Pak. J. Statist.
2017 Vol. 33(4), 269-276

NON-DIRECTIONAL TWO-DIMENSIONAL NEIGHBOR DESIGNS

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SUMMARY

This paper introduces two new procedures for the construction of non-directional two-dimensional neighbor designs which can be used in agriculture, biometrics, and other fields. The first method that we introduce generates equi-neighbor balanced designs and partially neighbor balanced designs for bordered and un-bordered plots simultaneously. The second method generates Latin squares and partially neighbor balanced designs using properties of primitive roots. The proposed designs are balanced and partially balanced with respect to nearest neighbors in any of four main directions, north, south, east and west, say. Our constructed designs are recommended for the cases where the performance of treatment is affected by its immediate neighbors.

KEYWORDS

Two-dimensional neighbor design; primitive roots; equi-neighbor balanced design.

1. INTRODUCTION

A design, in which interaction effects are minimized in four main directions, say, north, south, east and west, is called a two-dimensional neighbor design. In two-dimensional neighbor designs, any fixed treatment may be influenced by its immediate neighbors. These deigns are commonly used in agriculture, forestry and other fields [Besag and Kempton (1986)]. In a fertilizer experiment, nutrients may poach to neighboring treatments through irrigation or spraying, for example. Two-dimensional neighbor designs exist for bordered and un-bordered plots. In an un-bordered plot setup, the designs are regarded as linear in each direction whereas in bordered plots, the designs are regarded as torus. In bordered two-dimensional neighbor designs, the neighbor effects of the outer boundary of block are taken into account. A two-dimensional design is non-directional if the influence of neighboring plots in rows and columns are equal. The designs considered here are non-directional and two-dimensional, set out in p rows and q columns. The proposed designs are either balanced or partially balanced with respect to nearest neighbors. Without loss of generality, we refer to the directions as north, south, east, and west, although in practice the plots may have different orientations. To avoid complexity, intermediate directions are not considered here.

For the construction of two-dimensional neighbor designs, Olesen (1976) constructed four complete 4×4 Latin squares side by side. Freeman (1979) suggested different methods for the construction of row-column neighbor balanced designs with and without borders. Street and Street (1985) suggested some row-column partially neighbor balanced designs. Street (1986) constructed some families of un-bordered two-dimensional nearest neighbor balanced designs using complete and quasi complete Latin squares. Morgan (1988) presented methods for the construction of non-directional neighbor balanced designs. Federer and Basford (1991) formulated the statistical response models for two-dimensional layouts. Morgan and Uddin (1991) constructed two-dimensional designs on the torus and in the plane for correlated errors. Chan and Eccleston (1998) constructed bordered and un-bordered nearest neighbor balanced designs. Chan and Eccleston (2003) proposed designs with unequal row and column sizes. Uddin (2007) developed some classes of balanced two-dimensional designs for main and intermediate directions over toruses and cylinders.

Neighbor designs have vast applications and can play a vital role in upcoming research. Since each specific problem needs a particular neighbor design hence a general recommendation cannot be given for all situations. Before this research, two-dimensional neighbor designs were developed separately for bordered and un-bordered plots. It is first time in literature that a method is introduced which generates neighbor balanced and partially neighbor balanced designs for bordered and un-bordered plots simultaneously.

Definition:

A two-dimensional design is nearest neighbor balanced if each treatment has every other treatment as its neighbor in an adjacent plot for fixed number of times in rows and fixed number of times in columns, i.e., λ_p times in rows and λ_q times in columns. It is not necessary that $\lambda_p = \lambda_q$ in this case. The design will be equi-neighbor balanced if $\lambda_p = \lambda_q$. A design is partially neighbor balanced if for a fixed treatment, the rest of the treatments are nearest neighbor;

1. λ_{p_ξ} times in rows and λ_{q_ξ} times in columns, ($\xi = 1, 2, \ldots, v-1$);
2. λ_p times in rows and λ_{q_ξ} times in columns;
3. λ_{p_ξ} times in rows and λ_q times in columns.

