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APPROXIMATE EQUIVALENCE BASED ON SYMBOLIC COMPUTATION AND NUMERICAL CALCULATION FOR LINEAR ALGEBRA TRANSITION SYSTEMS

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ABSTRACT

In order to optimize linear algebra transition systems, a kind of hybrid system, approximate equivalence semantics for linear algebra transition systems is proposed. The semantics is called approximation ready trace semantics. The work is based on the ready trace semantics for concrete process algebra and numerical approximate calculation theory in algebra. The advantage of the work is that it makes the traditional formal verification theory can be applied to verification of the linear algebraic transition system. Then the approximate ready-trace semantics axiom system, which can be used to reasoning of linear algebraic transition system, is put forward. Finally, the advantage of approximate ready trace semantics is verified through a traffic lights control vehicle flow system example.

KEY WORDS

Axiom system, Numerical calculation, Approximation ready traces equivalence, Linear algebra transition systems, Ready trace semantics.

1. INTRODUCTION

On the basis of algebra predecessors, R. van Glabbeek has carried on the induction summary to various equivalence, 14 kinds of semantic equivalence [13] were presented. J.C.M.B AETEN [7] put forward ready-trace semantics for Concrete Process Algebra (CPA) [14], and pointed out that in the ready and failure semantic priority operator which was introduced to axiom system was incomplete. On this basis, the ready trace axiom systems with priority operator semantics were given. Pnueli [9] called ready trace semantic barbed semantics. Traditional formal models use abstract actions, which cannot meet the demand of data information description of the algebraic transition systems as following system in example 1.

Example 1: A program: multiply two numbers
"*int j,s where (s = 0 ∧ j =s0)*
s0 : while (j>0) do
(s, j) := (s+i0, j−1)
invariant s=i0j0"

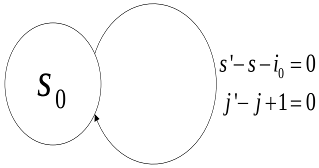


Fig. 1: Multiply Two Numbers

Sriram Sankaranareyanan defined algebra transition system [6]. Hao Yang proposed a method to approximate compute linear algebra transition system and did work for complete trace equivalence [2]. Deng Hui made approximate bisimulation semantics [12]. Studying for algebra transition system, researchers generally concentrated in the invariant generation algorithm [6, 11]. On the approximation to matrix [1, 3, 4, 5], the work was done. In the section 2, some theories of algebra are introduced, which are used in the technique, and a computational model of linear algebra transition systems was present. In the section 3, approximate ready trace semantics for linear algebra transition system based on the ready-trace equivalence for the concrete process algebra (CPA) is defined. And the completeness of the axiom system is provided and proven. Finally, the advantage of approximate ready trace equivalence for linear algebra transition systems that it can optimize linear algebra transition systems are verified through the traffic lights control vehicle flow system example.

2. LINEAR ALGEBRA TRANSITION SYSTEM AND APPROXIMATE CALCULATION OF MATRIX

In this section, some theories of algebra were introduced, including norm of matrix and the series concepts on linear algebra transition systems, which were used in the technique of the paper. And then a computational model of linear algebra transition systems was present.

Norm of Matrix

Definition 1:

$\|\cdot\|$ is a norm function of matrix $A \in R^{m\times n}$, if $\|A\|$ satisfies following:

(1) $\|A\| \geq 0$ ($\|A\| = 0$ if and only if $A=0$) (positive definiteness) ;
(2) $\|\alpha A\| = |\alpha| \, \|A\|$, $\forall \alpha \in R$; (Linearity);
(3) $\|A+B\| \leq \|A\| + \|B\|$, (Triangle Inequality);
(4) $\|AB\| \leq \|A\| \, \|B\|$, (Consistency).

In the paper, Frobenius Norm would be used. For approximate calculation of matrix, two methods were introduced. In the paper the Jordan approximation was used for approximate calculation.

Definition 2:

Let $A \in R^{m\times n}$, and the error $\varepsilon > 0$, $\varepsilon \in R$, if $B \in R^{m\times n}$, satisfy, $\| A-B \|_F \le \varepsilon$, $A, B \in R^{n\times n}$ is called as approximate about ε under Frobenius Norm.

Definition 3:

Let $A \in R^{m\times n}$, $A = PJ_A P^{-1}$, J_A is the Jordan norm, and the error $\varepsilon > 0$, $\varepsilon \in R$, if $B \in R^{m\times n}$, $B = PJ_A{}' P^{-1}$, where $J_A{}'$ is approximate matrix of J_A, satisfy $\| A-B \|_F \le \varepsilon$, $A,\ B \in R^{n\times n}$ is called as Jordan approximate about ε under Frobenius Norm.

