Pak. J. Statist.
2017 Vol. 33(3), 207-222

CONSTRUCTION OF HYPER GRAECO LATIN SUDOKU SQUARE DESIGN

Sundus Hussain[1§], **Sareer Badshah**[1] and **Sher Aslam**[2]
[1] Department of Statistics, Islamia College, Peshawar, Pakistan
[2] Govt. Degree College Ara Khel, FR Kohat, Pakistan
[§] Corresponding author Email: sndshussain@gmail.com

ABSTRACT

We have introduced a new experimental design i.e. Hyper Graeco Latin Sudoku Square Design or Hyper GLaSS Design, where blocking property of Sudoku square design is applied to Hyper Graeco Latin Square Design. By introducing the Block Sum of Squares and thereby reducing the Error Sum of Square. The purpose of Hyper GLaSS Design is to test three sets of treatments simultaneously in one experiment and allow the investigation of six factors. We have discussed Hyper GLaSS Design construction with the fixed effect model. The efficiency of Hyper GLaSS Design with that of Hyper Graeco Latin Square Design is checked through numerical example and relative efficiency. It is concluded that the proposed new design with the addition of blocking property of Sudoku Square Design is more efficient than Hyper Graeco Latin Square Design based on minimum mean squares error.

KEYWORDS

Graeco Latin Square Design (GLSD); Hyper Graeco Latin Square Design; Hyper GLaSS Design; Sudoku Design.

1. INTRODUCTION

In the last few decades the use of Graeco Latin Square design and other experimental designs have increased considerably in different fields like industry, agriculture, medicine and health related institutions (Keppel, 1991). In experimental field the researchers have great interest in reducing the experimental error and also to get maximum information with minimum resources (Kirk, 1982). A good design is one which has a minimum experimental error, but due to different situations and environmental conditions perfect design is still not available (Lakic, 2002).

Researchers keep on trying to introduce new designs simply to reduce the error (Sorana et al., 2009). In the experimental field when the variation is to be controlled for columns and rows then comparative designs like Latin Square, Graeco Latin Square and Hyper Graeco Latin squares, are used (Cochran, 1974). Fisher (1926) worked on the designs like Latin Square and Mutually Orthogonal Latin squares. Yates (1936) and Fisher (1926) revealed that such designs exist for prime order but not for non-prime order of higher terms and also they exist for 4, 8 and 9 orders.

Kishen (1950) generalized orthogonal Latin Square to Hyper-Graeco Latin Square. The extended form of GLSD "Graeco-Latin Square Design" is HGLSD "Hyper-Graeco

Latin Square Design" for controlling irritant factors from four sides with the restriction that factor levels are similar (Montgomery, 1984). It is the efficient design in the presence of irritant factors from four sides with one treatment (Colbourn and Dinitz, 2006).

HGLSD "Hyper-Graeco Latin Square Design" is the result of OLSD "Orthogonal Latin Square Design" of three types with three different types of treatment when they are overlaid on one another. The three different types of treatment are treatment 1, treatment 2 and treatment 3 (Mann, 1942). It has the capability to test treatments of three sets at the same time in the same experiment i.e., it permits five factors examination at the same time namely columns, rows, and three treatments (Colbourn and Dinitz, 2006). It is a square design and is composed of $p \times p$ horizontal grid where p is the complete figure of treatment factor levels. It permits to use a blocking factor of four in number (Bose, 1938).

A special type of Latin Square is Sudoku design developed by Harold Garns in 1979, derived from the Sudoku puzzles (Subramani and Ponnuswamy, 2009). The total experimental area of $(n_1.n_2)^2$ plots is divided into $(n_1.n_2)$ areas, each having $(n_1.n_2)$ plots and application of $(n_1.n_2)$ treatments to these plots are arranged in a way that each row or each column has no repeated digit. The empty blocks in Sudoku grids are called Sudoku puzzles where in Sudoku design the Sudoku grids are complete (Subramani and Ponnuswamy, 2009). Aslam (2008) presented the $(m_1.m_2)^2$ Sudoku square design with $m_1.m_2$ rows, $m_1.m_2$ columns, and $m_1.m_2$ blocks, as m_1 *and* m_2 digits which are greater than 1.

Designs accuracy can be increased by introducing blocks (Sorana et al., 2009). In order to further reduce the mean square error, an attempt has been made to introduce a new efficient design i.e., Hyper Graeco Latin Sudoku Square Design (Hyper GLaSS Design), by applying a blocking property of Sudoku Square Design to Hyper Graeco Latin Square Design.

The purpose of this paper is to

1. Merge the blocking property of Sudoku square design in Hyper Graeco Latin Square Design and construct a new efficient design.
2. Hyper GLaSS Design Construction with fixed effect model, its analysis with the help of numerical example and relative efficiency is discussed and compared with Hyper Graeco Latin Square Design.

