Pak. J. Statist.
2017 Vol. 33(1), 63-74

EFFECTS OF ADDITIVE OUTLIERS ON ASYMMETRIC GARCH MODELS

Abdul Raziq[1§], **Farhat Iqbal**[1] and **Ghulam Hyder Talpur**[2]
[1] Statistics Department, University of Balochistan, Quetta, Pakistan
[2] Statistics Department, University of Sindh, Jamshoro, Pakistan
[§] Corresponding author Email: raziqkakar@yahoo.com

ABSTRACT

This study examines effects of additive outliers on asymmetric generalized autoregressive conditional heteroscedastic (GARCH) models. The outlier's detection and correction method is developed for asymmetric GARCH models. We focus on the estimates of parameters and forecasts of out-of-sample volatility when data is contaminated with outliers. The performance of the proposed method is assessed through Monte Carlo simulations and an application to real data. Our results show that the asymmetric GARCH model fitted to outlier corrected data yields better estimates of both parameters and volatility forecasts than the asymmetric models for original returns.

KEYWORDS

Additive Outliers, Conditional Heteroscedasticity, GJR, Volatility clustering.

Mathematics Subject Classification: 62M10, 62P20.

1. INTRODUCTION

General time series models including autoregressive, moving average and autoregressive moving average (ARMA) models have the assumption of homoscedasticity, i.e., the variance of the error remains constant over time. This assumption does not hold for financial time series data where the variance or volatility is time-varying. Financial time series primarily focuses on the modelling of stock indices, share prices and foreign exchange rates. Some of the well-known characteristics of financial time series, also known as stylised facts, are heavy-tailedness, volatility clustering and leverage effect.

To capture these stylised facts of financial time series, the autoregressive conditional heteroscedastic (ARCH) model was first introduced by Engle (1982). The ARCH model has been extended to generalised ARCH (GARCH) model by Bollerslev (1986).Since then various extensions of the GARCH model have been proposed. Numerous applications of the GARCH model have found that the GARCH (1, 1) model can provide a good fit to the financial data. One of the drawbacks of the GARCH model is that it cannot model the asymmetric feature commonly found in many assets returns, i.e., it responds equally to both positive and negative shocks. To overcome this disadvantage, various asymmetric GARCH models have been proposed. Among these, one of the widely used asymmetric model is the GJR model of Glosten, Jagannathan and Runkle (1993).

Time series models are often contaminated with outliers. The identification of ARMA model is effected in the existence of outliers. Outliers may also introduce bias in the estimated parameters of ARMA models. Besides, outliers can affect the forecasts of these models; especially outliers near the start of the forecast period can have severe consequences. Fox (1972) and Tsay (1988) provided a detail discussion on outliers and their effects. Several studies exist on outlier's detection and correction procedures for ARMA models (see Chang et al., 1988 and Chen and Liu, 1993, among others).

The parameter estimates of GARCH models are also affected in the existence of additive outliers. The method of maximum likelihood in the presence of outliers may cause increase in ARCH and considerable decrease in GARCH estimates. The distribution of the unconditional returns is effected and the conditional normality assumption cannot be justified. The volatility forecasts of GARCH models are also influenced by outliers. Isolated observations, if not taken into account, reduce the forecasting performance which significantly reduces out-of-sample predictive power of GARCH models. Few studies exist in literatures which deal with the effect of outlier on GARCH models (see Franses and Ghijsels, 1999, Charles and Darne, 2005, Charles, 2008 and Carnero et al., 2012). Recently, Granéand Veiga (2014) studied the impact of additive outliers on the estimation of risk measures using GARCH-type models by comparing wavelet-based detection method of Grané and Veiga (2010) with various alternative proposals. Besides, robust methods are also developed for GARCH-type models (see Muler and Yohai, 2008, Mukherjee, 2008 and Iqbal and Mukherjee, 2010).

One of the main objectives of the present study is to investigate effects of additive outliers on GARCH-type models. More specifically, we suggest an additive outlier's detection and correction method for the asymmetric GJR model and aim to observe the effects of this type of outlier on the estimates of parameters and out-of-sample forecasts of volatility. We use Monte Carlo simulations to assess the effectiveness of the proposed technique of outlier correction method. Various evaluation measures are employed to evaluate the parameter estimates and volatility forecasts of both contaminated and corrected series. The method is also applied to estimate and forecast the volatility of Karachi Stock Exchange.