Federer and Basford (1991) have given the following two-dimensional model for p rows and q columns.

$$Y_{ijhabcd} = \mu + p_i + q_j + \tau_h + \varphi_a + \varphi_b + \varphi_c + \varphi_d + \varepsilon_{ijhabcd} \qquad (\mathrm{M}_1)$$

$Y_{ijhabcd}$ is response from the *i*th row and *j*th column of *h*th treatment having neighbors of a, b, c, d directions. The neighbor effect of any treatment is independent of the position occupied by the treatment. The Model (M_1) is assumed to be most plausible when plots are square because for rectangular plots, diagonal neighbors touch plot ijh on a corner only and cause a little effect (for details, see Federer and Basford (1991)).

2. EQUI-NEIGHBOR BALANCED DESIGN FOR BORDERED AND UN-BORDERED PLOTS SIMULTANEOUSLY

In this section, two theorems are introduced which generate infinite series of two-dimensional nearest neighbor designs for bordered and un-bordered plots simultaneously. The neighbor design consists of blocks of $p \times q$ size where $p = q$. In this design any two distinct treatments may appear as nearest neighbors equally often in p rows and q columns but no like neighbors are allowed in rows and in columns.

Theorem 2.1:

Let $v = 2n+1$ is odd prime, $n \geq 1$, then there are n Latin squares $\left(D_1, D_2, D_3 \ldots, D_n\right)$ under modulo v which generate non-directional two-dimensional nearest neighbor balanced design. The developed design is an equi-neighbor balanced for bordered and un-bordered plots simultaneously. This two-dimensional neighbor balanced design has parameters:

1. $v; r = nv;\ \lambda_p = \lambda_q = 2n$ for un-bordered plots;
2. $v; r = nv; \lambda_p = \lambda_q = v$ for bordered plots.

Proof:

Let v distinct treatments, $v = 2n+1 = \text{odd prime}$, appear in each of p rows and each of q columns of blocks $D_1, D_2, D_3 \ldots, D_n$ under modulo v. The row neighbor differences from the first row of D_1 are ± 1, from D_2 are ± 2 and so on from D_n are $\pm n$. Each neighbor difference appears $2n$ times. The remaining rows of D_1 is obtained by adding 1 to previous row, remaining rows of D_2 by adding 2 to previous row and so on remaining rows of D_n by adding n to previous row. The differences of any two nearest neighboring treatments are from ± 1 to $\pm(v-n)$ in D_1 to D_n respectively. The nv rows together give row neighbor balance with row neighbor count $= 2n$ of an un-bordered plot design. This is true since $\pm 1, \pm 2, \ldots, \pm n$ (mod v) are $1, 2, \ldots, 2n$. Similar argument exists for column neighbor balance.

If $D_1, D_2, D_3 \ldots, D_n$ are considered blocks of bordered plots then there is one more row (column) neighbor difference. It means there are $(2n+1) = v$ neighbor counts. In all blocks, no same treatment occurs side by side from east to west or north to south directions. These blocks give exactly r replicates of each treatment and all pairs of two neighboring treatments appear equally in all blocks. All these blocks generate a bordered and un-bordered equi-neighbor balanced design simultaneously. This equi-neighbor balanced design for un-bordered plots has parameters v; $r = nv$; $\lambda_p = \lambda_q = 2n$ and for bordered plots, parameters are; $v, r = nv,\ \lambda_p = \lambda_q = v$.