2. HYPER GLASS DESIGN CONSTRUCTION

Hyper GLASS design (HGD) is a special type of Hyper Graeco Latin square design with some more constraints, in which the experimental units are laid out in 'm' rows, 'm'columns, 'm' blocks and 'm' different types of treatments are allocated to these experimental units in such a way that every treatment must place only once in each row, once in each column and once in each region or block. The division of the experimental area requires vertically as well as horizontally for the layout of Hyper GLaSS design (HGD) of any order (even or odd). The vertical division called Vertical Grids and the horizontal division known to be Horizontal Grids. For treatment type (1) the Vertical Grids consists 1 to m^2 digits in a matrix form consecutively which is starting from column 1 to column $(m-1)m+1$, the second column is starting from $m-1$ to $(m-1)m+2$ and so on. Similarly, the Horizontal Grids consist 1 to m^2 digits in such a way that first row starts from 1 to row *m*, the second row starts from $m+1$ to $2m$, and so on. Due to this vertically as well as horizontally division, the experimental area is divided into $m.m$ blocks and every block is a square of $m.m$ plots. The two different types of orthogonal Sudoku square design with treatment type (2) and treatment type (3) are allocated in these Vertical as well as Horizontal Grids by following the rules for treatment type (1) Sudoku square design.

3. LIMITATIONS OF HYPER GLaSS DESIGN

Limitation of the proposed design are that the blocks as blocking factor in Hyper GLaSS Design are only effective if the variance between blocks is significantly larger than the variance within blocks, otherwise the appropriate design may be the Hyper-Graeco-Latin Square Design, because unnecessary loss of degrees of freedom will increase the error mean square and will make the results insignificant**.** The blocks are in the form of square. There is no interaction case in the study. The proposed ANOVA in this research is based on the assumption that the errors are uncorrelated. The useful Hyper GLaSS Design is when $m \geq 6$.

The layout of Hyper GLaSS Design for Even and odd Order is in Table 1 as; (See Appendix).

Example:

Let the first row of the treatments type (1) Sudoku square design is {1, 5, 9, 13, 2, 6, 10, 14, 3, 7, 11, 15, 4, 8, 12 and 16}. Then the first row of its orthogonal treatments type (2) Sudoku square design is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16} and the first row of its third orthogonal treatments type (3) Sudoku square design is {16, 12, 8, 4, 15, 11, 7, 3, 14, 10, 6, 2, 13, 9, 5 and 1}. Table 2 shows the complete Sudoku square design of treatments type (1) with the initial row {1, 5, 9, 13, 2, 6, 10, 14, 3, 7, 11, 15, 4, 8, 12 and 16}.

Table 2 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (2) with the initial row {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16} is in Table 3 as; (See Appendix).

Table 3 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (3) with the initial row {16 12, 8, 4, 15, 11, 7, 3, 14, 10, 6, 2, 13, 9, 5, 1} is in Table 4 as; (See Appendix).

By super imposing the three types of Sudoku square designs given in Table 2, Table 3 and Table 4, we get the $(4X4)^2$ Hyper GLaSS design given in Table 5 (See Appendix).

Note: The proposed sequential method of constructing Hyper GLaSS design is valid for any value of m^2 (odd or even).

4. HYPER-GLaSS DESIGN Statistical Analysis

In Hyper GLaSS Design the three types of treatments, columns and rows are orthogonal with each other but neither blocks and columns nor blocks and rows are orthogonal. Thus some of the contrasts on region or blocks are confounded with columns or with rows, which is resulting in the number of degree of freedom and the sum of squares for blocks.

The three treatments i.e. treatment 1, treatment 2 and treatment 3 tested in *m*.*m* Sudoku Square design with *m* columns, *m* rows and *m* blocks. Statistical analysis of fixed effect model without any interaction term is as;

$y_{ij(klrp)}$ is the experimental field having i^{th} row, j^{th} column, k^{th} treatments of 1^{st} type, l^{th} block, r^{th} treatments of 2^{nd} type and p^{th} treatment of 3^{rd} type are applied.