This article is organized as follows: Section 2 introduces additive outlier's detection and correction procedure for the asymmetric GJR model. In Section 3, results of simulations and empirical application is presented and discussed. Finally, Section 4 concludes the paper.

2. ADDITIVE OUTLIERS IN GARCH-TYPE MODELS

Franses and Ghijsels (1999) presented a method for detecting additive outliers in the GARCH model. Charles and Darne (2005) extended this method and considered the effect of innovative outliers in the GARCH model. The volatility forecasting of the symmetric GARCH model in the presence of outliers were studied by Charles (2008).

Consider the returns series $y_t = \log P_t - \log P_{t-1}$, where P_t is the closing price at time t.

The GARCH (1, 1) model is

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (1)$$

The model's parameters are supposed to meet the assumptions $\alpha_0 > 0, \alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$. These assumptions are required for conditional variance σ_t^2 to be positive and second-order stationary, see Bollerslev (1986). It is also assumed that the disturbance term ϵ_t is distributed identically and independently with zero mean and unit variance.

The GARCH (1, 1) model deals with conditional volatility of the returns, and has been widely used for volatility modelling and forecasting. The model can be written as

$$y_t^2 = \alpha_0 + (\alpha_1 + \beta_1) y_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1} \qquad (2)$$

Under the assumption that, $\eta_t = y_t^2 - \sigma_{t.}^2$ Eq. (2), corresponds to an ARMA (1, 1) model for y_t^2 , see Bollerslev (1986).

One of the widely-used asymmetric GARCH model is the GJR (1, 1) model defined as

$$y_t = \sigma_t \epsilon_t \qquad (3)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \gamma_1 I(y_{t-1} < 0) y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (4)$$

The indicator function $I(\cdot)$ takes the value 1 when the condition in the parenthesis is satisfied and 0 otherwise. The parameters of this model are supposed to meet the restrictions $\alpha_0, \alpha_1, \beta_1 > 0$ and $\left(\alpha_1 + \beta_1 + \frac{1}{2}\gamma_1\right) < 1$.

Let $\eta_t = y_t^2 - \sigma_t^2$, then $\sigma_t^2 = y_t^2 - \eta_t$, and Eq.(4) becomes

$$y_t^2 - \eta_t = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 y_{t-1}^2 - \beta_1 \eta_{t-1} + \gamma_1 I(.) y_{t-1}^2$$

$$y_t^2 = \alpha_0 + (\alpha_1 + \beta_1) y_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1} + \gamma_1 I(.) y_{t-1}^2$$

$$y_t^2 = \alpha_0 + (\alpha_1 + \beta_1 + \gamma_1 I(.)) y_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1}$$

Next, we develop additive outlier's detection and correction procedure for the GJR model. The detailed procedure and applications of the method can be found in Raziq (2012). The procedure works as follows:

Step 1

Using the method of maximum likelihood, estimate the parameters in Eq. (4). This will provide the vector of estimated parameters $\hat{\theta} = (\hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1)'$, the estimated volatility $\hat{\sigma}_t^2$ and the estimated residuals $\hat{\epsilon}_t$. Next, build $\hat{\eta}_t = y_t^2 - \hat{\sigma}_{t,}^2$

Step 2

The estimated residuals can be expressed as $\hat{\epsilon}_t = \hat{\pi}(L)y_t$, where L is the lag-operator and

$$\hat{\pi}(L) = \frac{\left[\left(1-(\hat{\alpha}_1+\hat{\beta}_1+\frac{1}{2}\hat{\gamma}_1)\right)L\right]}{(1-\hat{\beta}L)}$$

This, after simplification, reduces to

$$\hat{\pi}_j = \left(\hat{\alpha}_1 + \frac{1}{2}\hat{\gamma}_1\right)\hat{\beta}_1^{j-1} \text{for} j = 1,2, \ldots \qquad (5)$$

For each $t = \tau$, perform a regression $\hat{\eta}_t = \omega x_t + \nu_t$ for estimated residuals $\hat{\eta}_t$. with

$x_t = 0$ for $t < \tau$

$x_t = 1$ for $t = \tau$ and

$x_{t+k} = -\hat{\pi}_k$ for $t > \tau$ and $k = 1,2, \ldots,$

where $\hat{\pi}_k$ is defined as in Eq. (5).

