Pak. J. Statist.
2015 Vol. 31(6), 683-696

EFFICIENT CLASS OF EXPONENTIAL ESTIMATORS FOR POPULATION MEAN IN TWO-STAGE CLUSTER SAMPLING

Riffat Jabeen[1§], Aamir Sanaullah[1] and **Muhammad Hanif[2]**
[1] COMSATS Institute of Information Technology, Lahore, Pakistan
Email: riffat.jabeen79@gmail.com; chaamirsanaullah@yahoo.com
[2] National College of Business Administration and Economics
Lahore, Pakistan. Email: drmianhanif@gmail.com
[§] Corresponding author

ABSTRACT

For the purpose to estimate the population mean, when the available population is structured as clusters, we have proposed the class of efficient estimators in this paper under two stage cluster sampling. In this paper we have also discussed the estimators when the weights of clusters are either known or unknown and also when the clusters are of equal sizes. The expressions mean square error (MSE) and equations have been derived for the proposed class of exponential estimators. Some theoretical conditions have been identified for which the proposed class of estimators is more efficient than Sukhatme et al. (1984) and Mishara (2012). An empirical study has also been carried out in order to demonstrate the performance of the proposed estimators. The proposed estimators are more efficient than usual call of estimators.

1. INTRODUCTION

In planning surveys, it is beneficial to take advantage from some auxiliary information either at stage of estimation or survey planning, in order to estimate a finite population mean with higher degree of precision. For this purpose many researchers suggested several ratio, product, and regression estimators by considering the relationship between the study and auxiliary variables, e.g. Cochran (1940), Robson (1957), Bahl and Tuteja (1991) Upadhyaya et al. (2011) Singh et al. (2013). Noor-ul-Amin and Hanif (2012), Sanaullah et al. (2012) and Sanaullah et al. (2014) provided some exponential-type ratio-cum-ratio and product-cum-product estimators using two auxiliary variables.

In survey sampling, when the available population is in the form of clusters, it is useful to use two-stage sampling design in order to reduce the cost of the survey [Cochran 1977, Kalton 1983, and Sarndal et al. 1992]. Other remarkable work in the field of two stage sampling design are Brewer and Hanif (1970), Seber (1982), Srivastva and Grag (2009) and Nafiu (2012), Saini and Bahl (2012). Jabeen et al. (2014) advised a generalized separate-type estimator in two-stage sampling design using the information of single auxiliary variable under three different cases in two-stage sampling.

Let a population consists of N first stage units (fsu's) and each fsu consists of M_i second stage units (ssu's). Let a sample of n fsu's is selected and a sample of m_i ssu's from each of n fsu's is selected and then assigned weights $\eta_i = \dfrac{M_i}{\bar{M}}$ to i^{th} fsu's. Let $\bar{M}$ be the average number of ssu's belonging to each fsu. Further we assume that the selection of units at each stage has been done using simple random sampling. Let y_{ij} be the value of j-th second-stage unit in the i-th fsu $(j=1,2,...,M_i;\ i=1,2,...,N)$ and $\bar{Y}_{i.}=\frac{1}{M_i}\sum_{j=1}^{M_i}y_{ij}$ and $\bar{y}_i=\frac{1}{m_i}\sum_{j=1}^{m_i}y_{ij},$ are the means belonging to the i^{th} fsu respectively in the population and sample. Also let $\bar{Y}_s^{'}=\frac{1}{N}\sum_{i=1}^{N}\eta_i\bar{y}_i$, and $\bar{y}_s^{'}=\frac{1}{n}\sum_{i=1}^{n}\eta_i\bar{y}_{i.}$ be the means respectively for a population and sample. Let $S_{yb}'^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_{ij}-\bar{Y}_s^{'}\right)^2$ be the population variance between the clusters and $S_{ywi}'^2=\frac{1}{M_i-1}\sum_{i=1}^{n}\left(y_{ij}-\bar{Y}_{i.}\right)^2$ be the population variance within the cluster for study variable.