The model of Hyper GLaSS Design is

$$Y_{ij(klrp)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma_l + \theta_r + \lambda_p + \varepsilon_{ij(klrp)}$$

$$i, j, k, l, r, p = 1, 2, 3..........m$$

where,

$\mu =$ Mean of the population

$\alpha_i =$ Effect of the i^{th} row,

$\beta_j =$ Effect of the j^{th} column,

$\tau_k =$ Effect of the k^{th} treatments type (1)

$\gamma_l =$ Effect of the l^{th} block,

$\theta_r =$ Effect of the r^{th} treatments type (2)

$\lambda_p =$ Effect of the p^{th} treatments type (3)

$\varepsilon_{ij(klrp)} =$ Effect of random error

The constraints are

$$\sum\alpha_i = \sum\beta_j = \sum\tau_k = \sum\gamma_l = \sum\theta_r = \sum\lambda_p = 0$$

with the assumptions that $y_{ij(klrp)}$ is linear function, and $\mu, \alpha_i, \beta_j, \tau_k, \gamma_l, \theta_r, \lambda_p, \varepsilon_{ij(klrp)}$ are identically and independently distributed as $N\left(0, \sigma^2\right)$.

The notations used in the statistical analysis of HGD are following:

$$\sum_{j=1}^{m} y_{ij(klrp)} = R_i = i^{th} \text{ row total}$$

$$\sum_{i=1}^{m} y_{ij(klrp)} = C_j = j^{th} \text{ column total}$$

$$\sum_{l=1}^{m} y_{ij(klrp)} = T_k = k^{th} \text{ treatments type (1) total}$$

$$\sum_{r=1}^{m} y_{ij(klrp)} = B_l = l^{th} \text{ block total}$$

$$\sum_{p=1}^{m} y_{ij(klrp)} = \theta_r = r^{th} \text{ treatment type (2) total}$$

$$\sum_{k=1}^{m} y_{ij(klrp)} = \lambda_p = p^{th} \text{ treatment type (3) total}$$

Then,

$$\bar{Y}_{i.(....)} = \bar{Y}_i = \frac{R_i}{m} = i^{th} \text{ row mean}$$

$$\bar{Y}_{.j(....)} = \bar{Y}_j = \frac{C_j}{m} = j^{th} \text{ column mean}$$

$$\bar{Y}_{..(k...)} = \bar{Y}_k = \frac{T_k}{m} = k^{th} \text{ treatments type (1) mean}$$

$$\bar{Y}_{..(.l..)} = \bar{Y}_l = \frac{B_l}{m} = l^{th} \text{ block mean}$$

$$\bar{Y}_{..(..r.)} = \bar{Y}_r = \frac{\theta_r}{m} = r^{th} \text{ treatments type (2) mean}$$

$$\bar{Y}_{..(...p)} = \bar{Y}_p = \frac{\lambda_p}{m} = r^{th} \text{ treatments type (3) mean}$$

$$\sum_{i=1}^{m} R_i = \sum_{j=1}^{m} C_j = \sum_{k=1}^{m} T_k(1) = \sum_{l=1}^{m} B_l = \sum_{r=1}^{m} T_r(2) = \sum_{p=1}^{m} T_p(3) = G$$

where, i^{th} row, j column, k^{th} treatment type 1, l^{th} block, r^{th} treatment type 2, p^{th} treatment type 3 are from 1 to m

$$\bar{Y} = \frac{G}{m^2} = \text{Grand mean}$$

$$C.F. = \frac{G^2}{m^2} = \text{Correction factor}$$

$$\text{Total Sum of Square} = \text{TSS} = \sum_{i=1}^{m}\sum_{j=1}^{m}\left(y_{ij(klrp)} - \bar{Y}\right)^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} y^2{}_{ij(klrp)} - C.F.$$

with $m^2 - 1$ $d.f$

$$\text{Rows Sum of Square} = \text{SSR} = m\sum_{i=1}^{m}\left(\bar{Y}_i - \bar{Y}\right)^2 = \frac{1}{m}\sum_{i=1}^{m} R_i^2 - C.F.$$

with m-1 $d.f$

$$\text{Columns Sum of Square} = \text{SSC} = m\sum_{j=1}^{m}\left(\bar{Y}_j - \bar{Y}\right)^2 = \frac{1}{m}\sum_{j=1}^{m} C_j^2 - C.F.,$$

with m-1 $d.f$

$$\text{Treatment (1) Sum of Square=SST(1)} = m\sum_{k=1}^{m}(\bar{Y}_k - \bar{Y})^2 = \frac{1}{m}\sum_{k=1}^{m} T_k^2 - C.F.$$

with m-1 $d.f$

$$\text{Blocks Sum of Square} = \text{SSB} = m\sum_{l=1}^{m}\left(\bar{Y}_l - \bar{Y}\right)^2 = \frac{1}{m}\sum_{l=1}^{m} B_l^2 - C.F.,$$

with m-1 $d.f$

$$\text{Treatment (2) Sum of Square=SST(2)} = m\sum_{r=1}^{m}\left(\bar{Y}_r - \bar{Y}\right)^2 = \frac{1}{m}\sum_{r=1}^{m} \theta_r^2 - C.F.$$