The impact ω of additive outlier at time$t = \tau$ can be estimated as

$$\hat{\omega}_{(\tau)} = \frac{\sum_{t=\tau}^{T}\hat{\eta}_t x_t}{\sum_{t=\tau}^{T} x_t^2} \qquad (6)$$

The $\hat{\tau}$ statistic,$\hat{\tau} = \left[\frac{\hat{\omega}(\tau)}{\hat{\sigma}_\nu}\right] / [\sum_{t=\tau}^{T} x_t^2]^{\frac{1}{2}}$, is calculated for $\hat{\eta}_t$, and then compared with C. Note that $\hat{\sigma}_\nu$ is the estimated standard deviation of the residual process and obtained from the 'omit-one' method (Franses and Ghijsels, 1999).

Step 3

At $t = \tau, \hat{\eta}_t$ is replace by $\hat{\eta}_t^* = \hat{\eta}_t - \hat{\omega}_t I_t(\tau)$, when the largest value of the $\hat{\tau}$ statistic exceeds C, where I_t is an indicator function taking the value 1 when $t = \tau$ and $\hat{\omega}_{(\tau)}$ is the estimated weight as in Eq. (6).

Step 4

Build y_t^{*2} via $y_t^{*2} = \hat{\eta}_t^* + \hat{\sigma}_t^2$ at time $t = \tau$ use $\hat{\eta}_t^*$ and $\hat{\sigma}_t^2$ and obtain the additive outlier (AO)-corrected returns as

$y_t^* = y_t$ for $t \neq \tau$ and

$y_t^* = \text{sign}(y_t).(y_t^{*2})^{\frac{1}{2}}$ for $t = \tau$

The expression above indicates that the sign of y_t^* is equivalent to that of y_t at $t = \tau$. This means that for additive outlier corrected returns there is no change of sign.

Step 5

Go to step 1 with the corrected series y_t^*, and repeat all steps until no $\hat{\tau}$ test statistic value exceeds C, where C is a predetermined critical value. In other words, perform all steps until there are no more additive outliers in the data. As suggested in Charles and Darne (2005) a critical value of C=10 is used in this study. This value was initially proposed by Verhoeven and McAleer (2000).

In the end, we have $\hat{\alpha}_0^*, \hat{\alpha}_1^*, \hat{\beta}_1^*, \hat{\gamma}_1^*$ and y_t^* and $\hat{\sigma}_t^{*2}$ and hence one-step ahead forecasts for $\hat{\sigma}_{t+1}^{*2}$ can be obtained.

3. RESULTS AND DISCUSSION

In this section, the GJR model's performance is assessed in existence of additive outliers through Monte Carlo simulations and an application to real data. The main focus is to assess the effect of additive outliers on estimates of parameters and volatility. In addition, out-of-sample volatility forecasts for the contaminated and corrected returns are also compared. All computations are performed using MATLAB software.

First, we use Monte Carlo simulations to assess the precision and consistency of the proposed additive outlier's detection and correction method. Next, an application to a real data set is also provided.

3.1 Simulation of the GJR (1, 1) Model

The return series from the GJR (1, 1) model is simulated for two sample sizes ($T = 520, 3020$). Last 20 observations are left for the evaluation of volatility forecasts. The errors are generated from normal (standard normal), heavy-tailed (Student's-t with 4 degrees of freedom) and heavy-tailed skewed (skewed-t with 4 degrees of freedom and skewness of 0.20) distributions. In order to diminish the influence of starting observations, initial 2000 observations are discarded. A total of $K = 1000$ Monte Carlo sample are generated in this way.

The GJR (1, 1) model

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \gamma_1 y_{t-1}^2 I(y_{t-1} < 0) + \beta_1 \sigma_{t-1}^2$$

is used and the values of true parameters considered are $\theta_0 = (0.01, 0.1, 0.1, 0.7)'$. This choice for true parameter vector is based on the values of parameters commonly found in application of the GJR model.

Next, the return series is contaminated by plugging three random additive outliers (given T and ω) of different magnitudes ($\omega = 5\sigma, \omega = 9\sigma, \omega = 15\sigma$), where σ is the standard deviation of the returns.