Sukhatme et al. (1984) discussed two estimators in two-stage sampling as

1) $$t_1=\frac{1}{n}\sum_{i=1}^{n}\eta_i\bar{y}_i\ . \tag{1.1}$$

The mean square error (MSE) may be given as

$$MSE\left(\mathrm{t}_1\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_{yb}^2+\frac{1}{nN}\sum_{i=1}^{N}\left(\frac{1}{m_i}-\frac{1}{M_i}\right)S_{ywi}^2\ . \tag{1.2}$$

2) $$t_2=\frac{\bar{y}_s}{\bar{x}_s}\bar{X}_s \tag{1.3}$$

The *MSE* may be given as

$$MSE(\mathrm{t}_2)\approx\left(\frac{1}{n}-\frac{1}{N}\right)\left(S_{by}{}^2-2RS_{bxy}+R^2S_{bx}{}^2\right)$$
$$+\frac{1}{nN}\sum_{i=1}^{N}\eta_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left(S_{wiy}{}^2-2RS_{wixy}+R^2S_{wix}{}^2\right). \tag{1.4}$$

Mishra (2012) proposed an estimator for population mean in two-stage sampling design.

$$t_3=\frac{\frac{1}{n}\sum_{i=1}^{n}\eta_i y_i}{\frac{1}{n}\sum_{i=1}^{n}\eta_i x_i}\bar{X}\ , \tag{1.5}$$

The MSE of t_3 for two-stage sampling design is,

$$MSE(\mathrm{t}_3)\approx\left(\frac{1}{n}-\frac{1}{N}\right)\left(S_{by}{}^2-2RS_{bxy}+R^2S_{bx}{}^2\right)$$
$$+\frac{1}{nN}\sum_{i=1}^{N}\eta_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\left(S_{wiy}{}^2-2RS_{wixy}+R^2S_{wix}{}^2\right), \tag{1.6}$$

Jabeen et al. (2014) proposed a separate-type weighted generalized estimator for unequal fsu's in two-stage sampling design as,

$$t_4 = \frac{1}{n}\sum_{i=1}^{n}\alpha_i t_i^G , \tag{1.7}$$

where t_4 is proposed weighted generalized estimator in two-stage sampling design, α_i is weighting known constant and t_i^G is proposed ratio type estimator for population mean for ssu's belonging to the i^{th} fsu's as,

$$t_i^G = \bar{y}_i\left[\frac{\left(a_i\bar{X}_i + b_i\right)}{\beta\left(a_i\bar{x}_i + b_i\right) + \left(1-\beta\right)\left(a_i\bar{X}_i + b_i\right)}\right]^g, \quad i = 1, 2, \ldots, n , \tag{1.8}$$

where β and g are assumed to be the unknown constants whose values are to be estimated. $a_i(\neq 0)$, and b_i are assumed to be known as either real numbers or (linear or non-linear) functions of some known parameters of auxiliary variable x such as standard deviation (σ_{xi}), coefficient of variation (C_{xi}), skewness $\left(\beta_{1i}(x)\right)$, kurtosis $\left(\beta_{2i}(x)\right)$ and correlation coefficient (ρ_i) for ssu's belonging to i^{th} fsu's from the population.

The MSE of t_4^G is

$$MSE\left(t_4^G\right) = \frac{f}{N-1}\sum_{i=1}^{N}\left(\alpha_i\bar{Y}_i - \frac{\sum_{i=1}^{N}\alpha_i\bar{Y}_i}{N}\right) + \frac{1}{nN}\sum_{i=1}^{n} f_{m_i}\alpha_i^2\bar{Y}_i^2\left(C_{y_i}^2 + \beta^2 g^2\lambda_i^2 C_x^2 - 2\beta g\lambda_i\rho_i C_{x_i}C_{y_i}\right). \tag{1.9}$$

2. PROPOSED GENERALIZED EXPONENTIAL ESTIMATORS IN TWO STAGE SAMPLING

In this study, some exponential ratio and product type estimators have been generalized under two-stage sampling design. The proposed class of estimators is useful for the estimation of population mean using single auxiliary information in two-stage sampling.

i) The exponential ratio-type estimator may be defined as,

$$t_{\exp}^R = \bar{y}_s \exp\left\{1 - \frac{2\bar{x}_s}{\bar{X}_s + \bar{x}_s}\right\}, \tag{2.1}$$

ii) The exponential product -type estimator may be defined as,

$$t_{\exp}^P = \bar{y}_s \exp\left\{-\left(1 - \frac{2\bar{x}_s}{\bar{X}_s + \bar{x}_s}\right)\right\}, \tag{2.2}$$

