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ON GENERALIZED ALTERNATIVE HYPER-POISSON DISTRIBUTION

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ABSTRACT

In this paper a three parameter generalized alternative hyper-Poisson (GAHP) distribution is defined and discussed. The GAHP distribution contains, as special sub models, several important discrete distributions such as alternative hyper-Poisson (AHP) distribution derived by Kumar and Nair (2012) and Poisson distribution. We develop here the stuttering generalized alternative hyper-Poisson (SGAHP) distribution and discuss some important properties of SGAHP distributions. We also develop the Conway-Maxwell generalized alternative hyper-Poisson (CMGAHP) distribution in the form of H-power series function and find some of its important properties. Finally, a real data application of CMGAHP distribution is also provided.

KEY WORDS

Generalized alternative hyper-Poisson distribution, Stuttering generalized alternative hyper-Poisson distribution, Alternative hyper-Poisson distribution, Conway-Maxwell generalized alternative hyper-Poisson distribution, Poisson distribution.

1. INTRODUCTION

Family of Poisson distributions such as Poisson distribution, hyper-Poisson distribution, alternative hyper-Poisson have several applications in civil engineering, in earth quake engineering, in occurrence of cyclones in a particular time period, in the study of vehicular traffic and in many real life situations such as accidents, geology and agriculture.

Some queuing theory associated with hyper-Poisson distribution arrivals had been worked out by Nisida (1962). Katz (1963) generated family of distributions by including more parameters in the class of discrete distributions. Bardwell and Crow (1964) derived and discussed the hyper-Poisson distribution and found the application in accident and contagion phenomena. Staff (1964) developed the displaced Poisson distribution.

Crow and Bardwell (1965) estimated the parameters of the hyper-Poisson distribution. Kemp (1968) found that many discrete distributions may be write in the form of the hyper-geometric series. Ahmad (1992) estimated the parameters of incomplete bivariate hyper-Poisson distribution. Kemp (2002) developed a q-analogue of the hyper-Poisson distribution. Roohi and Ahmad (2003a) estimated the parameters of the hyper-Poisson distribution using negative moments.

Roohi and Ahmad (2003b) derived expressions for ascending factorial moments and further obtained certain recurrence relations for negative moments and ascending factorial moments of the hyper-Poisson distributions. Anwar (2008) characterized the discrete distributions by using different methods. Kumar and Nair (2012) developed an alternative form of hyper-Poisson distribution.

$$f(x)=\frac{\theta^x}{(\gamma)_x}\,{}_1F_1\left[1+x;\gamma+x;-\theta\right],\ \gamma>0,\theta>0, \tag{1}$$

where $(\gamma)_x=\dfrac{\Gamma(\gamma+x)}{\Gamma(\gamma)}, x=0,1,2...,$ is the ascending factorial.

They (2012) found factorial moments, raw moments, recurrence relation of raw moments for AHP distribution, and also estimated the parameters of AHP distribution by using the three methods such as maximum likelihood (ML) estimation, mixed moments (MM) estimation, and factorial moment (FM) estimation. They discussed the application of AHP distribution on the epileptic seizure counts and corn borers data.

Shoaib et al. (2013) developed a generalized alternative hyper-Poisson distribution. This family of distributions is widely used in the field of accident, contagions phenomena, and biological, birth and death processes, and life testing. The probability generating function (pgf) of generalized alternative hyper-Poisson distribution is

$$G(t)={}_1F_1\left[\alpha;\gamma;\theta(t-1)\right] \tag{2}$$

and corresponding probability mass function (pmf) of generalized alternative hyper-Poisson distribution is

$$f(x)=\frac{(\alpha)_x\theta^x}{x!(\gamma)_x}\,{}_1F_1\left[\alpha+x;\gamma+x;-\theta\right],\ x=0,1,2,... \qquad \alpha,\theta,\gamma>0 \tag{3}$$

When $\alpha=1$, $f(x)$ reduces to AHP distribution developed by Kumar and Nair (2012).