The GJR (1, 1) model for contaminated returns become

$$y_t^* = \sigma_t^* \epsilon_t$$

$$\sigma_t^{*2} = \alpha_0^* + \alpha_1^* y_{t-1}^2 + \gamma_1^* {y^*}_{t-1}^2 I(y_{t-1}^* < 0) + \beta_1^* \sigma_{t-1}^{*2}$$

The quasi maximum likelihood (QML) estimation method is employed to obtain $\hat{\theta}^* = (\hat{\alpha}_0^*, \hat{\alpha}_1^*, \hat{\gamma}^*, \hat{\beta}_1^*)'$, the vector of estimated parameters of the contaminated series model. Next, the proposed method of outlier's detection and correction is applied and the corrected series is estimated to obtain $\hat{\theta}_c = (\hat{\alpha}_{c,0}, \hat{\alpha}_{c,1}, \hat{\gamma}_c, \hat{\beta}_{c,1})'$, the vector of estimated parameters of corrected series and the corrected returns ($\dot{y}_t$).

3.2 Evaluation of Estimated Parameters

The performance of estimated parameters for both contaminated and corrected returns are evaluated using evaluation measures such as mean squared error (MSE), root mean squared error (RMSE) and mean absolute error (MAE).

The MSE is defined as

$$\mathrm{MSE} = \frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}_k - \theta_0)^2$$

where $\hat{\theta}_k$, is vector of estimated parameters and θ_0 is the vector of true parameters. Similarly, RMSE and MAE are defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}_k - \theta_0)^2}$$

$$\mathrm{MAE} = \frac{1}{K}\sum_{k=1}^{K}|\hat{\theta}_k - \theta_0|$$

Table 1 represents the result of the parameter's evaluation when the GJR (1, 1) model is fitted to contaminated and corrected returns for a small sample of size T=500. It can be seen from the results of the parameters evaluation that in the presence of outliers, the estimated parameters of the GJR (1, 1) model are not found close to their true values. It is also observed that as the size of the outliers increases the GJR (1, 1) model fitted to contaminated series produces large biases in estimated parameters. The estimated parameters from the corrected returns under all error distributions are found better than the contaminated returns.

Table 1
Evaluation of the Estimated Parameters of the GJR (1, 1) Model Fitted to Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

T=500		Standard Normal		Student-t (4)		Skewed-t (4,0.2)	
		y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
5σ	MSE	0.0165	**0.0003**	0.0205	**0.0182**	0.0155	**0.0124**
	RMSE	0.1284	**0.0186**	0.1432	**0.1351**	0.1244	**0.1112**
	MAE	0.0817	**0.0093**	0.0840	**0.0691**	0.0745	**0.0598**
9σ	MSE	0.0808	**0.0014**	0.0756	**0.0281**	0.0671	**0.0208**
	RMSE	0.2843	**0.0374**	0.2749	**0.1676**	0.2590	**0.1443**
	MAE	0.1843	**0.0213**	0.1682	**0.0875**	0.1618	**0.0696**
15σ	MSE	0.1451	**0.0009**	0.1314	**0.0140**	0.0951	**0.0028**
	RMSE	0.3809	**0.0297**	0.3625	**0.1185**	0.3038	**0.0532**
	MAE	0.2568	**0.0171**	0.2463	**0.0520**	0.2094	**0.0309**

Values in bold represent the least values of evaluation criteria

Table 2 shows the result of the parameter's evaluation for a large sample of size T=3000. The errors in the estimated parameters have decreased as compared to the results of the previous table where a small sample size was used. The GJR (1, 1) model fitted to the corrected series is found to estimate the parameters correctly than the same model for contaminated returns and hence for consistent and accurate estimates of the parameters of the model, the outlier's effect must be removed.

Table 2
Evaluation of the Estimated Parameters of the GJR (1, 1) Model Fitted to Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

T=3000		Standard Normal		Student-t (4)		Skewed-t (4,0.2)	
		y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
5σ	MSE	0.0005	**0.0001**	0.0005	**0.0033**	0.0006	**0.0026**
	RMSE	0.0231	**0.0076**	0.0214	**0.0577**	0.0237	**0.0507**
	MAE	0.0160	**0.0046**	0.0139	**0.0355**	0.0130	**0.0320**
9σ	MSE	0.0048	**0.0001**	0.0262	**0.0068**	0.0276	**0.0117**
	RMSE	0.0696	**0.0076**	0.1618	**0.0825**	0.1661	**0.1082**
	MAE	0.0446	**0.0047**	0.0991	**0.0473**	0.1020	**0.0555**
15σ	MSE	0.0180	**0.0001**	0.0392	**0.0052**	0.0403	**0.0099**
	RMSE	0.1340	**0.0076**	0.1980	**0.0721**	0.2007	**0.0095**
	MAE	0.0877	**0.0049**	0.1931	**0.0323**	0.1145	**0.0415**