The estimators in (2.1) and (2.2) lead to the generalized form as by introducing constants λ and a as:

$$t_{\exp}^{G}=\lambda\bar{y}_s\exp\left\{\alpha\left(1-\frac{a\bar{x}_s}{\bar{X}_s+(a-1)\bar{x}_s}\right)\right\},\qquad 0<\lambda\ \leq 1 \tag{2.3}$$

where $a(\neq 0)$ and $\lambda(\neq 0)$ in (2.3) are suitably chosen constants to be determined such as MSE of $t_{\exp}^{G}$ is minimum and α being constant takes the values (0,-1,1)for designing different ratio-type and product-type estimators. It is to be mentioned that for a different choice of a, λ and α we get different estimators under two-stage sampling design.

It is observed that by choosing suitable values of the constants in (2.3), various exponential-type ratio and product estimators can be derived as a class of estimators of $t_{\exp}^{G}$. Some of these are presented in Table 1 and their respective MSE's are presented in Table 2.

Now in order to derive the bias and MSE of the proposed class of estimator, we define,

$$\bar{y}_s=\bar{Y}_s\left(1+e_0\right),\ \bar{x}_s=\bar{X}_s\left(1+e_1\right), \tag{2.4}$$

where e_0 and e_1 are random error terms. Further we assume that $E\left(e_0\right)=E\left(e_1\right)=0$, and some expectations under two-stage sampling design are obtained in order to obtain the bias and mean square error as,

$$\left.\begin{aligned}
&E\left(e_0\right)=E\left(e_1\right)=0\\
&E\left(e_0{}^2\right)=V_{02},E\left(e_1{}^2\right)=V_{20},E\left(e_2{}^2\right)=V_{02},E\left(e_0e_1\right)=V_{11},\\
&\text{where}\\
&V_{02}=\frac{1}{\bar{Y}_s^2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S^2{}_{yb}+\frac{1}{nN}\sum_{i=1}^{N}\eta_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)S^2{}_{ywi},\right\},\\
&V_{20}=\frac{1}{\bar{X}_s^2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S^2{}_{xb}+\frac{1}{nN}\sum_{i=1}^{N}\eta_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)S^2{}_{xwi}\right\},\\
&\text{and}\quad V_{11}=\frac{1}{\bar{Y}_s\bar{X}_s}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{xyb}S_{yb}S_{xb}+\frac{1}{nN}\sum_{i=1}^{N}\eta_i^2\left(\frac{1}{m_i}-\frac{1}{M_i}\right)\rho_{xywi}S_{ywi}S_{xwi}\right\}.
\end{aligned}\right\} \tag{2.5}$$

We rewrite (2.3) as

$$t_{\exp}^{G}=\lambda\bar{Y}_s\left(1+e_0\right)\exp\left[-\frac{\alpha}{a}e_1\left(1+\frac{(a-1)}{a}e_1\right)^{-1}\right]. \tag{2.6}$$

We assume that $|e_1|<1$, so that we may expand $\left(1+\frac{(a-1)}{a}e_1\right)^{-1}$. We get $t_{\exp}^{G}$ as,

$$t_{\exp}^{G}=\lambda\bar{Y}_s\left(1+e_0\right)\exp\left[-\frac{\alpha}{a}e_1\left(1-\frac{(a-1)}{a}e_1+\frac{(a-1)^2}{a^2}e_1^{\,2}+\ldots\right)\right]. \tag{2.7}$$

It is assumed that the contribution of the terms involving powers in e_0, and e_1 higher than two is negligible. It is therefore expanding the exponentials and ignoring terms in e_0', and e_1' of order higher than one, we have

$$t_{\exp}{}^{G}-\bar{Y}_s\approx\lambda\bar{Y}_s\left[e_0-\frac{\alpha}{a}e_1+\frac{\alpha(a-1)}{a^2}e_1^{\,2}+\frac{1}{2}\frac{\alpha^2}{a^2}e_1^2-\frac{\alpha}{a}e_1e_0\right]+\left(\lambda-1\right)\bar{Y}_s, \tag{2.8}$$