When $\alpha=1$ and $\gamma=1$ then $f(x)$ becomes a Poisson distribution.

When $\gamma=1$ then $f(x)$ an alternative hyper-Poisson distribution (Shoaib et al. 2013)

$$f(x)=\frac{(\alpha)_x\theta^x}{(x!)^2}\,{}_1F_1\left[\alpha+x;1+x;-\theta\right],\ \ \alpha,\theta,\gamma>0$$

In section 2, we develop the stuttering generalized alternative hyper-Poisson (SGAHP) distribution and discuss some of its important properties. In section 3, we generate the Conway-Maxwell generalized alternative hyper-Poisson distribution and find its some properties. We also apply the CMGAHP distribution on a real data set used in Allen (2006).

2. STUTTERING GENERALIZED ALTERNATIVE HYPER-POISSON DISTRIBUTION (SGAHPD)

Stuttering is defined as an order k version of the distribution; it may be helpful for structuring real world situations arising from numerous fields of research such as actuarial science, biological sciences, operations research and physical sciences.

Here, we start by presenting an explanation of the major development that has occurred in the area of stuttering (order k version of the distribution). Galliher et al. (1959) discussed the dynamics of two classes of continuous review inventory systems. Philippou and Muwafi (1982) developed the properties of the order k version of the geometric distribution.

Philippou et al. (1983) discussed the order k version of the Poisson distribution. Aki (1985), Philippou (1988), Moothathu and Kumar (1995) and Kumar (2009) derived the stuttering Poisson distribution. Kumar and Nair (2013) developed the order k version of the of the hyper-Poisson distribution and discussed some of its properties.

Kumar and Nair (2013) derived the order k version of the alternative hyper-Poisson distribution and discussed some of its important properties.

We develop the stuttering generalized alternative hyper-Poisson distribution by Consider a sequence $\{S_n, n \geq 1\}$ of independent and identically distributed discrete random variable, where S_n has a k-point distribution with probability generating function (pgf).

$$\varphi(s) = \sum_{j=1}^{k} q_j s^j \tag{4}$$

where $q_j \geq 0$ for $j = 1, 2, ..., k$ such that $q_k \neq 0$ and $\sum_{j=1}^{k} q_j = 1$. Let X be a non-negative integer valued random variable following generalized alternative hyper-Poisson distribution (GAHPD) with pgf.

$$g(s) = \varphi\left[\alpha;\gamma;\theta(s-1)\right] \tag{5}$$

Set $q_j = \dfrac{\theta_j}{\theta}$ for $j = 1, 2, ..., k$ with $\theta = \sum_{j=1}^{k} \theta_j$. Assume that $\{S_n, n \geq 1\}$ and X are statistically independent. Define $T_0 = 0$. Then the pgf of $T_X = \sum_{n=0}^{X} S_n$ is

$$G(s) = g\left[\varphi(s)\right] \tag{6}$$

$$G(s) = \varphi\left[\alpha;\gamma;\theta\left(\sum_{j=1}^{k} q_j s^j - 1\right)\right] \tag{7}$$

Set $q_j = \frac{\theta_j}{\theta}$ in (7)

$$G(s) = \varphi\left[\alpha;\gamma;\left(\sum_{j=1}^{k}\theta_j s^j - \theta\right)\right]. \tag{8}$$

When $\alpha = 1$ in equation (8), SGAHPD converts into stuttering alternative hyper-Poisson distribution (SAHPD) Kumar and Nair (2013).When $\alpha = 1,\ \gamma = 1$ in equation (8), SGAHPD converts into stuttering Poisson distribution (PD). When $\alpha = 1,\ \gamma = 1,\ \theta_j = \theta$ in equation (8), SGAHPD converts into Poisson distribution (PD) of order *k*.