with m-1 $d.f$

$$3^{rd}\text{ Treatment Sum of Square=SST(3)} = m\sum_{p=1}^{m}\left(\bar{Y}_p - \bar{Y}\right)^2 = \frac{1}{m}\sum_{p=1}^{m} \lambda_p^2 - C.F.$$

with m-1 $d.f$

SSE = Error Sum of Square

$$= \sum_{i=1}^{m}\sum_{j=1}^{m}\left(\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{r=1}^{m}\sum_{p=1}^{m}\right)\left(Y_{ij(klrp)} - \bar{Y}_i - \bar{Y}_j - \bar{Y}_k - \bar{Y}_l - \bar{Y}_r - \bar{Y}_p + 5Y\right)^2$$

with (m-1)(m-5) $d.f$

These results are summarized in Table 6 as; (See Appendix).

5. NUMERICAL ILLUSTRATION FOR COMPARISON

Hyper GLaSS Designs analysis and its comparison with Hyper Graeco Latin Square Design through hypothetical data is in Table 7 as; (See Appendix)

The ANOVA for Hyper Graeco Latin Square Design is in Table 8 as; (See Appendix)

The corresponding ANOVA Table for the Hyper-GLaSS Design is in Table 9 as; (See Appendix).

6. RELATIVE EFFICIENCY OF THE HYPER GLaSS DESIGN

A quantitative and more precise measure of the efficiency of Hyper GLaSS design over the HGLS design is the estimated relative efficiency (RE) obtained by the relation

Relative efficiency $= E_2 / E_1$

Relative Efficiency $= S^2{}_b + (m-4) S_e{}^2 / (m-3) S^2{}_e$

Since E_1 is the error mean square of HGLSD and E_2 is the error mean square of HGD, therefore the ratio between E_1 and E_2 will give the relative efficiency of HGD as compared to HGLSD.

Using the results of Table 8 and Table 9, (see Appendix)

$m_1 = 3,\ m_2 = 3,\ m \times m = 9,\ S^2{}_b = 78,\ E_2 = 5.20,\ E_1 = 6.117$

Relative Efficiency $= E_2 / E_1 = 0.85$

7. RESULTS AND DISCUSSION

The error mean squares of Hyper-Graeco-Latin square design (HGLSD) and Hyper GLaSS design (HGD) has been calculated in Table 8 and Table 9 (see appendix) for the same hypothetical data as shown in Table 7(see appendix). By removing the variability due to blocking in the proposed design i.e. HGD has decreased the experimental error. Error mean square of HGD is 5.2 is less than the error mean square of HGLSD i.e., 6.11. Further, the relative efficiency of the two designs i.e., HGLSD with that of HGD have been compared and it can be seen that relative efficiency is less than one, means that HGD is more efficient than HGLSD. As HGD possess the properties of both HGLSD and Sudoku Square design so the result of HGD are authentic because it gives less mean square for error.

8. CONCLUSION

Hyper Graeco Latin Sudoku Square Design or Hyper GLaSS Design (HGD) is the merger of two designs i.e., Hyper Graeco Latin Square Design (HGLSD) and Sudoku Square Design. Introducing the Block Sum of Square in the new proposed design, the error sum of square is further reduced. The purpose of HGD is to test three sets of treatments simultaneously in one experiment and allow investigation of six factors. Parameter estimation with the fixed effect model and its ANOVA is given in this paper. The efficiency of the new proposed design is checked through numerical example and concluded that the new proposed design is more efficient than HGLSD based on minimum Mean Square Error. Further, the relative efficiency of the two designs is checked and proved the efficiency of HGD.

Hence the additional blocking factor makes the result more authentic and with the use of HGD one can control variability from six sides with less error mean square.