In order to obtain the Bias $\left(t_{\exp}^{G}\right)$, we take expectation of (2.7) and using (2.5) for case-I, the bias of $t_{\exp}^{G}$ is

$$Bias\left(t_{\exp}^{G}\right)\approx\lambda\bar{Y}_s\left[\frac{\alpha(a-1)}{a^2}V_{20}+\frac{1}{2}\frac{\alpha^2}{a^2}V_{20}-\frac{\alpha}{a}V_{11}\right]+\left(\lambda-1\right)\bar{Y}_s, \tag{2.9}$$

In order to derive the MSE$\left(t_{\exp}^{G}\right)$, we take square of (2.6) and retain terms in e_0, and e_1 up to the order one. Then we have,

$$\left(t_{\exp}^{G}-\bar{Y}_s\right)^2\approx\lambda^2\bar{Y}_s^{\;2}\left[e_0^{\;2}+\frac{\alpha^2}{a^2}e^2{}_1-\frac{2\alpha}{a}e_1e_0\right]+\left(\lambda-1\right)^2\bar{Y}_s^{\;2}, \tag{2.10}$$

On taking expectation and using (2.10), the MSE$\left(t_{\exp}^{G}\right)$ is

$$MSE\left(t_{\exp}^{G}\right)\approx\bar{Y}_s^2\left(\lambda^2\left(V_{02}+\left(\frac{\alpha}{a}\right)^2V_{20}-2\left(\frac{\alpha}{a}\right)V_{11}\right)+\left(\lambda-1\right)^2\right), \tag{2.11}$$

In order to find the optimal value of λ and a ,we differentiate (2.11) with respect to λ and a , then equate to zero, we get

$$\lambda=\frac{1}{1+V_{02}+\left(\frac{\alpha}{a}\right)^2V_{20}-2\left(\frac{\alpha}{a}\right)V_{11}}, \tag{2.12}$$

and

$$a^{opt} = \frac{\alpha V_{11}}{V_{20}}, \tag{2.13}$$

By substituting (2.13) in (2.12) we obtain

$$\lambda^{opt} = \frac{1}{1+\left(V_{02} - \frac{V_{11}{}^2}{V_{20}}\right)}, \tag{2.14}$$

Now by substituting (2.13) and (2.14) in (2.9), we get minimum MSE as the MSE given in (2.9) is derived up to the first order in e's, as

$$MSE_{\min}\left(t_{\exp}^{G}\right) = \frac{\bar{Y}_s^{\,2} V_{02}\left(1-\rho^2\right)}{\bar{Y}_s^{\,2} + V_{02}\left(1-\rho^2\right)}, \tag{2.15}$$

We may observe from (2.15) that proposed generalized estimator gives us more precise results under the optimal conditions, as compare to its class of the estimators.

On substituting the optimal value λ^{opt} and a^{opt} in (2.3), we get optimal estimator as:

$$t_{\exp}^{G} \approx \lambda^{opt} \bar{y}_s \exp\left\{\alpha\left(1 - \frac{a^{opt}\bar{x}_s}{\bar{X}_s + \left(a^{opt}-1\right)\bar{x}_s}\right)\right\}, \quad 0 < \lambda \le 1 \tag{2.16}$$

In real life situations, it is not possible for the researcher to presume the value of λ' and a' by employ all the resources e.g. see Murthy (1967), Singh and Karpe (2010), Upadhyaya et al. (2011), Yadav and Kadilar (2013) and Sanaullah et al. (2014), Jabeen et al. (2014), so it is better to replace these by their consistent estimates as,

$$\hat{\lambda}^{opt} = \frac{1}{1+\hat{V}_{02} + \left(\frac{\alpha}{\hat{a}^{opt}}\right)^2 \hat{V}_{20} - 2\left(\frac{\alpha}{\hat{a}'^{opt}}\right)\hat{V}_{11}} \text{ and } \hat{a}^{opt} = \frac{\alpha \hat{V}_{11}}{\hat{V}_{20}} \tag{2.17}$$

So (2.3) may be written as

$$\hat{t}_{\exp}^{G} \approx \hat{\lambda}^{opt} \bar{y}_s \exp\left\{\alpha\left(1 - \frac{\hat{a}^{opt}\bar{x}_s}{\bar{X}_s + (\hat{a}^{opt}-1)\bar{x}_s}\right)\right\}, \qquad 0 < \hat{\lambda}^{opt} \le 1 \tag{2.18}$$