In the linear algebra migration process, the two polynomials transition relation involves multiplying of the matrix and will be involved in the transition matrix power. So investigating multiplying of two matrices and the power of the matrix approximation error analysis is necessary.

Lemma 1:

Let $A, B \in R^{n\times n}$, $\| A^k - B^k \|_F \le \sum_{i=1}^{k} C_k^i \| (A-B) \|_F{}^i \| B \|_F{}^{k-i}$, what is more, let $\| A-B \| = \beta \| B \|$. Then $\| A^k - B^k \|_F \le \| B \|_F{}^k \left[(1+\beta)^k - 1 \right]$.

Proof:

$$\| \left(A^k - B^k \right) \|_F = \| (A-B+B)^k - B^k \|_F$$

Case 1: k=1

$$\| A-B \|_F = \| B \|_F \, [(1+\beta)-1] \le \| B \|_F \, [(1+\beta)-1]$$

Case 2: k =n,

Let $\| \left(A^n - B^n \right) \|_F \le \| B \|_F{}^n \, [(1+\beta)^n - 1]$ hold

Case 3: k=n+1,

$$\| \left(A^{n+1} - B^{n+1} \right) \|_F = \| \left(A^n A - B^n B \right) \|_F$$

$$\| \left(A^n (A-B+B) - B^n B \right) \|_F = \| A^n (A-B) + \left(A^n - B^n \right) B \|_F$$

$$\le \beta \| B \|_F \| A-B+B \|_F{}^n + \| B \|_F{}^{n+1} \left[(1+\beta)^n - 1 \right]$$

$$\le \beta \| B \|_F \left(\| A-B \|_F + \| B \|_F \right)^n + \| B \|_F{}^{n+1} \, [(1+\beta)^n - 1]$$

$$= \| B \|_F{}^{n+1} \left[\beta(\beta+1)^n + (1+\beta)^n - 1 \right]$$

$$= \| B \|_F{}^{n+1} \left[(\beta+1)^{n+1} - 1 \right]$$

So for each k, $\| A^k - B^k \|_F \le \| B \|_F{}^k \, [(1+\beta)^k - 1]$ hold.

Linear Algebra Transition System

Definition 4:

LATS (linear algebra transition system) is a 6-tuple $S = (V, Q, q_0, X_0, \Gamma)$, where:

(1) V is a finite set of variables over real field;
(2) Q is a finite set of locations or states;
(3) q_0 is the initial location or initial state;
(4) X_0 denotes the initial values of variables over V in q_0;
(5) Γ is the set of transitions. Each τ of Γ is a 3-tuple (q, q', f), where $q, q' \in Q$ are the pre-location and post-location, f is a polynomial transition relation over $X \cup X'$, where X, X' are the sets of variables in q, q'. f can be denoted as following :

$$f: \begin{cases} x_1 := f_1(x_1, x_2 \cdots, x_n) \\ \cdots \\ x_n := f_n(x_1, x_2 \cdots, x_n) \end{cases}$$

Let $X = (x_1, x_2 \cdots, x_n)$ denote the set of variables in q, and $X' = (x_1, x_2 \cdots, x_n)$ denote the set of variables in q'. Then f can be denoted as following:

$X' = F(X)$, if $F(X)$ is linear polynomial $S = (V, Q, q_0, X_0, \Gamma)$ is called linear algebraic transition system. Then f can be denoted as following:

$$f: \begin{cases} x_1 := a_{11}x_1 + a_{22}x_2 \cdots a_{nn}x_n + b_1 \\ \cdots \\ x_n := a_{n1}x_1 + a_{n2}x_2 \cdots + a_{nn}x_n + b_n \end{cases}$$

i.e. $f : X = AX'$,

where $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$, $\vec{b} = (b_1, b_2, \cdots b_n)^T$.

Moreover, if $\vec{b} = \vec{0}$, S is called linear homogeneous algebra transition system.

IP denotes the set of all linear polynomial transition relations.