Probability Mass Function of stuttering generalized alternative hyper-Poisson Distribution.

$$f_{TSX}(\gamma) = \sum_{j_x}\left(\frac{u!}{(\gamma)_u}\varphi\left(\alpha+u;\gamma+u;-\sum_{j=1}^{k}\theta_j\prod_{j=1}^{k}\left(\frac{\theta_j^{\gamma_j}}{\gamma_j!}\right)\right)\right), \tag{9}$$

where $u = \sum_{j=1}^{k} r_j$ and $\sum_{J_r}$ denote the k tuple sum over the set $J_r = \left\{(r_1, r_2, ..., r_k)\Big/\sum_{j=1}^{k} jr_j = r\right\}$.

Now we obtain mean and variance of stuttering generalized alternative hyper-Poisson Distribution by using the p.g.f (8)

$$\mu = \frac{\alpha}{\gamma}\sum_{j=1}^{k} j\theta_j, \qquad \alpha,\theta,\gamma > 0 \tag{10}$$

$$\sigma^2 = \frac{\alpha}{\gamma}\left(\frac{(\alpha+1)}{(\gamma+1)} - \frac{\alpha}{\gamma}\right)\left(\sum_{j=1}^{k} j\theta_j\right)^2 + \frac{\alpha}{\gamma}\sum_{j=1}^{k} j^2\theta_j \tag{11}$$

Result 3.1:

Recurrence Formula for Factorial Moment of SGAHP Distribution

Recurrence Formula for Factorial Moments $\mu_{[n]}(\alpha,\gamma)$ of the SGAHPD

$$\mu_{[n+1]}(\alpha,\gamma) = \frac{\alpha}{\gamma}\sum_{j=1}^{k}\sum_{i=0}^{j-1}\binom{j-1}{i} j\theta_j n^{(i)}\mu_{[n-1]}(\alpha+1,\gamma+1) \tag{12}$$

where $n^{(i)} = n(n-1)(n-2)...(n-i+1)$ for $n^{(0)} = 1$.

Proof:

The factorial moment generating function $F(s)$ of the SGAHPD with p.g.f (8) is the following

$$F(s) \quad = G(1+s)$$

$$F(s) = \varphi\left(\alpha;\gamma;\left(\sum_{j=1}^{k}\theta_j(1+s)^j - \theta\right)\right) \tag{13}$$

Put $\theta = \sum_{j=1}^{k}\theta_j$ in (13)

$$F(s) = \varphi\left(\alpha;\gamma;\left(\sum_{j=1}^{k}\theta_j\left[(1+s)^j - 1\right]\right)\right) \tag{14}$$

$$F(s) = \sum_{n=0}^{\infty}\mu_{[n]}(\alpha,\gamma)\frac{s^n}{n!} \tag{15}$$

On differentiating (15) with respect to s obtain

$$\frac{\alpha}{\gamma}\sum_{j=1}^{k}\theta_j\, j(1+s)^{j-1}\left[\varphi\left(\alpha+1;\gamma+1;\left(\sum_{j=1}^{k}\theta_j\left[(1+s)^j - 1\right]\right)\right)\right]$$

$$= \sum_{n=1}^{\infty}\mu_{[n]}(\alpha,\gamma)\frac{s^{n-1}}{(n-1)!} \tag{16}$$

From (16) replace α by $\alpha+1$ and γ by $\gamma+1$

$$\varphi\left(\alpha+1;\gamma+1;\left(\sum_{j=1}^{k}\theta_j\left[(1+s)^j - 1\right]\right)\right) = \sum_{n=0}^{\infty}\mu_{[n]}(\alpha+1,\gamma+1)\frac{s^n}{n!} \tag{17}$$

$$\frac{\alpha}{\gamma}\sum_{j=1}^{k} j\theta_j\sum_{i=0}^{j-1}\binom{j-1}{i}s^i\sum_{n=0}^{\infty}\mu_{[n]}(\alpha+1,\gamma+1)\frac{s^n}{n!} = \sum_{n=0}^{\infty}\mu_{[n+1]}(\alpha,\gamma)\frac{s^n}{n!} \tag{18}$$