Also the minimum MSE may be written as:

$$MSE_{\min}\left(\hat{t}_{\exp}^{G}\right) \approx \frac{\bar{Y}^2 \hat{V}_{02}\left(1-\hat{\rho}^2\right)}{\bar{Y}^2 + \bar{Y}^2 \hat{V}_{02}\left(1-\hat{\rho}^2\right)}. \tag{2.19}$$

COROLLARY 1:

In previous sections, we considered $\bar{Y}_s = \frac{1}{N}\sum_{i=1}^{N}\eta_i \bar{y}_{i.}$ and $\bar{y}_s = \frac{1}{n}\sum_{i=1}^{n}\eta_i \bar{y}_{i.}$ as population and sample means respectively. At the moment, if we ignore the weighting constant η_i, so that $\bar{Y}_s = \frac{1}{N}\sum_{i=1}^{N}\bar{y}_{i.}$ and $\bar{y}_s = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_{i.}$ are population mean and sample mean respectively. Similarly we may have $\bar{X}_s = \frac{1}{N}\sum_{i=1}^{N}\bar{x}_{i.}$ and $\bar{x}_s = \frac{1}{n}\sum_{i=1}^{n}\bar{x}_{i.}$ will be population mean and sample mean for auxiliary information respectively.

It is to be illustrious that there will be no change in the construction of the estimator given in (2.3) and similarly the expression for the bias and MSE may be obtained directly by ignoring into (2.9) and (2.11) respectively.

COROLLARY 2:

In some situations of practical importance when we have first stage units of equal sizes i.e. $M_1 = M_2 = M_3 = ... = M_N = M = \bar{M}$, we put $\eta_i = \frac{M_i}{\bar{M}} = 1$, $m_i = m$, $M_i = M$, and $\bar{y}_i = \sum_{j=1}^{m}\frac{y_{ij}}{m}$ in (2.9), (2.11) and get the expression for the bias and MSE. It is also to be mentioned that there will be no change in the construction of the estimator.

3. EFFICIENCY COMPARISON

To compare the efficiency among the generalized exponential estimator $t_{\exp}^{G}$ with the other estimators of the class of proposed generalized estimator, we use the following:

$$A = V_{02} + \left(\frac{\alpha}{a}\right)^2 V_{20} - 2\left(\frac{\alpha}{a}\right)V_{11}\ C = V_{02} + V_{20} - 2V_{11},\quad H = \frac{V_{02}\left(1-\rho^2\right)}{\bar{Y}_s^2 + V_{02}\left(1-\rho^2\right)}$$

The efficiency conditions may be written as:

i) Firstly, we may compare our proposed generalized estimator with Sukhatme et al. (1984) estimator for population mean.

$$MSE\left(t_{\exp}^{G}\right) - MSE\left(t_{sukh}\right) \le 0$$

If

$$\min\left(\frac{1 \pm \sqrt{\left(1-\left(1+A\right)\left(1-V_{02}\right)\right)}}{\left(1+A'\right)}\right) \le \lambda \le \max\left(\frac{1 \pm \sqrt{\left(1-\left(1+A\right)\left(1-V_{02}\right)\right)}}{\left(1+A\right)}\right), \tag{3.1}$$

ii) Secondly, we may compare our proposed generalized estimator with Mishra (2012).

$$MSE\left(t_{\exp}^{G}\right)-MSE\left(t_{Mish}^{Rat}\right)\le 0 \text{ If}$$

$$\min\left(\frac{1\pm\sqrt{\left(1-(1+A)\left(\frac{1}{(1+C)}\right)\right)}}{(1+A)}\right)\le\lambda\le\max\left(\frac{1\pm\sqrt{\left(1-(1+A)\left(\frac{1}{(1+C)}\right)\right)}}{(1+A)}\right), \tag{3.2}$$

iii) We also compare our optimal proposed generalized estimator with generalized estimator;

$$\min MSE\left(t_{\exp}^{G}\right)-MSE\left(t_{\exp}^{G}\right)\le 0 \text{ If}$$

$$\min\left(\frac{1\pm\sqrt{\left(1-(1+A)(1-H)\right)}}{(1+A)}\right)\le\lambda\le\max\left(\frac{1\pm\sqrt{\left(1-(1+A)(1-H)\right)}}{(1+A)}\right). \tag{3.3}$$

4. EMPIRICAL RESULTS AND DISCUSSION

We have proposed a generalized exponential estimator under two-stage sampling using single auxiliary information. The proposed generalized estimator may be modified for generalized ratio exponential estimator and generalized product exponential estimator. We have made the comparison of our proposed class of generalized ratio exponential estimator with existing estimators i.e. Sukhatme et al. (1984) and Mishra (2012).