Example 2:

Let $S = (V, Q, q_0, X_0, \Gamma)$ be a LATS, as Figure 2, where $Q = \{s_0, s_1, s_2, s_3\}$, $V = \{x_1, x_2\}$ initial states s_0, initial values of variables $X_0 = (1.000, 1.000)^T$, the set of transitions

$$\Gamma = \{(s_0, f_1, s_1), (s_0, f_2, s_2), (s_2, f_3, s_3)\}$$

where

$$f_1 : x_1 := 1.000x_1 + 0.999x_2, x_2 := 2.000x_1 + 3.001x_2;$$
$$f_2 : x_1 := 3.000x_1 + 1, x_2 := 2.000x_1 + 3.001x_2;$$
$$f_3 : x_1 := 1.001x_1 + x_2, x_2 := 2.001x_1 + 3.000x_2;$$

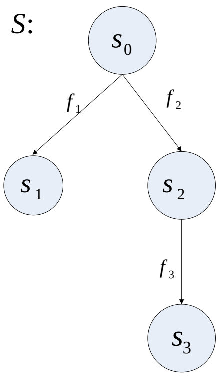


Fig. 2

Lemma 2:

An arbitrary $\vec{b} = (b_1, b_2, \cdots b_n)^T \neq \vec{0}$ linear algebra of transition system can be turned into homogeneous algebraic linear transfer system, so, for linear algebra system change, we only study the transition of homogeneous algebra system.

Proof:

Let $S = (V, Q, q_0, X_0, \Gamma)$ be a linear algebra transition system, we just need proof for each linear algebra transition $f : \overrightarrow{X} = A\overrightarrow{X'} + \vec{b}$ relations can be exchanged for another a homogeneous transition $f' : \overrightarrow{Y} := B\overrightarrow{Y'}$. Assume $\overrightarrow{b_0}$ is one solution of the equation $(A - E)\vec{x} = \vec{b}$, and let $Y = X + \overrightarrow{b_0}$, then $f \in \Gamma : X = AX' + \vec{b}$ can be replaced by $f \in \Gamma : Y = AY'$ where $Y' = X' + \overrightarrow{b_0}$.

From Lemma 2, it is known that only homogeneous linear algebra transition systems for linear algebra transition systems need to be studied.

3. APPROXIMATE READY TRACE EQUIVALENCE FOR LINEAR ALGEBRAIC TRANSITION SYSTEM

Next, the ART equivalence of transition in algebraic system will be introduced. All processes are finite here. Let H be process graph domain marked by polynomial transition set F. in the domain H, the operation symbol sym $\oplus$, ., $\|$, $\lfloor\!\lfloor$, $|$, ∂_H, π_n, θ [7] were defined and the polynomial expression operator $\oplus$, (.). $\|$ is the parallel composition operator, x$\|$y will interleave the transitions of x and y, except for the communication transitions, called merge. $\lfloor\!\lfloor$ is left parallel composition operator, called left merge. x $\lfloor\!\lfloor$ y is sub graph of x||y, but with the restriction that communication start from x. x|y is communication between the first step of the x and the first step of the y, called communication merge. $\oplus$ is alternative operator, x $\oplus$ y denote the process that chose x or y, and then executes the chosen process. .denoting sequential composition, if x and y is a two processes, x.y said it first to the implementation of the x and then to the implementation of y. . is higher order than $\oplus$. So x.y $\oplus$ z denotes (x.y) $\oplus$ z. δ denotes deadlock, which means that the process of an algebraic cannot continue to carried out, nor can it be selected. ∂_H is encapsulation operator. Here H is a set of algebra transitions or actions, and ∂_H blocks those transitions, renames them into δ . nth projection operator ($n \geq 1$) π_n which means that the step-by-step of the executive to nth step cuts off all branches after n steps. + + indicates the addition of the relationship between of the polynomial domain, (.) means that the multiplication of the relationship between of the polynomial domain.

Example 3:

Let $\tau_1 : (s_1, f_1, s_3)$, $\tau_2 : (s_1, f_2, s_2)$ be two linear algebra transitions, as Figure 3, Figure 4.

Figure 5, Figure 6, Figure 7 and Figure 8 present $f_1 \oplus f_2$, $f_1 . f_2$, $f_1 + f_2$ and $f_1 . f_2$.

Example 4:

Let p, q (as Figure 9, Figure 10) be two linear algebra transition systems, $f_i \in \mathrm{IP}, i = 1,2,3,4$, IP is the set of all one order linear polynomial row $p \| q$. And δ -normal form of $p \| q$ is present in Figure 11 and Figure 12. $p \lfloor\!\lfloor q$ is the graph in Figure 13, $p|q$ is the graph in Figure 14, $\pi_2(p \lfloor\!\lfloor q)$ is the graph in Figure 15, and $\partial_{\{f_1, f_3\}}(p)$ is the graph in Figure 16.