$$\frac{\alpha}{\gamma}\sum_{j=1}^{k} j\theta_j\sum_{i=0}^{j-1}\binom{j-1}{i}\sum_{n=i}^{\infty}\mu_{[n-i]}(\alpha+1,\gamma+1)\frac{n!}{(n-i)!}\frac{s^n}{n!} = \sum_{n=0}^{\infty}\mu_{[n+1]}(\alpha,\gamma)\frac{s^n}{n!} \tag{19}$$

On equating the coefficients of $\frac{s^n}{n!}$ on both sides of (19), we get (12)

$$\frac{\alpha}{\gamma}\sum_{j=1}^{k}\sum_{i=0}^{j-1}\binom{j-1}{i}j\theta_j n^{(i)}\mu_{[n-1]}(\alpha+1,\gamma+1) = \mu_{[n+1]}(\alpha,\gamma).$$

Case:

When $\alpha = 1$ in (12), then it is converted into the recurrence formula for factorial moments of SAHPD developed by Kumar and Nair (2013).

Result 3.2:

Recurrence Formula for Raw Moments of the SGAHP Distribution

Recurrence Formula for raw moments $\mu_{[n]}(\alpha,\gamma)$ of the SGAHPD

$$\mu_{[n+1]}(\alpha,\gamma)=\frac{\alpha}{\gamma}\sum_{m=0}^{n}\sum_{j=1}^{k}\binom{n}{m}j^{m+1}\theta_j\mu_{n-m}(\alpha+1,\gamma+1)\,. \tag{20}$$

Proof:

The characteristic function $H_w(s)$ of the SGAHPD with p.g.f (8) is the following

$$H_w(s)=P\left(e^{is}\right)$$

$$=\varphi\left(\alpha;\gamma;\left(\sum_{j=1}^{k}\theta_j\left[e^{ijs}-1\right]\right)\right) \tag{21}$$

$$=\sum_{n=0}^{\infty}\mu_n(\alpha;\gamma)\frac{(is)^n}{n!} \tag{22}$$

On differentiating (21) with respect to s obtain

$$\frac{\alpha}{\gamma}\sum_{j=1}^{k}\theta_j ije^{ijs}\varphi\left(\alpha+1;\gamma+1;\sum_{j=1}^{k}\theta_j\left(e^{ijs}-1\right)\right)=\sum_{n=1}^{\infty}\mu_n(\alpha;\gamma)\frac{(it)^{n-1}}{(n-1)!} \tag{23}$$

From (22) replace α by $\alpha+1$ and γ by $\gamma+1$

$$\varphi\left(\alpha+1;\gamma+1;\left(\sum_{j=1}^{k}\theta_j\left[e^{ijs}-1\right]\right)\right)=\sum_{n=0}^{\infty}\mu_n(\alpha+1;\gamma+1)\frac{(is)^n}{(n)!} \tag{24}$$

$$\sum_{n=0}^{\infty}\mu_{(n+1)}(\alpha;\gamma)\frac{(is)^n}{(n)!}=\frac{\alpha}{\gamma}\sum_{j=1}^{k}\sum_{m=0}^{n}\sum_{n=0}^{\infty}j^{m+1}\theta_j\mu_{n-m}(\alpha+1,\gamma+1)\binom{n}{m}\frac{(is)^n}{n!} \tag{25}$$

Comparing coefficient of $\frac{(is)^n}{(n)!}$ on both sides of (25) and we get (20)

$$\mu_{[n+1]}(\alpha,\gamma)=\frac{\alpha}{\gamma}\sum_{m=0}^{n}\sum_{j=1}^{k}\binom{n}{m}j^{m+1}\theta_j\mu_{n-m}(\alpha+1,\gamma+1)\,.$$

Case:

When $\alpha=1$ in (20), then it is converted into the recurrence formula for raw moments of SAHPD developed by Kumar and Nair (2013).