Values in bold represent the least values of evaluation criteria

3.3 Evaluation of Out-of-Sample Volatility Forecasts

Subsequently, the effect of outliers on volatility forecasts is studied. The out-of-sample volatility forecasts of both contaminated and corrected returns are evaluated through measures like $\mathrm{MSE_f}$, $\mathrm{RMSE_f}$ and $\mathrm{MAE_f}$. The estimates of $\mathrm{MSE_f}$, $\mathrm{RMSE_f}$ and $\mathrm{MAE_f}$ are obtained.

$$\mathrm{MSE_f} = \frac{1}{h}\sum_{s=1}^{h}(\hat{\sigma}_{T+s}^2 - y_{T+s}^2)^2$$

where $\hat{\sigma}_{T+s}^2$ are estimated forecast of volatilities from contaminated or corrected returns. As a proxy, the squared returns y_{T+s}^2 are traditionally used. This enable volatility forecasts to be assessed in the similar fashion as the forecasts of return series. Similarly, $\mathrm{RMSE_f}$ and $\mathrm{MAE_f}$ are defined as

$$\mathrm{RMSE_f} = \sqrt{\frac{1}{h}\sum_{s=1}^{h}(\hat{\sigma}_{T+s}^2 - y_{T+s}^2)^2}$$

$$\mathrm{MAE_f} = \frac{1}{h}\sum_{s=1}^{h}|\hat{\sigma}_{T+s}^2 - y_{T+s}^2|$$

The result of out-of-sample volatility forecasts of the GJR (1, 1) model fitted to a small sample (T=500) of contaminated and corrected series is presented in Table 3. It is observed from the findings of the table that additive outliers can affect volatility forecasts and this effect can be decreased by removing the outliers from the data. The errors in volatility forecasts of the corrected returns for all error distributions and for magnitude of small, medium and large outliers are smaller than that of contaminated returns.

Table 3
Evaluation of volatility forecasts of the GJR (1, 1) Model fitted to Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

T=500		Standard Normal		Student-t (4)		Skewed-t (4,0.2)	
		y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
5σ	MSE	0.0097	**0.0094**	0.0910	**0.0876**	0.1067	**0.0917**
	RMSE	0.0985	**0.0969**	0.3017	**0.2959**	0.3266	**0.3082**
	MAE	0.0648	**0.0632**	0.1136	**0.0856**	0.1198	**0.0955**
9σ	MSE	0.0847	**0.0124**	0.3004	**0.2474**	0.3115	**0.2646**
	RMSE	0.2911	**0.1114**	0.5481	**0.4973**	0.5581	**0.5144**
	MAE	0.1933	**0.0699**	0.2245	**0.0961**	0.2302	**0.0987**
15σ	MSE	0.2527	**0.0097**	0.3557	**0.0360**	0.3633	**0.0961**
	RMSE	0.5027	**0.0987**	0.5964	**0.1898**	0.6028	**0.3100**
	MAE	0.4171	**0.0647**	0.4553	**0.0734**	0.4777	**0.0942**

Values in bold represent the least values of evaluation criteria

Table 4 shows the result of volatility forecasts of a large sample (T=3000). Again, GJR model applied to outlier corrected series show better results as errors in out-of-sample forecasts are found smaller. These are true for all error distributions and all magnitudes of outliers. These findings show that the volatility forecasts in the presence of additive outliers may be misleading and for reliable volatility estimates and forecasts the data need to be cleaned from outliers.