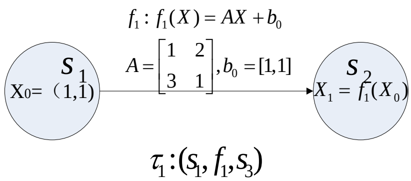


Fig. 3

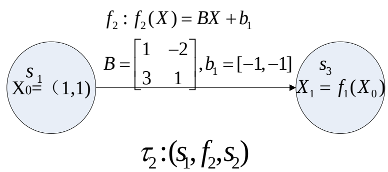


Fig. 4

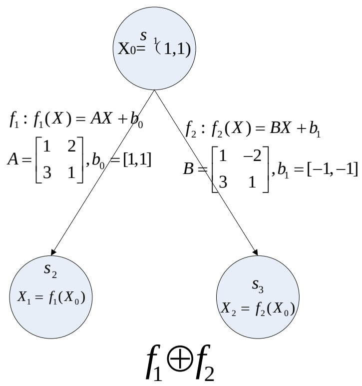


Fig. 5

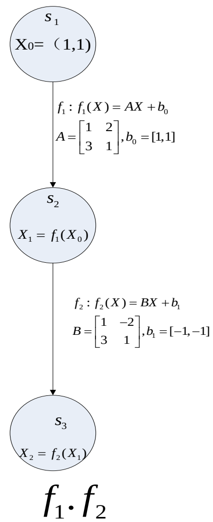


Fig. 6

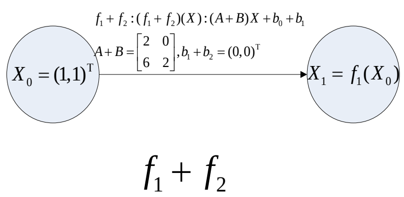


Fig. 7

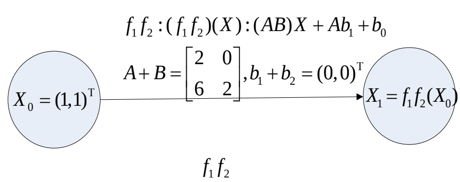


Fig. 8

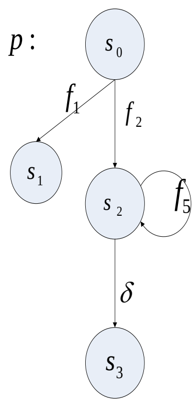


Fig. 9

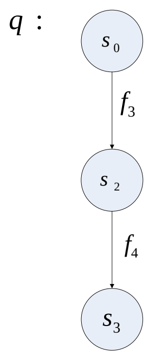


Fig. 10

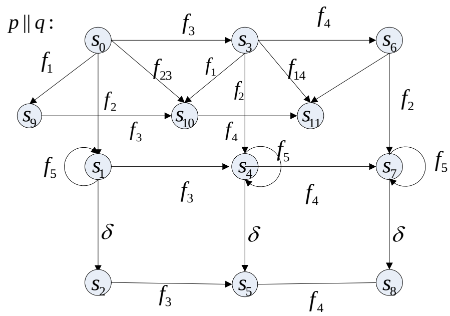


Fig. 11

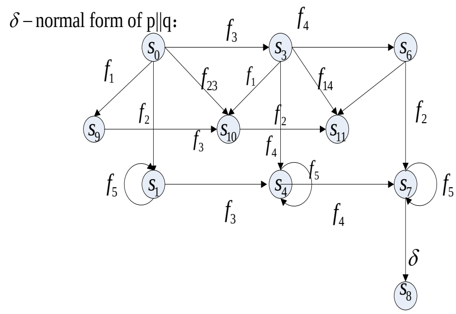


Fig. 12

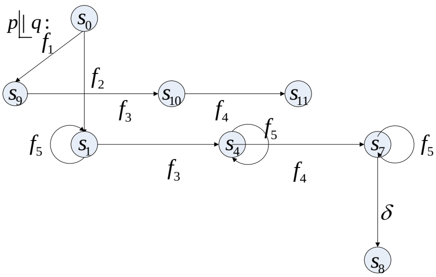


Fig. 13

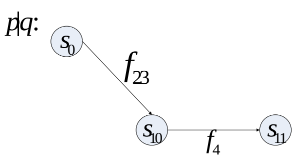


Fig. 14

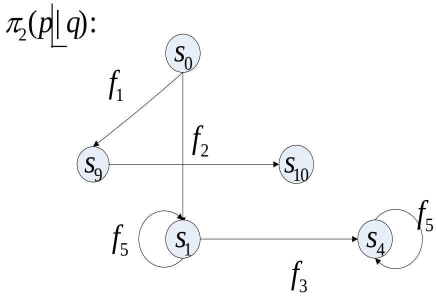


Fig. 15

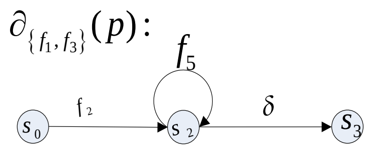


Fig. 16

Let p be an algebra transition process or sub graph of a algebra process graph. I(p) denote the set of all first algebra transitions. Such as in Figure 9, $I(p) = \{f_1, f_2\}$. If p is only a deadlock δ operator, then $I(p) = \varnothing$. If p is $\varnothing$, $I(p) = \varnothing$ denotes the process is successfully terminated.