REFERENCES

1. Aslam, M. (2008). *Sudoku Experimental Design*, (M.Phil. Thesis), National College of Business Administration & Economics, Lahore, pp 91.
2. Bose, R.C. (1938). On the application of the properties of Galois fields to the problem of construction of hyper-Graeco-Latin squares. *Sankhyā: The Indian Journal of Statistics*, 3, 323-338.
3. Cochran, W.G. (1974). Early Development of Techniques in Comparative Experimentation (No. TR-44). Harvard Univ Cambridge Mass Dept. of Statistics.
4. Colbourn, C.J. and Dinitz, J.H. (Eds.). (2006). Handbook of combinatorial designs. Chapman and Hall/CRC, Taylor and Francis Group, Boca Raton London New York.
5. Fisher, R.A. (1926). The Arrangement of Field Experiments. *Journal of Ministry of Agriculture*, 33, 503-513.
6. Keppel, G. (1991). *Design and Analysis: A Researcher's Handbook*. Prentice-Hall, Inc.
7. Kirk, R.E. (1982). *Experimental design*. John Wiley & Sons, Inc.
8. Kishen, K. (1950). On the construction of Latin and hyper-Graeco-Latin cubes and hyper cubes. *J. Ind. Soc. Agric. Statist*, 2, 20-48.
9. Lakić, N.S. (2002). Classification of Latin squares. *Journal of Agricultural Sciences*, 47(1), 105-112.
10. Mann, H.B. (1942). The construction of orthogonal Latin squares. *The Annals of Mathematical Statistics*, 13(4), 418-423.
11. Montgomery, D.C. (1984). *Design and Analysis of Experiments*, John Wiley, New York, 2th Edition.
12. Sorana, D.B., Jäntschi, L. and Radu, E. (2009). Statistical approaches in analysis of variance: from random arrangements to Latin square experimental design. *Leonardo Journal of Sciences*, 8(15), 71-82
13. Subramani, J. and Ponnuswamy, K.N. (2009). Construction and analysis of Sudoku designs. *Model Assisted Statistics and Applications*, 4(4), 287-301.
14. Yates, F. (1936). Incomplete randomized blocks. *Annals of Eugenics*, 7(2), 121-140.

APPENDIX

Table 1: $m\times m$ Hyper GLaSS Design for any Values of *m* (Odd, Even)

		Columns										
		1	2	…	m	1	2	…	m	…	1		m
Rows	1	$H_{1,1,m^2}$	$H_{m+1,2,(m-1)m}$	…	$H_{(m-1)m+1,m,m}$	$H_{2,m+1,m^2-1}$	$H_{m+2,m+2,m^2-2m+2}$	…	$H_{(m-1)m+2,2m,2}$	…	$H_{m,(m-1)m+1,(m-1)m+1}$	…	$H_{m^2,m^2,1}$
	2	$H_{2,m+1,1}$	$H_{m+2,m+2,(m-1)m+1}$	…	$H_{(m-1)m+2,2m,m+1}$	$H_{3,2m+1,m^2}$	$H_{m+3,2m+2,m^2-2m+3}$	…	$H_{(m-1)m+3,3m,m}$	…	$H_{m+1,1,(m-1)m+2}$	…	$H_{1,m,2}$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	…	⋮	…	⋮	⋮	⋮
	m	$H_{m,(m-1)m+1,m-1}$	$H_{2m,(m-1)m+2,m^2-1}$	…	$H_{m^2,m^2,2m-1}$	$H_{m+1,1,m-2}$	$H_{2m+1,2,(m-1)m+1}$	…	$H_{1,m,m+1}$	…	H_{2m-1,m^2-2m+1,m^2}	…	$H_{(m-1),m(m-1),m}$
	1	$H_{m+1,2,m}$	H_{2m+1,m,m^2}	…	$H_{1,m+1,2m}$	$H_{m+2,m+2,m-1}$	$H_{2m+2,m+3,(m-1)m+2}$	…	$H_{2,2m+1,m+2}$	…	$H_{2m,(m-1)m+2,1}$	…	$H_{m,1,m+1}$
	2	$H_{m+2,m+2,m+1}$	$H_{2m+2,2m,1}$	…	$H_{2,2m+1,2m+1}$	$H_{m+3,2m+2,m}$	$H_{2m+3,2m+3,(m-1)m+3}$	…	$H_{3,3m+1,m+3}$	…	$H_{2m+1,2,2}$	…	$H_{m+1,m+1,m+2}$
	⋮	⋮	⋮	…	⋮	⋮	⋮	…	⋮	…	⋮	⋮	⋮
	m	$H_{2m,(m-1)m+2,2m-1}$	$H_{3m,m^2,m-1}$	…	$H_{m,1,3m-1}$	$H_{2m+1,2,m+1}$	$H_{3m+1,3,m-2}$	…	$H_{m+1,m+1,3m-2}$	…	$H_{3m-1,m^2-2m+2,m}$	…	$H_{2m-1,(m-1)m+1,2m}$
	⋮	⋮	⋮	…	⋮	⋮	⋮	…	⋮	…	⋮	⋮	⋮
	1	$H_{(m-1)m+1,m,(m-1)m}$	…	…	$H_{m+1,2m-1,m^2}$	$H_{(m-1)m+2,2m,m+2}$	…	…	$H_{m+2,3m-1,m^2-1}$	…	H_{m^2,m^2,m^2-2m+1}	…	$H_{(m-1)m,2,(m-1)m+1}$
	⋮	⋮	⋮	…	⋮	⋮	⋮	…	⋮	…	⋮	⋮	⋮
	m	H_{m^2,m^2,m^2-1}	…	…	$H_{m(m-1),m-1,m-1}$	$H_{1,m,2m+1}$	…	…	$H_{(m-1)m+1,2m-1,m-2}$	…	$H_{m-1,(m-1)m,m(m-1)}$	…	$H_{m^2-1,(m-1)m+2,m^2}$

Note: *H* indicates Hyper GLaSS design, where the first subscripts represent treatments type (1), the second subscripts represent treatments type (2) and the third subscripts represent treatments type (3).