Table 4
Evaluation of Volatility Forecasts of the GJR (1, 1) Model Fitted to Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

T=3000		Standard Normal		Student-t (4)		Skewed-t (4,0.2)	
		y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
5σ	MSE	0.0086	**0.0079**	0.0299	**0.0190**	0.0520	**0.0401**
	RMSE	0.0927	**0.0899**	0.1729	**0.1378**	0.2281	**0.2002**
	MAE	0.0587	**0.0451**	0.0715	**0.0521**	0.0795	**0.0693**
9σ	MSE	0.0291	**0.0113**	0.0262	**0.0168**	0.0276	**0.0177**
	RMSE	0.1706	**0.1061**	0.1618	**0.1296**	0.1661	**0.1330**
	MAE	0.0822	**0.0664**	0.0991	**0.0473**	0.1020	**0.0555**
15σ	MSE	0.0419	**0.0097**	0.0388	**0.0152**	0.0396	**0.0169**
	RMSE	0.1050	**0.0985**	0.1970	**0.1233**	0.1990	**0.1300**
	MAE	0.0799	**0.0632**	0.1305	**0.0404**	0.1201	**0.0523**

Values in bold represent the least values of evaluation criteria

We also calculated the percentage of correct and false detection of outliers. The percentage of correct and false detections varies from 91% – 100% to 7% – 35%, respectively in all six scenarios considered in this study. Note that higher percentage of false detection was observed when Student-t and skewed-t were used as these distributions are heavy-tailed and may contain aberrant observations.

3.4 Empirical Application to Karachi Stock Indices

The daily closing prices of Karachi Stock Exchange (KSE) for fourteen years (January 1998 to December 2011), obtained from (http://www.finance.yahoo.com), are used in this empirical application. For indices, at time t the log-returns are defined as

$$y_t = (\log P_t - \log P_{t-1}) \times 100\%, \qquad t = 1,2,\cdots,T$$

where P_t represents the closing price of KSE at time t. The KSE data set consists of total $(T = 3369)$ observations. Preliminary $T - h$ observations are used for the estimation of the model and last $h = 1000$ observations are left for out-of-sample volatility forecast evaluation.

Table 5 below shows the summary of descriptive statistics of KSE log-returns for both estimation and forecast periods. It is observed that the data set demonstrates excess kurtosis and asymmetry. Both estimation and forecast periods of KSE are found slightly negatively skewed. Significantly large values of Jarque-Bera (JB) statistic reject the normality hypothesis in the data. High values of the Ljung-Box $Q^2(10)$ statistic for the squared returns up to lag-10, clearly indicate ARCH effect, i.e., dependence in squared returns.

Table 5
Descriptive Statistics of KSE Daily Log-Returns

	Estimation Period	***Forecast Period***
Sample size	2369	1000
Mean	0.0148	-0.0351
Median	0.0587	0.0140
Minimum	-5.7626	-2.3165
Maximum	5.5184	3.5608
Standard deviation	0.7792	0.6366
Skewness	0.3930	-0.1952
Kurtosis	8.4542	5.6865
JB	2997.4	307.0668
$Q^2(10)$	742.6466	396.8315

JB: Jarque-Bera test statistic. $Q^2(10)$ is the Ljung-Box statistic for squared log-returns

The method of QML is used to fit the GJR (1, 1) model to the estimation period and the estimates of parameters are obtained. Next, the outlier's detection and correction method is applied and the effects of outliers are removed. The GJR (1, 1) model is then fitted to the corrected series and again estimates of parameters are obtained. Results of parameters estimates are not reported here for the sake of brevity.

Descriptive statistics in Table 6 show that the residuals of corrected series have smaller skewness and kurtosis as compared to contaminated series. The Jarque-Bera (JB) statistic is significantly larger in contaminated series indicating that the residuals are far from normality. The value for Ljung-Box Q^2 statistic for the squared residuals of both series shows no ARCH effect. These findings confirm that the residuals obtained from the corrected series have better statistics then the contaminated series.

Table 6
Descriptive Statistics of Residuals Obtained from Fitting the GJR (1, 1) Model to Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

	Standard Normal		**Student-t**		**Skewed-t**	
	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
Skewness	-0.4903	-0.3409	-0.4567	-0.3478	-0.4735	-0.3601
Kurtosis	5.9831	4.4507	6.5734	4.6803	6.4231	4.6085
JB	973.31	253.63	1342.70	326.45	1245.1	306.60
$Q^2(10)$	3.8252	4.6922	5.3174	5.6485	5.1703	5.7628

Finally, we compare the out-of-sample volatility forecasts of KSE for both contaminated and corrected returns using various measures of assessment.

