + \varepsilon_i \qquad \text{where} \qquad \varepsilon_i \sim N(0, w_{xi}) \tag{6.7}$$

The variability for the variable of interest y is proportional to the auxiliary variable itself in model (6.6) while in model (6.7) the variance is proportional to the within-network mean level of the auxiliary variable. Thus, the within network variances for the variable of interest in the networks comprising of greater than one quadrates are a lot higher in the population produced with model (6.6). The model (6.8) is used to generate the values of the study variable with the auxiliary variables x and z. Let y_i, x_i and z_i indicates the i-th values for the study variable y, auxiliary variables x *and* z respectively.

$$y_i = 4x_i + 4z_i + \varepsilon_i \qquad \text{where} \qquad \varepsilon_i \sim N(0, x_i + z_i) \tag{6.8}$$

In simulated population (Table 6.3) the variability of variable of interest is proportional to the summation of the auxiliary variables. Consequently, within network variances of the study variable in the networks comprising two or more units are expected to be greatly higher in the population simulated with model (6.8).

Table 6.3
Simulated Values (*y*)

0	0	12	20	0	0	0	0	0	0
0	0	0	93	54	0	0	173	713	0
0	0	0	0	8	11	9	0	63770	4
0	0	0	0	0	0	0	0	52	486
0	0	0	0	0	0	7	0	0	820

There found a very high correlation 0.999 between the variable of interest and auxiliary variables and this correlation remains same in the transformed population (Tables 6.4 -6.6) as well. Thus, there is a high correlation between the sampling unit level as well as network (region) level. Dryver and Chao (2007) investigated that for very high correlation at unit level the traditional estimators in *srswor* execute well than ACS estimators and the traditional estimators execute poorer when there is very high correlation at network level.

Table 6.4
Average Values of *x* (*Wx*)

0	0	7.57	7.57	0	0	0	0	0	0
0	0	0	7.57	7.57	0	0	2009.4	2009.4	0
0	0	0	0	7.57	7.57	7.57	0	2009.4	2009.4
0	0	0	0	0	0	0	0	2009.4	2009.4
0	0	0	0	0	0	2	0	0	2009.4

Table 6.5
Average Values of *z* (*Wz*)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	343.14	343.14	0
0	0	0	0	0	0	0	0	343.14	343.14
0	0	0	0	0	0	0	0	343.14	343.14
0	0	0	0	0	0	0	0	0	343.14

Table 6.6
Average Values of y (Wy)

0	0	29.57	29.57	0	0	0	0	0	0
0	0	0	29.57	29.57	0	0	9429.7	9429.7	0
0	0	0	0	29.57	29.57	29.57	0	9429.7	9429.7
0	0	0	0	0	0	0	0	9429.7	9429.7
0	0	0	0	0	0	7	0	0	9429.7

Table 6.8
Comparative Percentage Relative Efficiencies

$E(v)$	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_{11}	t_{12}	t_{13}
18.90	100	*	161.8	453.2	133.9	*	221.8	309.3	274.1	145.7	364.8	332.3
28.76	100	*	161.8	328.6	136.7	*	228.3	339.8	281.7	146.6	407.4	341.6
34.13	100	*	161.8	261.8	148.6	0.01	235.9	543.3	359.9	162.6	646.4	423.7
37.44	100	*	161.8	217.4	164.5	49708.7	268.2	879.1	460.4	177.3	944.1	549.7
39.91	100	3541	161.8	185.9	187.8	177606	309.9	1347.1	592.5	202.7	1831.4	696.1

Table 6.9
Estimated Relative Bias for Different Sample Sizes

n	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_{11}	t_{12}	t_{13}
5	*	-0.02	-0.49	-0.01	*	0	-0.56	-0.25	-0.01	-0.52	-0.22
10	*	0.01	-0.43	0.00	*	0	-0.28	-0.11	0.00	-0.26	-0.10
15	*	0.00	-0.36	0.00	*	0	-0.16	-0.05	0.00	-0.15	-0.06
20	*	0.00	-0.30	0.00	0	0	-0.10	-0.03	-0.01	-0.09	-0.03
25	0.05	0.00	-0.25	0.00	0	0	-0.07	-0.02	0.00	-0.03	-0.02

7. CONCLUSION

The outcomes of simulations study are shown in Table (6.8) for 15 diverse estimators. The first 3 estimators are from conventional sampling design and remaining estimators from adaptive cluster sampling design. The results of relative efficiencies in Table (6.8) have revealed the poorer performance of usual estimators in *srswor* because of the large network-variances of the study variable. Table (6.8) point outs the improved performance estimators in ACS as compare to usual estimators due to high correlation between study and auxiliary variables at the network level. The usual ratio estimator and ratio estimator in ACS did not perform and return no value (*) for the initial sample sizes 5, 10 and 15. Dryver and Chao (2007) assumed 0/0 as zero for the modified ratio estimator in ACS. In this simulation study 0/0 is not assumed 0. The estimated relative bias is given in Table (6.9), with increasing the sample size the bias decreases. The estimator ratio estimators returns no values (*) and t_{16} gives infinity (**). In adaptive cluster sampling, the proposed estimators are much more efficient as compare to Dryver and Chao (2007) ratio estimator. The use of difference cum exponential estimators is better in adaptive cluster sampling than assuming an unlikely assumption for the ratio estimator in adaptive cluster sampling.

ACKNOWLEDGEMENTS

The authors are indebted to Yves G. Berger, University of Southampton, UK for the guidance to generate population, simulation studies and recommendations to improve this paper.

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