Table 2
Complete Sudoku Square Design of Treatments Type (1) with the Initial Row {1, 5, 9, 13, 2, 6, 10, 14, 3, 7, 11, 15, 4, 8, 12 and 16}

1	5	9	13	2	6	10	14	3	7	11	15	4	8	12	16
2	6	10	14	3	7	11	15	4	8	12	16	5	9	13	1
3	7	11	15	4	8	12	16	5	9	13	1	6	10	14	2
4	8	12	16	5	9	13	1	6	10	14	2	7	11	15	3
5	9	13	1	6	10	14	2	7	11	15	3	8	12	16	4
6	10	14	2	7	11	15	3	8	12	16	4	9	13	1	5
7	11	15	3	8	12	16	4	9	13	1	5	10	14	2	6
8	12	16	4	9	13	1	5	10	14	2	6	11	15	3	7
9	13	1	5	10	14	2	6	11	15	3	7	12	16	4	8
10	14	2	6	11	15	3	7	12	16	4	8	13	1	5	9
11	15	3	7	12	16	4	8	13	1	5	9	14	2	6	10
12	16	4	8	13	1	5	9	14	2	6	10	15	3	7	11
13	1	5	9	14	2	6	10	15	3	7	11	16	4	8	12
14	2	6	10	15	3	7	11	16	4	8	12	1	5	9	13
15	3	7	11	16	4	8	12	1	5	9	13	2	6	10	14
16	4	8	12	1	5	9	13	2	6	10	14	3	7	11	15

Table 2 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (2) with the initial row {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16} is in Table 3 as;

Table 3
Orthogonal Sudoku Square Design of Treatments Type (2) with the Initial Row {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1
6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5
10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9
14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13
3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2
7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6
11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10
15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14
4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3
8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7
12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11
16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 3 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (3) with the initial row {16 12, 8, 4, 15, 11, 7, 3, 14, 10, 6, 2, 13, 9, 5, 1} is in Table 4 as;

Table 4
Orthogonal Sudoku Square Design of Treatments Type (3) of Order $(4x4)^2$

16	12	8	4	15	11	7	3	14	10	6	2	13	9	5	1
1	13	9	5	16	12	8	4	15	11	7	3	14	10	6	2
2	14	10	6	1	13	9	5	16	12	8	4	15	11	7	3
3	15	11	7	2	14	10	6	1	13	9	5	16	12	8	4
4	16	12	8	3	15	11	7	2	14	10	6	1	13	9	5
5	1	13	9	4	16	12	8	3	15	11	7	2	14	10	6
6	2	14	10	5	1	13	9	4	16	12	8	3	15	11	7
7	3	15	11	6	2	14	10	5	1	13	9	4	16	12	8
8	4	16	12	7	3	15	11	6	2	14	10	5	1	13	9
9	5	1	13	8	4	16	12	7	3	15	11	6	2	14	10
10	6	2	14	9	5	1	13	8	4	16	12	7	3	15	11
11	7	3	15	10	6	2	14	9	5	1	13	8	4	16	12
12	8	4	16	11	7	3	15	10	6	2	14	9	5	1	13
13	9	5	1	12	8	4	16	11	7	3	15	10	6	2	14
14	10	6	2	13	9	5	1	12	8	4	16	11	7	3	15
15	11	7	3	14	10	6	2	13	9	5	1	12	8	4	16

By super imposing the three types of Sudoku square designs given in Table 2, Table 3 and Table 4, we get the $(4x4)^2$ Hyper GLaSS design given in Table 5.

Table 5

Layout of Hyper GLaSS Design $(4\times4)^2$, when $n=4$ (where $m=n^2$)