Out-of-sample volatility forecasts results of the GJR (1, 1) model fitted to KSE data is shown in Table 7. It can be clearly seen that the outlier corrected returns produce smaller errors in out-of-sample volatility than the contaminated returns. In other words, a volatility forecast of the KSE has improved when the return series is cleaned from additive outliers. This holds for all error distributions.

Table 7
Out-of-Sample Volatility Evaluation of KSE for Contaminated Returns (y_t^*) and Corrected Returns ($\dot{y}_t$)

	Standard Normal		Student-t		Skewed-t	
	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$	y_t^*	$\dot{y}_t$
MSE_f	0.6482	**0.6391**	0.6744	**0.6425**	0.6720	**0.6534**
$RMSE_f$	0.8051	**0.7994**	0.8212	**0.8016**	0.8198	**0.8083**
MAE_f	0.4488	**0.4390**	0.4519	**0.4402**	0.4690	**0.4541**

Values in bold represent the least values of evaluation criteria

Results of this empirical study confirm the findings of our Monte Carlo study and it is concluded that the outliers have severe effects on the parameter estimates and volatility forecasts of GARCH-type models and to get reliable parameters and volatility estimates, the effect of additive outliers must be removed.

4. CONCLUSION

This paper deals with effects of additive outliers on the parameter estimates and volatility forecasts of the asymmetric GJR model. A method for detecting and correcting additive outliers in the GJR model is developed. Monte Carlo simulations are used to assess the performance of the proposed method under different error distributions and various magnitudes of outliers. The results of simulation study show that outliers produce bias in the estimated parameters and volatility forecasts. When the outlier correction procedure is applied, the parameter estimates and volatility forecasts of corrected series improved. An empirical application to KSE also confirms these findings and it is suggested that in order to produce consistent estimates of parameters and reliable forecasts of volatility, the return series should be cleaned from additive outliers.

REFERENCES

1. Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327.
2. Carnero, M., Peña, D. and Ruiz, E. (2012). Estimating GARCH volatility in the presence of outliers. *Economics Letters*, 114(1), 86-90.
3. Chang, I., Tiao, G.C. and Chen, C. (1988). Estimation of time series parameters in the presence of outliers. *Technometrics*, 30(2), 192-204.

4. Charles, A. and Darne, O. (2005). Outliers and GARCH models in financial data. *Economics Letters*, 86(3), 347-352.
5. Charles, A., (2008). Forecasting volatility with outliers in GARCH models. *Journal of Forecasting*, 27(7), 551-565.
6. Chen, C. and Liu, L.M. (1993). Joint estimation of model parameters and outlier effects in time series. *Journal of American Statistical Society*, 88(421), 284-297.
7. Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4), 987-1007.
8. Fox, A.J. (1972). Outliers in time series. *Journal of the Royal Statistical Society (Series B,)* 34, 350-363.
9. Franses, P.H. and Ghijsels, H. (1999). Additive outliers, GARCH and forecasting volatility. *International Journal of Forecasting*, 15(1), 1-9.
10. Glosten, L.R., Jagannathan, R. and Runkle, D. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48(5), 1779-1801.
11. Grané, A. and Veiga, H. (2010). Wavelet-based detection of outliers in financial time series. *Computational Statistics and Data Analysis*, 54(11), 2580-2593.
12. Grané, A. and Veiga, H. (2014). Outliers, GARCH-type models and risk measures: A comparison of several approaches. *Journal of Empirical Finance*, 26, 26-40.
13. Iqbal, F. and Mukherjee, K. (2010). M-estimators for some GARCH-type models; Computation and application. *Statistics and Computing*, 20(4), 435-445.
14. Mukherjee, K. (2008). M-estimation in GARCH model. *Econometric Theory*, 24(6), 1530-1553.
15. Muler, N. and Yohai, Y.J. (2008). Robust estimates for GARCH models. *Journal of Statistical Planning and Inference*, 138(10), 2918-2940.
16. Raziq, A. (2012). *Effects of Additive Outliers on Asymmetric GARCH Models*, Published M.Phil. Thesis, Department of Statistics, University of Balochistan, Quetta, Pakistan.
17. Tsay, R.S. (1988). Outlier, level shift and variance changes in time series. *Journal of Forecasting*, 7(1), 1-20.
18. Verhoeven, P. and McAleer, M. (2000). *Modelling outliers and extreme observations for ARMA-GARCH processes*. Working Paper, University of Western Australia.