Example 2.1:

The blocks for $v = 2n+1 = 7, n = 3$ is

$$D_1 = \begin{array}{|ccccccc|}\hline 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 & 0 \\ 2 & 3 & 4 & 5 & 6 & 0 & 1 \\ 3 & 4 & 5 & 6 & 0 & 1 & 2 \\ 4 & 5 & 6 & 0 & 1 & 2 & 3 \\ 5 & 6 & 0 & 1 & 2 & 3 & 4 \\ 6 & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline \end{array}\cdot\ D_2 = \begin{array}{|ccccccc|}\hline 0 & 2 & 4 & 6 & 1 & 3 & 5 \\ 2 & 4 & 6 & 1 & 3 & 5 & 0 \\ 4 & 6 & 1 & 3 & 5 & 0 & 2 \\ 6 & 1 & 3 & 5 & 0 & 2 & 4 \\ 1 & 3 & 5 & 0 & 2 & 4 & 6 \\ 3 & 5 & 0 & 2 & 4 & 6 & 1 \\ 5 & 0 & 2 & 4 & 6 & 1 & 3 \\ \hline \end{array}\cdot\ D_3 = \begin{array}{|ccccccc|}\hline 0 & 3 & 6 & 2 & 5 & 1 & 4 \\ 3 & 6 & 2 & 5 & 1 & 4 & 0 \\ 6 & 2 & 5 & 1 & 4 & 0 & 3 \\ 2 & 5 & 1 & 4 & 0 & 3 & 6 \\ 5 & 1 & 4 & 0 & 3 & 6 & 2 \\ 1 & 4 & 0 & 3 & 6 & 2 & 5 \\ 4 & 0 & 3 & 6 & 2 & 5 & 1 \\ \hline \end{array}$$

Each treatment is the nearest neighbor of every other treatment 6 times for un-bordered plots and 7 times for bordered plots in each of four directions. For un-bordered plots, these blocks yield an equi-neighbor balanced design with parameters; $v = 7, r = 21, \lambda p = \lambda q = 6$. Similarly, for bordered plots, these blocks yield an equi-neighbor balanced design with parameters; $v = 7, r = 21, \lambda p = \lambda q = 7$.

Corollary (Theorem 2.1):

If there are $D_1, D_2, D_3 \ldots, D_{2n}$ blocks of Theorem 2.1, we get $2n$ blocks in total. These blocks generate an equi-neighbor balanced design in which every treatment occurs with every other treatment as its nearest neighbor $4n$ times in all rows and $4n$ times in all columns for un-bordered plots and similarly $2v$ times in all rows and $2v$ times in all columns for bordered plots. The developed equi-neighbor balanced design for un-bordered plots has parameters $v, r = 2nv, \lambda p = \lambda q = 4n$. For bordered plots, parameters of equi-neighbor balanced design are; $v, r = 2nv, \lambda p = \lambda q = 2v$.

Theorem 2.2:

Let $v = m-1$ treatments, where $m = 2n+1$ is an odd prime, then there are n blocks $(D_1, D_2, D_3 \ldots., D_n)$ under modulo m which yield non-directional two-dimensional partially neighbor balanced design with parameters:

1. $v, r = nv$, $\lambda_{p_1} = \lambda_{q_1} = v-1$ and $\lambda_{p_2} = \lambda_{q_2} = 2(v-1)$ for un-bordered plots;
2. $v, r = nv$, $\lambda_{p_1} = \lambda_{q_1} = v$ and $\lambda_{p_2} = \lambda_{q_2} = 2v$ for bordered plots.

Proof:

The row neighbor differences from the first row of D_1 are ± 1, from D_2 are ± 2 and so on from D_n are $\pm n$. The remaining rows of D_1 is obtained by adding 1 to previous row, remaining rows of D_2 by adding 2 to previous row and so on remaining rows of D_n by adding n to previous row. If D_1 to D_n are blocks of un-bordered plot then $\binom{v}{2} - n$ neighbor differences appear $(v-1)$ times and n neighbor differences appear