In order to demonstrate the performance of generalized ratio exponential estimator, we have used two real populations to compare the proposed generalized exponential estimator with Sukhatme et al. (1984) and Mishra (2012) under two-stage sampling. The populations are:

Population I: (Source: Srivastva and Garg (2009) pg#116)

The population I consists of four clusters of unequal size and the relationship between study variable and auxiliary information is positive. The population I has been used to show the performance for Case I

Population II: (Source: Srivastva and Garg (2009) pg#116)

The population II have four clusters of equal sizes. The study variable and auxiliary information are positively correlated with each other. The population II has been used to show the performance for Case II

Further descriptive of the population is given in Table 3.

To illustrate the performance of the proposed generalized exponential estimator, we have calculated Percentage relative efficiency (PRE) for each estimator to compare it with the estimators mentioned above. The MSE and PRE values for each of the estimators are given in Table 4 and Table 5 respectively. The PRE is calculated as

$$PRE\left(t_{\exp}^{j}\right)=\frac{MSE\left(\bar{y}_s\right)}{MSE\left(t_{\exp}^{j}\right)}\times 100\,.$$

The discussion about the performance of the estimators is illustrated is given for both above mentioned cases.

a) Case-I (Unequal FSU's)
   From Table 3, we may observe that proposed generalized estimator $t_{\exp}^{opt}$ has minimum MSE value as compare to Mishra et al. (2012) and Sukhatme et al. (1984). The PRE for the proposed generalized estimator $t_{\exp}^{opt}$ is maximum among the proposed class of estimators. We have also observed the performance of $t_{\exp}^{5}$ is good among the class of estimators.

b) Case-II (Equal FSU's)
   We have used Population –I that is based on equal FSU's in order to show the efficiency of our proposed generalized estimator over Sukhatme et al. (1984) and Mishra (2012). The proposed generalized estimator provides maximum PRE among the class of proposed generalized estimator. Also we see that MSE of the proposed estimator is least as compare to Sukhatme et al. (1984) and Mishra (2012). Among the class of proposed generalized estimator, we see that $t_{\exp}^{3}$ is equally efficient as Mishra (2012) i.e. t_3 .

6. CONCLUSION

From the empirical comparison, it may be concluded that the performance of the proposed class of estimators is more efficient as compared to its class of estimator in two-stage sampling and usual unbiased estimator and Mishra (2012). As a conclusion the proposed class of estimators is acceptable for the real life application in two-stage sampling design.

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APPENDIX

Table 1
Some special cases of the generalized estimator $t_{\exp}^{G}$

Ratio-type estimator $\alpha=1$	**Product-type estimators** $\alpha=-1$	a	λ
$t_{\exp}^{1}=\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s+\bar{x}_s}\right)$	$t_{\exp}^{2}=\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{x}_s+\bar{X}_s}\right)$	2	1
$t_{\exp}^{3}=\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s}\right)$	$t_{\exp}^{4}=\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{X}_s}\right)$	1	1
$t_{\exp}^{5}=\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s+(a-1)\bar{x}_s}\right)$	$t_{\exp}^{6}=\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{x}_s+(a-1)\bar{X}_s}\right)$	a	1
$t_{\exp}^{7}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s+\bar{x}_s}\right)$	$t_{\exp}^{8}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{x}_s+\bar{X}_s}\right)$	2	λ
$t_{\exp}^{9}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s}\right)$	$t_{\exp}^{10}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{X}_s}\right)$	1	λ
$t_{\exp}^{11}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{X}_s-\bar{x}_s}{\bar{X}_s+(a-1)\bar{x}_s}\right)$	$t_{\exp}^{12}=\lambda\bar{y}_s\exp\left(\dfrac{\bar{x}_s-\bar{X}_s}{\bar{x}_s+(a-1)\bar{X}_s}\right)$	a	λ