1,1,16	5,2,12	9,3, 8	13,4,4	2,5,15	6,6,11	10,7,7	14,8,3	3,9,14	7,10,10	11,11,6	15,12,2	4,13,13	8,14,9	12,15,5	16,16,1
2,5,1	6,6,13	10,7,9	14,8,5	3,9,16	7,10,12	11,11,8	15,12,4	4,13,15	8,14,11	12,15,7	16,16,3	5,1,14	9,2,10	13,3,6	1,4,2
3,9,2	7,10,14	11,11,10	15,12,6	4,13,1	8,14,13	12,15,9	16,16,5	5,1,16	9,2,12	13,3,8	1,4, 4	6,5,15	10,6,11	14,7,7	2,8,3
4,13,3	8,14,15	12,15,11	16,16,7	5,1,2	9,2,14	13,3,10	1,4,6	6,5,1	10,6,13	14,7,9	2,8,5	7,9,16	11,10,12	15,11,8	3,12,4
5,2,4	9,3,16	13,4,12	1,5, 8	6,6,3	10,7,15	14,8,11	2,9,7	7,10,2	11,11,14	15,12,10	3,13,6	8,14,1	12,15,13	16,16,9	4,1,5
6,6,5	10,7,1	14,8,13	2,9,9	7,10,4	11,11,16	15,12,12	3,13,8	8,14,3	12,15,15	16,16,11	4,1,7	9,2,2	13,3,14	1,4,10	5,5,6
7,10,6	11,11,2	15,12,14	3,13,10	8,14,5	12,15,1	16,16,13	4,1,9	9,2,4	13,3,16	1,4,12	5,5,8	10,6,3	14,7,15	2,8,11	6,9,7
8,14,7	12,15,3	16,16,15	4,1,11	9,2,6	13,3,2	1,4,14	5,5,10	10,6,5	14,7,1	2,8,13	6,9,9	11,10,4	15,11,16	3,12,12	7,13,8
9,3,8	13,4,4	1,5,16	5,6,12	10,7,7	14,8,3	2,9,15	6,10,11	11,11,6	15,12,2	3,13,14	7,14,10	12,15,5	16,16,1	4,1,13	8,2,9
10,7,9	14,8,5	2,9,1	6,10,13	11,11,8	15,12,4	3,13,16	7,14,12	12,15,7	16,16,3	4,1,15	8,2,11	13,3,6	1,4,2	5,5,14	9,6,10
11,11,10	15,12,6	3,13,2	7,14,14	12,15,9	16,16,5	4,1,1	8,2,13	13,3,8	1,4,4	5,5,16	9,6,12	14,7,7	2,8,3	6,9,15	10,10,11
12,15,11	16,16,7	4,1,3	8,2,15	13,3,10	1,4,6	5,5,2	9,6,14	14,7,9	2,8, 5	6,9,1	10,10,13	15,11,8	3,12,4	7,13,16	11,14,12
13,4,12	1,5,8	5,6,4	9,7,16	14,8,11	2,9,7	6,10,3	10,11,15	15,12,10	3,13, 6	7,14,2	11,15,14	16,16,9	4,1,5	8,2,1	12,3,13
14,8,13	2,9,9	6,10,5	10,11,1	15,12,12	3,13,8	7,14,4	11,15,16	16,16,11	4,1, 7	8,2,3	12,3,15	1,4,10	5,5,6	9,6,2	13,7,14
15,12,14	3,13,10	7,14,6	11,15,2	16,16,13	4,1,9	8,2,5	12,3,1	1,4,12	5,5, 8	9,6,4	13,7,16	2,8,11	6,9,7	10,10,3	14,11,15
16,16,15	4,1,11	8,2,7	12,3,3	1,4,14s	5,5,10	9,6,6	13,7,2	2,8,13	6,9,9	10,10,5	14,11, 1	3,12,12	7,13,8	11,14,4	15,15,16

Table 6
ANOVA Table for $m \times m$ Hyper GLaSS Design

Sources of Variation	d.f	SS	MS
Rows	$m-1$	$SSR = \frac{1}{m}\sum_{i=1}^{m} R_i^2 - C.F.$	$S^2{}_R = \frac{SSR}{m-1}$
Columns	$m-1$	$SSC = \frac{1}{m}\sum_{j=1}^{m} C_j{}^2 - C.F.$	$S^2{}_C = \frac{SSC}{m-1}$
Treatments Type (1)	$m-1$	$SST(1) = \frac{1}{m}\sum_{k=1}^{m} T_k{}^2(1) - C.F.$	$S^2_{T(1)} = \frac{SST}{m-1}$
Blocks	$m-1$	$SSB = \frac{1}{m}\sum_{l=1}^{m} B_l{}^2 - C.F.$	$S^2{}_B = \frac{SSB}{m-1}$
Treatments Type (2)	$m-1$	$SST(2) = \frac{1}{m}\sum_{r=1}^{m} T_r^2(2) - C.F.$	$S^2{}_{T(2)} = \frac{SST(2)}{m-1}$
Treatments Type (3)	$m-1$	$SST(3) = \frac{1}{m}\sum_{p=1}^{m} T_p^2(3) - C.F.$	$S^2{}_{T(3)} = \frac{SST(3)}{m-1}$
Error	$(m-1)(m-5)$	By Subtraction	$S^2{}_E = \frac{SSE}{(m-1)(m-5)}$
Total	m^2-1	$TSS = \sum_{i,j=1}^{m}\sum_{k,l,r,p=1}^{m} y^2{}_{ij(klrp)} - C.F.$	