Definition 6(1):

Let $p \in IP$. A path π of p is defined as an alternating sequence of states of p and labeled transition edges in g, starting from the root s_0 of p. As following:

$$\pi = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} \ldots \xrightarrow{f_{n-1}} s_n$$

where $f_i \in IP, i = 1,\ 2,\ \dots,\ n-1$, if n=0, we called it empty path. If the path can't be prolonged or ending in the initial node of a δ -step, we call it maximal path.

Definition 6(2):

Let $p \in H_{IP}$, $\pi = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} \dots \xrightarrow{f_{n-1}} s_n$, $n \geq 0$ is a path of $p \in H_{IP}$, a ready trace $rt(\pi)$ of π is the alternating sequence of $I(s_i)$ and f_i :

$$I(s_0), f_0, I(s_1), f_1, \dots, f_{n-1}, I(s_n)$$

and we can use the notation $(\sigma, \overrightarrow{X})$ for such a ready trace, where $\sigma = f_0 f_1,\ \dots,\ f_{n-1},\ \overrightarrow{X} = I(s_0),\ f_0,\ I(s_1),\ f_1,\ \dots,\ f_{n-1},\ I(s_n)$. The ready trace of P, notation RT(p), is $\{rt(\pi) \mid \pi\ a$ path in p, starting from the root}. If $RT(p) = RT(g)$, we say p, g are ready trace equivalence, notation $p =_{rt} g$.

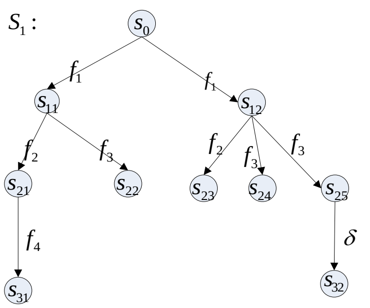
Example 5:

Let S_1, S_2 be two linear algebra transition systems. They are ready trace equivalence. Where $S_1 = (\overrightarrow{X} = (x_1, x_2),\ Q = \left\{s_0, s_{11}, \dots, s_{32}\right\},\ s_0,\ \overrightarrow{X_0} = (1,1), \Gamma),\ S_2 = (\overrightarrow{X} = (x_1, x_2)$

$Q = \left\{s_0,\ s_{11},\ s_{12},\ \dots,\ s_{32}\right\},\ s_0,\ \overrightarrow{X_0} = (1,1), \Gamma)$. $S_1,\ S_2$ are the processes in Figure 17 Figure 18

$$f_1 : f_1(X) = A_1 X \qquad f_2 : f_2(X) = A_2 X \qquad f_3 : f_3(X) = A_3 X \qquad f_4 : f_4(X) = A_4 X$$

$$A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$



$S_1 =_{RT} \quad S_2$

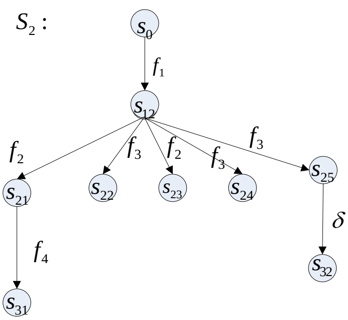


Fig. 17 **Fig. 18**

Absolutely, the number of states of S_2 is less than S_1 . And it is easier to compute.

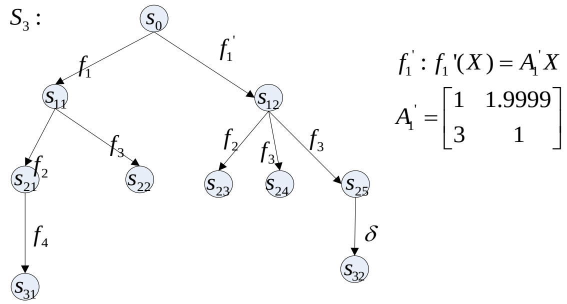


Fig. 19

In Figure 19, the difference between S_3 and S_1 is the edge $s_0 \xrightarrow{f_1} s_{12}$ and $s_0 \xrightarrow{f_1'} s_{12}$. As a result, the state cannot be rounded down because of only small differences in output. However, the result still satisfies the given reasonable error in computing, the result can be seen as same. So if f_1 can take place of f_1', which would bring the state reduction, the calculation is simple. So next, an approximate ready trace equivalence of the linear algebra transition system was defined.