Table 2

MSE's of some special cases of the generalized estimator $t'^{G}_{\exp}$

Ratio-type estimator $\alpha=1$	**Product-type estimators** $\alpha=-1$	a	λ
$MSE\left(t^{1}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+\left(\frac{1}{4}\right)V_{20}-V_{11}\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+\left(\frac{1}{4}\right)V_{20}+V_{11}\right)$	2	1
$MSE\left(t^{3}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+V_{20}-2V_{11}\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+V_{20}+2V_{11}\right)$	1	1
$MSE\left(t^{5}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+\left(\frac{1}{a}\right)^2V_{20}-2\left(\frac{1}{a}\right)V_{11}\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(V_{02}+\left(\frac{1}{a}\right)^2V_{20}+2\left(\frac{1}{a}\right)V_{11}\right)$	a	1
$MSE\left(t^{7}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+V_{20}-V_{11}\right)+\left(\lambda-1\right)^2\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+V_{20}+V_{11}\right)+\left(\lambda-1\right)^2\right)$	1	λ
$MSE\left(t^{9}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+\left(\frac{1}{4}\right)V_{20}-V_{11}\right)+\left(\lambda-1\right)^2\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+\left(\frac{1}{4}\right)V_{20}+V_{11}\right)+\left(\lambda-1\right)^2\right)$	2	λ
$MSE\left(t^{11}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+\left(\frac{1}{a^2}\right)V_{20}-\frac{1}{a}V_{11}\right)+\left(\lambda-1\right)^2\right)$	$MSE\left(t^{G}_{\exp}\right)\approx \bar{Y}_s^2\left(\lambda^2\left(V_{02}+\left(\frac{1}{a^2}\right)V_{20}+\frac{1}{a}V_{11}\right)+\left(\lambda-1\right)^2\right)$	a	λ

Table 3
Data Statistics for Population-I and Population-II

Cluster	Population-I(equal fsu's)				Population-II(unequal fsu's)			
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**
M_i	16	16	16	16	18	14	12	20
m_i	8	8	8	8	9	7	6	10
$\bar{Y}_i$	26.20625	24.12313	26.68875	22.11438	25.77722	22.79286	28.43500	23.0905
$\bar{X}_i$	50.96019	50.35994	62.70413	55.75731	51.06389	46.49700	67.00217	57.11855
$C_{y_i}^2$	0.62364	0.33905	0.32637	0.36886	0.58025	0.39297	0.34783	0.31545
$C_{x_i}^2$	0.47888	0.28038	0.38836	0.49081	0.43322	0.29984	0.41947	0.40689
ρ_{i1}	0.88451	0.85254	0.84212	0.80242	0.88373	0.83895	0.82425	0.82113

Table 4
***MSEs* of the class of $t_{\exp}^{G}$ for population I and Population II for equal and unequal fsu's**

MSE using Population-I (equal fsu's) (CASE-II)		MSE using Population-II (unequal fsu's) (CASE-I)
t_{sukh}	9.214122	9.817826
t_3	4.72486	4.399081
$t_{\exp}^{1}$	4.578562	4.152578
$t_{\exp}^{3}$	4.72486	4.399081
$t_{\exp}^{5}$	4.051741	3.531721
$t_{\exp}^{7}$	4.653287	4.125222
$t_{\exp}^{9}$	4.688791	4.368393
$t_{\exp}^{11}$	4.025188	3.511914
$t_{\exp}^{opt}$	**4.025188**	**3.511914**

Table 5

***PREs* of the class of $t_{\exp}^{G}$ for population I and Population II for equal and unequal first stage units**

PRE using Population-II (equal fsu's) Case-II		PRE using Population-II (unequal fsu's) (CASE-I)
t_{sukh}	100	100
t_3	209.4556	207.7908
$t_{\exp}^{1}$	221.8891	214.4303
$t_{\exp}^{3}$	209.4556	207.7908
$t_{\exp}^{5}$	260.896	246.3287
$t_{\exp}^{7}$	223.3606	210.9869
$t_{\exp}^{9}$	210.927	209.3893
$t_{\exp}^{11}$	262.3675	243.9098
$t_{\exp}^{opt}$	262.3675	243.9098