$2(v-1)$ times. These blocks generate two-dimensional partially neighbor balanced design in which $h_1 = \binom{v}{2} - n$ pairs occur $(v-1)$ times and n pairs occur $2(v-1)$ times as nearest neighbor for un-bordered plots. Similarly, If D_1 to D_n are blocks of bordered plot then these blocks generate two-dimensional partially neighbor balanced design in which $h_1 = \binom{v}{2} - n$ pairs occur v times and n pairs occur $2v$ times as nearest neighbor. The partially neighbor balanced design for un-bordered plots has parameters; $v, r = nv$, $\lambda_{p_1} = \lambda_{q_1} = v-1$ and $\lambda_{p_2} = \lambda_{q_2} = 2(v-1)$. For bordered plots, parameters of partially neighbor balanced design are; $v, r = nv$, $\lambda_{p_1} = \lambda_{q_1} = v$ and $\lambda_{p_2} = \lambda_{q_2} = 2v$.

Example 2.2:

The blocks for $v = m-1 = 4, m = 2n+1 = 5, n = 2$ is

$$D_1 = \begin{array}{|cccc|} \hline 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ \hline \end{array}\,. \quad D_2 = \begin{array}{|cccc|} \hline 2 & 4 & 1 & 3 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \\ 3 & 2 & 4 & 1 \\ \hline \end{array}\,.$$

Remember that $4 \equiv 0$ in all blocks. In above blocks, 4 pairs of neighboring treatments occur 3 times for un-bordered plots and 4 times for bordered plots whereas 2 pairs occur 6 times for un-bordered plots and 8 times for bordered plots in main directions. These blocks yield a partially neighbor balanced design for un-bordered plots with parameters: $v = 4,\ r = 8$, $\lambda_{p_1} = \lambda_{q_1} = 3$ and $\lambda_{p_2} = \lambda_{q_2} = 6$. Similarly for bordered plots, blocks yield partially neighbor balanced design with parameters: $v = 4,\ r = 8$, $\lambda_{p_1} = \lambda_{q_1} = 4$ and $\lambda_{p_2} = \lambda_{q_2} = 8$.

3. LATIN SQUARES AND PARTIALLY NEIGHBOR BALANCED DESIGNS FOR BORDERED PLOTS THROUGH PRIMITIVE ROOTS

Bose (1938) developed designs for prime number of treatments using primitive roots. In this section Latin squares are developed by the modification of Bose (1938) construction procedure. Latin squares are helpful in measuring neighbor effect where experimental units are influenced by neighboring units. The procedure given below generates Latin square for $v = m-1$ treatments through primitive root, where m is a prime number. The generated Latin square through primitive root is partially neighbor balanced design for bordered plots.

Theorem 3.1:

Let '$v = m-1$' treatments, m be the prime number and x be the primitive root of m. Consider the row (column) under mod (m) is:

$$\left\{ x^{m-1}, x^{m-2}, x^{m-3}, \ldots, x^{1} \right\}.$$

Now develop this row (column) under mod (v). Placing this new row (column) as the first row (column) of the following block we get Latin square. This Latin square is non-directional partially neighbor balanced design with parameters: $v, r = v$, $\lambda_{p_1} = \lambda_{q_1} = 2$ and $\lambda_{p_2} = \lambda_{q_2} = 4$.

Proof:

First row (column) of the block is developed under mod (m) and then again developed under mod (v). The primitive root x generates a complete cycle of v treatments. The row neighbor differences of first row are $\pm 1, \pm 2, \pm 3, \ldots, \pm v/2$. Each neighbor difference appears 2 times. Second row is obtained by adding the neighbor difference of $\left(x^{m-2} - x^{m-1}\right)$ to first row, third row is obtained by adding the neighbor difference of $\left(x^{m-3} - x^{m-2}\right)$ to second row and so on developed under mod (v). Since number of treatments are even so neighbor difference $\left(\pm v/2\right)$ appears 4 times. The block is Latin square because it contains complete cycle of primitive root x in its each row and in each column. This Latin square is also a partially neighbor balanced design. Each distinct pair of neighboring treatments occurs once while those pairs whose difference is $v/2$ occur twice. In this partially neighbor balanced design, each treatment is a neighbor of every other treatment once in east, west, north and south directions except those neighboring treatments whose difference is $v/2$ occur twice in all directions. Thus $v/2$ pairs of neighboring treatments occur twice. This partially neighbor balanced design has parameters: $v, r = v$, $\lambda_{p_1} = \lambda_{q_1} = 2$ and $\lambda_{p_2} = \lambda_{q_2} = 4$.