Definition 7:

A matrix $A \in R^{n\times n}$, rand(A)=r, the error ε, and a norm $\|\cdot\|$ of matrix, then [||A||+n, ||A||+(n+1) is a division for all matrixes whose rand is r, denotation $(\|A\|, \varepsilon, \|\ \|)$ For $\forall B, C \in R^{n\times n}$, If $\|B\|, \|C\| \in [\|A\| + i\varepsilon, \|A\| + (i+1)\varepsilon)$ and $\|B - C\| < \varepsilon$ then B and C is approximate equivalence under division $(\|A\|, \varepsilon, \|\ \|)$, denoted $B_{(\|A\|,\varepsilon,\|\ \|)}C$.

Definition 8:

Let f and m are two linear transition, and the transition matrix is A, M, and a division $(\|A\|, \varepsilon, \|\ \|)$ if the following is hold $M_{(\|E\|,\varepsilon,\|\ \|)}A$.

Then f and m are approximate equivalence under division $(\|A\|, \varepsilon, \|\ \|)$, denoted $f_{(\|E\|,\varepsilon,\|\ \|)}m$

1. $A_{(\|E\|,\varepsilon,\|\ \|)}A$
2. If $M_{(\|E\|,\varepsilon,\|\ \|)}A$, then $A_{(\|E\|,\varepsilon,\|\ \|)}M$
3. If $M_{(\|E\|,\varepsilon,\|\ \|)}A$ and $A_{(\|E\|,\varepsilon,\|\ \|)}N$, then $M_{(\|E\|,\varepsilon,\|\ \|)}N$.

Definition 9:

Let rt and rt' be two ready trace of linear algebra transition systems, where $rt : I(n_0) f_1 I(f_1) f_2 \cdots I(f_n)$, $rt' : I(n_0) f_1' I(f_1') f_2' \cdots I(f_n')$. The division $(\|A\|, \varepsilon, \|\ \|)$ and the transition matrixes are $A_1 A_2 \cdots A_n, A_1' A_2' \cdots A_n'$, if for each positive integer k, $\prod_{i=1}^{k} A_{i(\|A\|,\varepsilon,\|\ \|)} \prod_{i=1}^{k} A_i'$ hold. Then rt and rt' are approximate ready trace equivalence under division $(\|A\|, \varepsilon, \|\ \|)$, denoted $rt\ {}_{RT_{(\|A\|,\varepsilon,\|\ \|)}}\ rt'$

Definition 10:

Let two linear algebra transition system $S=(N,X,P,\Gamma,X_0,n_0)$, $S'=(N',X,P',\Gamma',X_0,n_0)$, and division $(\|A\|,\varepsilon,\|\ \|)$. If for $\forall rt\in RT(S)$, $\exists rt'\in RT(S')$, s.t $rt_{\ RT_{(\|A\|,\varepsilon,\|\ \|)}}rt'$, then S is included by S' under $(\|A\|,\varepsilon,\|\ \|)$, denoted $RT(S)_{(\|A\|,\varepsilon,\|\ \|)}RT(S')$, if $RT(S)_{(\|A\|,\varepsilon,\|\ \|)}RT(S')$, $RT(S')_{(\|A\|,\varepsilon,\|\ \|)}RT(S)$, i.e. $RT(S)=_{(\|A\|,\varepsilon,\|\ \|)}RT(S')$. Then $S=(N,X,P,\Gamma,X_0,n_0)$ and $S'=(N',X,P',\Gamma',X_0,n_0)$ are approximate ready trace equivalence under division $(\|A\|,\varepsilon,\|\ \|)$, denoted $S_{\ (\|A\|,\varepsilon,\|\ \|)}S_i'$

Example 6:

Let S_1, S_2 and S_3 be the three linear algebra transition systems given in example 5, and the error $\varepsilon=0.1$. In MATLAB6.0, the following results were gain, $\|A_1\|_F=3.618034$, $\|A_1'\|_F=3.618006$, $\|A_1\|-\|A_1'\|_F=0.000028$, $\|A_1-A_1'\|_F=0.0001<\varepsilon$, $\|A_2A_1-A_2A_1'\|_F=0.000361<\varepsilon$, $\|A_2A_1\|_F-\|A_2A_1'\|_F=0.00024<\varepsilon$, $\|A_3A_1\|_F-\|A_3A_1'\|_F=0.000149<\varepsilon$ and $\|A_3A_1-A_3A_1'\|_F=0.000224<\varepsilon$. So $\|A_2A_1\|_F=13.5208$, $\|A_2A_1'\|_F=13.52056$, $\|A_3A_1\|_F=8.640901$, $\|A_3A_1'\|_F=8.640752$. From the definitions 7, 8, 9, 10, S_1, S_2 and S_3 are approximate ready trace equivalence. Then S_3 be taken place of S_1 when we compute the S_3. The states are cut subtraction by simple calculation under error conditions.