3. CONWAY-MAXWELL GENERALIZED ALTERNATIVE HYPER-POISSON (CMGAHP) DISTRIBUTION WHEN $\alpha=1$

Conway and Maxwell (1962) presented generalization of the Poisson distribution by including an additional parameter ν namely Conway-Maxwell Poisson distribution.

Boatwright et al. (2003) applied the Conway-Maxwell Poisson distribution to purchase quantity and timing theory. Shmueli et al. (2005) presented the importance of Conway-Maxwell Poisson distribution for fitting the discrete data.

Ahmad (2007) introduced the CMHP distribution and found some of its properties such as moments and the characterization of Conway-Maxwell hyper-Poisson distribution by using the property of proportionality. Kalsoom (2008) found the properties of Conway-Maxwell Poisson, CMHP distribution and characterized many discrete distributions.

In this section we develop the Conway-Maxwell generalized alternative hyper-Poisson distribution when $\alpha = 1$ by adding an additional parameter v. This type of distribution applied in the field of electric power supply, timing theory, and purchase quantity, map scoring, Bayesian theory. The probability mass function of CMGAHP distribution is

$$P(x) = \frac{\theta^x}{\left[(\gamma)_x\right]^{\nu}} \sum_{r=0}^{\infty} \frac{(1+x)_r (-\theta)^r}{(\gamma+x)_r (r!)^{\nu}}, \quad x = 0,1,2,\dots, \ \theta,\gamma,\nu > 0 \tag{26}$$

or

$$P(x) = \frac{\theta^x}{\left[(\gamma)_x\right]^{\gamma}} \left[1 + \frac{(x+1)(-\theta)^1}{(\gamma+x)(1!)^{\nu-1}(1!)} + \frac{(x+1)(x+2)(-\theta)^2}{(\gamma+x)(\gamma+x+1)(1.2)^{\nu-1}2!} + \dots\right] \tag{27}$$

Ahmad and Saboor (2009) defined the hyper geometric power series function or ${}_rH_s$ -function

$$\begin{aligned} &{}_rH_s\left[(a_1,m_1),(a_2,m_2),\dots,(a_r,m_r);(b_1,n_1),(b_2,n_2),\dots,(b_s,n_s);z\right] \\ &\quad = \sum_{i=o}^{\infty} \frac{\prod_{k=1}^{r}\left[(a_k)_i\right]^{m_k}}{\prod_{j=1}^{s}\left[(b_j)_i\right]^{n_j}} \cdot \frac{z^i}{i!} \end{aligned} \tag{28}$$

Equation (27) is represented by H power series function

$$P(x) = \frac{\theta^x}{\left[(\gamma)_x\right]^{\gamma}} \, {}_1H_2\left[1+x;\gamma+x;(1,v-1);-\theta\right] \tag{29}$$

When $\nu = 1$, equation (29) reduces to AHP distribution. When $\gamma = 1, \nu = 1$, equation (29) becomes Poisson distribution. When $\gamma = 1$, equation (29) provides Conway-Maxwell Poisson distribution Shmueli et al. (2005)

Result 3.1:

Probability Generating Function of CMGAHP Distribution

$$G(t)=\sum_{x=0}^{\infty} f(x)t^{x} \tag{30}$$

or

$$G(t)=\sum_{x=0}^{\infty} \frac{\theta^{x}}{\left[(\gamma)_{x}\right]^{\nu}}\, {}_{1}H_{2}\left[x+1; x+\gamma; (1, \nu-1); -\theta\right] t^{x} \tag{31}$$

Corollary:

When $\nu=1$, equation (31) converting into the probability generating function of the AHP distribution

$$G(t)={}_{1}F_{1}\left[1; \gamma; \theta(t-1)\right]$$

Now we characterize the Conway-Maxwell alternative hyper-Poisson distribution.