Example 3.1:

Let $m = v + 1 = 11$, primitive root $\left(x\right) = 2$ and treatments are $v = 10$, then first row (column) under mod (m) is (1, 6, 3, 7, 9, 10, 5, 8, 4, 2). Now taking mod (v) of this row (column) we get (1, 6, 3, 7, 9, 0, 5, 8, 4, 2).

We generate Latin square from first row and this Latin square is partially neighbor balanced design with bordered plots such that each treatment appears with every other treatment as nearest neighbor once in each of four directions except those pairs whose difference is $v/2 = 5$ appear twice in each direction.

$$D = \begin{array}{c|cccccccccc|c}
 & 2 & 7 & 4 & 8 & 0 & 1 & 6 & 9 & 5 & 3 & \\
\hline
2 & 1 & 6 & 3 & 7 & 9 & 0 & 5 & 8 & 4 & 2 & 1 \\
7 & 6 & 1 & 8 & 2 & 4 & 5 & 0 & 3 & 9 & 7 & 6 \\
4 & 3 & 8 & 5 & 9 & 1 & 2 & 7 & 0 & 6 & 4 & 3 \\
8 & 7 & 2 & 9 & 3 & 5 & 6 & 1 & 4 & 0 & 8 & 7 \\
0 & 9 & 4 & 1 & 5 & 7 & 8 & 3 & 6 & 2 & 0 & 9 \\
1 & 0 & 5 & 2 & 6 & 8 & 9 & 4 & 7 & 3 & 1 & 0 \\
6 & 5 & 0 & 7 & 1 & 3 & 4 & 9 & 2 & 8 & 6 & 5 \\
9 & 8 & 3 & 0 & 4 & 6 & 7 & 2 & 5 & 1 & 9 & 8 \\
5 & 4 & 9 & 6 & 0 & 2 & 3 & 8 & 1 & 7 & 5 & 4 \\
3 & 2 & 7 & 4 & 8 & 0 & 1 & 6 & 9 & 5 & 3 & 2 \\
\hline
 & 1 & 6 & 3 & 7 & 9 & 0 & 5 & 8 & 4 & 2 &
\end{array}$$

The above design is partially neighbor balanced with bordered plots having parameters; $v = 10, r = 10$, $\lambda_{p_1} = \lambda_{q_1} = 2$ and $\lambda_{p_2} = \lambda_{q_2} = 4$.

4. DISCUSSION

Two-dimensional Neighbor designs have not yet achieved sufficient attention to be used on routine basis. The designs proposed in this paper are equi-neighbor balanced and partially neighbor balanced in two dimensions for bordered and un-bordered plots. The suggested neighbor designs are non-directional in nature as great confusion arises over the different causes of association between neighboring units. Designs are either balanced or partially balanced with respect to nearest neighbors. Our constructed designs with bordered and un-bordered plots can be fruitful in the field of agriculture, forestry and biometrics. It is first time in literature that neighbor balanced and partially neighbor balanced designs are developed for bordered and un-bordered plots simultaneously which has never been considered before. This concept is relatively under-explored in the literature but if applied in agriculture and forestry, we can increase our yield by balancing out the neighbor effect of the treatments. A new method for constructing Latin square based design is described by using the primitive roots. Partially neighbor balanced designs are observed according to characteristics of primitive root. Our objective is to control neighbor effects through the proper construction (design).

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