Table 7
Hypothetical Data for Hyper GLaSS Design of Order 9

Row \ Col	1	2	3	4	5	6	7	8	9
1	$A_1A_2I_3$ (15)	$D_1B_2F_3$ (11)	$G_1C_2C_3$ (16)	$B_1D_2H_3$ (17)	$E_1E_2E_3$ (14)	$H_1F_2B_3$ (16)	$C_1G_2G_3$ (14)	$F_1H_2D_3$ (15)	$I_1I_2A_3$ (18)
2	$B_1D_2A_3$ (18)	$E_1E_2G_3$ (23)	$H_1F_2D_3$ (20)	$C_1G_2I_3$ (16)	$F_1H_2F_3$ (17)	$I_1I_2C_3$ (14)	$D_1A_2H_3$ (16)	$G_1B_2E_3$ (16)	$A_1C_2B_3$ (17)
3	$C_1G_2 B_3$ (15)	$F_1H_2H_3$ (10)	$I_1I_2E_3$ (20)	$D_1A_2A_3$ (18)	$G_1B_2G_3$ (15)	$A_1C_2D_3$ (17)	$E_1D_2I_3$ (15)	$H_1E_2F_3$ (18)	$B_1F_2C_3$ (16)
4	$D_1B_2C_3$ (17)	$G_1C_2I_3$ (15)	$A_1D_2F_3$ (18)	$E_1E_2B_3$ (16)	$H_1F_2H_3$ (15)	$B_1G_2E_3$ (11)	$F_1H_2A_3$ (16)	$I_1I_2G_3$ (14)	$C_1A_2D_3$ (17)
5	$E_1E_2D_3$ (19)	$H_1F_2A_3$ (20)	$B_1G_2G_3$ (17)	$F_1H_2C_3$ (20)	$I_1I_2I_3$ (18)	$C_1A_2F_3$ (23)	$G_1B_2B_3$ (14)	$A_1C_2H_3$ (17)	$D_1D_2E_3$ (16)
6	$F_1H_2E_3$ (20)	$I_1I_2B_3$ (23)	$C_1A_2H_3$ (19)	$G_1B_2D_3$ (20)	$A_1C_2A_3$ (15)	$D_1D_2G_3$ (10)	$H_1E_2C_3$ (17)	$B_1F_2I_3$ (15)	$E_1G_2F_3$ (18)
7	$G_1C_2F_3$ (18)	$A_1D_2C_3$ (20)	$D_1E_2I_3$ (17)	$H_1F_2E_3$ (19)	$B_1G_2B_3$ (16)	$E_1H_2H_3$ (15)	$I_1I_2D_3$ (19)	$C_1A_2A_3$ (20)	$F_1B_2G_3$ (18)
8	$H_1F_2G_3$ (17)	$B_1G_2D_3$ (19)	$E_1H_2A_3$ (18)	$I_1I_2F_3$ (17)	$C_1A_2C_3$ (15)	$F_1B_2I_3$ (14)	$A_1C_2E_3$ (17)	$D_1D_2B_3$ (19)	$G_1E_2H_3$ (17)
9	$I_1I_2H_3$ (15)	$C_1A_2E_3$ (17)	$F_1B_2B_3$ (16)	$A_1C_2G_3$ (18)	$D_1D_2D_3$ (16)	$G_1E_2A_3$ (18)	$B_1F_2 F_3$ (18)	$E_1G_2C_3$ (23)	$H_1H_2I_3$ (15)

Table 8
ANOVA Table for 9×9 Hyper Graeco-Latin Square Design

Sources of Variation	d.f	SS	MS	F
Rows	8	88.8801	11.1101	1.8162
Columns	8	64.4401	8.0550	1.3168
Treatments type (1)	8	41.7777	5.2222	0.8537
Treatments type (2)	8	38.2222	4.7777	0.7811
Treatments type (3)	8	62	7.7501	1.2669
Error	40	244.6801	6.1170	
Total	80	540		

Table 9
ANOVA Table for 9×9 Hyper GLaSS Design

Sources of Variation	d.f	SS	MS	F
Rows	8	88.8801	11.1101	2.1320
Columns	8	64.4401	8.0551	1.5460
Treatments type (1)	8	41.7777	5.2222	1.0021
Blocks	8	78	9.7501	1.8718
Treatments type (2)	8	38.2222	4.7777	0.9172
Treatments type (3)	8	62	7.7501	1.4879
Error	32	166.6801	5.2087	
Total	80	540		

* As the observed values are less than the tabulated values with 5% significance level hence these are insignificant effects.