Theorem 3.2:

X is a discrete CMAHP random variable if and only if

$$\frac{P(x)}{P(x-1)}=\frac{\theta}{(\gamma+x-1)^{\nu}} \frac{{}_{1}H_{2}\left[x+1; \gamma+x; (1, \nu-1); -\theta\right]}{{}_{1}H_{2}\left[x; \gamma+x-1; (1, \nu-1); -\theta\right]} \tag{32}$$

Proof:

$$P(x)=\frac{\theta^{x}}{\left[(\gamma)_{x}\right]^{\nu}}\, {}_{1}H_{2}\left[1+x; \gamma+x; (1, \nu-1); -\theta\right]$$

Since

$$(\gamma)_{x}=(\gamma+x-1)(\gamma)_{x-1}$$

$$P(x)=\frac{\theta}{(\gamma+x-1)^{\nu}} \frac{{}_{1}H_{2}\left[1+x; \gamma+x; (1, \nu-1); -\theta\right]}{{}_{1}H_{2}\left[x; \gamma+x-1; (1, \nu-1); -\theta\right]} P(x-1) \tag{33}$$

Put $x=1$ in (33)

$$P(1)=\frac{\theta}{(\gamma)^{\nu}} \frac{{}_{1}H_{2}\left[2; \gamma+1; (1, \nu-1); -\theta\right]}{{}_{1}H_{2}\left[1; \gamma; (1, \nu-1); -\theta\right]} P(0) \tag{34}$$

Put $x=2$ in (33)

$$P(2)=\frac{\theta}{(\gamma)^{\nu}(\gamma+1)^{\nu}} \frac{{}_{1}H_{2}\left[3; \gamma+2; (1, \nu-1); -\theta\right]}{{}_{1}H_{2}\left[1; \gamma; (1, \nu-1); -\theta\right]} P(0)$$

$$\vdots$$

$$P(x)=\frac{\theta^{x}}{\left[(\gamma)_{x}\right]^{\nu}} \frac{{}_{1}H_{2}\left[1+x; \gamma+x; (1, \nu-1); -\theta\right]}{{}_{1}H_{2}\left[1; \gamma; (1, \nu-1); -\theta\right]} P(0) \tag{35}$$

Applying summation on both sides of (35)

$$\sum_{x=0}^{\infty} P(x) = \frac{P(0)}{{}_1H_2\left[1;\gamma;(1,\nu-1);-\theta\right]} \sum_{x=0}^{\infty} \frac{\theta^x}{[(\gamma)_x]^\nu} {}_1H_2\left[1+x;\gamma+x;(1,\nu-1);-\theta\right]$$

$$P(0) = {}_1H_2\left[1;\gamma;(1,\nu-1);-\theta\right]$$

Putting the value $P(0)$ in (35) and we get (32).

The data in Table 1 show the number of branches emanating from non terminal concepts and the observed frequency of each number (for example, there were 5 nodes having 4 emanating branches) for students who completed concept maps in Class III (See Allen (2006)). A maximum likelihood technique is used to find the parameter estimate of θ by pre-fixing other parameters γ and ν. A chi-square test shows the empirical and truncated CMGAHP distributions are very close. One would have to accept a 82% chance of being wrong to conclude that the empirical and CMGAHP distributions were different.

Table I

No. Links	Obs. Freq.		CMGAHP Truncated Distribution	Exp. Freq.	$(o-e)^2/e$	p-value
1	24	$\hat{\theta}=1.4537$	0.4435	23.95	0.0000997	
2	16	$\gamma=1$	0.3222	17.41	0.1127	
3	8	$\nu=1$	0.1561	8.44	0.02169	
4	5		0.0772	4.20	0.75001	
5	1					
Total	54		1.000	54.00	0.8845	0.8221

Obs. Freq. = Observed frequency
Exp. Freq. = Expected frequency

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