Lemma 3:

Approximate ready trace equivalence is an equivalent relation.

Proof:

To prove that the definition 10 is an equivalence relation, only need to prove that the definition 9 is an equivalence relation, further, proved that the definition 10 is an equivalence relation; clearly the definition 8 is an equivalence relation, so the definition of 10 is an equivalent relation.

4. AXIOM SYSTEM FOR ART SEMANTICS OF LATS

In [7, 8, and 9] the ready trace semantics equivalence axioms system is studied, and its completeness is proven. Before giving axiom system, it is agreed that involved in the process of linear algebra or a single linear algebra transition is at the same initial state. f, g, h denoted linear algebra of a single transition system, *x*, *y*, *z* denoted linear algebra transition system or part process of the whole transition system.

Table 1

$BPA_\delta + R1,2 + PR1-4 + RTR$

$x \oplus y = y \oplus x$	A_1
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$	A_2
$x \oplus x = x$	A_3
$(x \oplus y).z = x.z \oplus y.z$	A_4
$(x.y).z = x.(y.z)$	A_5
$\delta \oplus x = x$	A_6
$\delta.x = \delta$	A_7
$f.(g.x \oplus u) \oplus f.(g.y \oplus v) = f.(g.x \oplus g.y \oplus u) \oplus f.(g.x \oplus g.y \oplus v)$	R_1
$f.(g \oplus u) \oplus f.(g.y \oplus v) = f.(g \oplus g.y \oplus u) \oplus f.(g \oplus g.y \oplus v)$	R_2
$f.x \oplus f.(y \oplus z) = f.x \oplus f.(x \oplus y) \oplus f.(y \oplus z)$	S
$x \parallel y = x \lfloor\!\lfloor y \oplus y \lfloor\!\lfloor x \oplus x \mid y$	CM_1
$f \lfloor\!\lfloor x = f.x$	CM_2
$(f.x) \lfloor\!\lfloor y = f.(x \parallel y)$	CM_3
$(x \oplus y) \lfloor\!\lfloor z = x \lfloor\!\lfloor z \oplus y \lfloor\!\lfloor z$	CM_4
$(f.x) \mid g = (f \mid g).x$	CM_5
$f \mid (g.x) = (f \mid g).x$	CM_6
$(f.x) \mid (g.y) = (f \mid g).(x \parallel y)$	CM_7
$(x \oplus y) \mid z = x \mid z \oplus y \mid z$	CM_8
$x \mid (y \oplus z) = x \mid y \oplus x \mid z$	CM_9
$\partial_F(f) = f \quad if \quad f \notin F$	D_1
$\partial_F(f) = \delta \quad if \quad f \in F$	D_2
$\partial_F(x \oplus y) = \partial_F(x) \oplus \partial_F(y)$	D_3
$\partial_F(x.y) = \partial_F(x).\partial_F(y)$	D_4
$\pi_m(f) = f$	PR_1
$\pi_1(f.x) = f$	PR_2
$\pi_{m+1}(f.x) = f\pi_m(x)$	PR_3
$\pi_m(x \oplus y) = \pi_m(x) \oplus \pi_m(y)$	PR_4
$\dfrac{\pi_1(x) = \pi(y)}{z.(x \oplus y) = z.x \oplus z.y}$	RTR

Lemma 4:

The axiom system $BPA_\delta + R1,2 + PR1-4 + RTR$ is a complete axiom system.

Proof:

From [7], it is known that the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR$ for concrete process algebra is a complete axiom system. Here, because x,y and f,g in

Table 1 for $BPA_\delta + R1,2 + PR1-4 + RTR$ is operated on process graph, so *x,y* and *f,g* table 1 can be seen as x,y and a,b in Ref.[14].

The axiom system for approximation ready trace semantics are presented in table 2.

Table 2:
$BPA_\delta + R1,2 + PR1-4 + RTR + Ap1-8$

$x \oplus y = y \oplus x$	A_1
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$	A_2
$x \oplus x = x$	A_3
$(x \oplus y).z = x.z \oplus y.z$	A_4
$(x.y).z = x.(y.z)$	A_5
$\delta \oplus x = x$	A_6
$\delta.x = \delta$	A_7
$f.(g.x \oplus u) \oplus f.(g.y \oplus v) = f.(g.x \oplus g.y \oplus u) \oplus f.(g.x \oplus g.y \oplus v)$	R_1
$f.(g \oplus u) \oplus f.(g.y \oplus v) = f.(g \oplus g.y \oplus u) \oplus f.(g \oplus g.y \oplus v)$	R_2
$f.x \oplus f.(y \oplus z) = f.x \oplus f.(x \oplus y) \oplus f.(y \oplus z)$	S
$x \parallel y = x \lfloor\!\lfloor y \oplus y \lfloor\!\lfloor x \oplus x \mid y$	CM_1
$f \lfloor\!\lfloor x = f.x$	CM_2
$(f.x) \lfloor\!\lfloor y = f.(x \parallel y)$	CM_3
$(x \oplus y) \lfloor\!\lfloor z = x \lfloor\!\lfloor z \oplus y \lfloor\!\lfloor z$	CM_4
$(f.x) \mid g = (f \mid g).x$	CM_5
$f \mid (g.x) = (f \mid g).x$	CM_6
$(f.x) \mid (g.y) = (f \mid g).(x \parallel y)$	CM_7
$(x \oplus y) \mid z = x \mid z \oplus y \mid z$	CM_8
$x \mid (y \oplus z) = x \mid y \oplus x \mid z$	CM_9
$\partial_F(f) = f \quad if \quad f \notin F$	D_1
$\partial_F(f) = \delta \quad if \quad f \in F$	D_2
$\partial_F(x \oplus y) = \partial_F(x) \oplus \partial_F(y)$	D_3
$\partial_F(x.y) = \partial_F(x).\partial_F(y)$	D_4
$\pi_m(f) = f$	PR_1
$\pi_1(f.x) = f$	PR_2
$\pi_{m+1}(f.x) = f\pi_m(x)$	PR_3
$\pi_m(x \oplus y) = \pi_m(x) \oplus \pi_m(y)$	PR_4
$\dfrac{\pi_1(x) = \pi(y)}{z.(x \oplus y) = z.x \oplus z.y}$	RTR

Theory 1:

The axiom system $BPA_\delta + R1,2 + PR1-4 + RTR$ is a complete axiom system.

Proof:

From Lemma 3, we know that the axiom system the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR$ is complete system. So the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR$ in table 2 is complete system. Then if we proof that the axiom Ap1-8 is not conflict with the axioms system $BPA_\delta + R1,2 + PR1-4 + RTR$, then the theory is proven.

For Ap1, the expression denotes that when two processes are equivalence then they are approximate equivalence, absolutely, it is not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR$.

For Ap2-4, the expression denotes that three characters of equivalence, there no doubt that they are not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR$.

For Ap5-8, because of the change of approximation is only on one step, recording to the definition of approximate ready trace equivalence, they are not they are not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR$.

Then the theory 1 is proven.

5. EXPERIMENTS

With the continuous economic and social development, the city traffic has already aroused society's widespread interest. In this section, approximate ready trace equivalence of linear algebraic transition systems can optimize linear algebraic programs as well as reduce states of system is verified by the traffic lights control vehicle flow system example.

The vehicles in the given road are divided into motor vehicles and non-motor vehicles. Many factors affect the vehicle flow, we only analyze traffic signal duration and weather factor. Weather is the sunny or rain. Traffic lights have green light, red light and yellow light. The real traffic lights control vehicle flow system as shown in Figure 20.

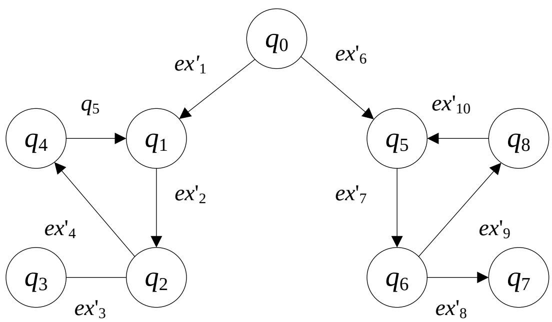


Fig. 20: The real traffic lights control vehicle flow system

Let x_1, x_2 respective be motor vehicle number and non-motor vehicle number, $X=(x_1,x_2)^{\mathrm{T}}$ be vehicle flow, $X_0=(5,5)^{\mathrm{T}}$, ε=0.045 be the given permission precision. rs=<q,X,f,Inv>, ex=<e,r,lab>. The weather in the left side is sunny and the weather in the right side is rain. At the q1 and q5, traffic light is yellow light and duration is 5 seconds. At the q_2 and q_6, traffic light is red light and duration is 10 seconds. At the q_3 and q_7 traffic light is green light and duration is 30 seconds. At the q_4 and q_8 traffic light is green light and duration is 45 seconds.

$$f_1' \text{ is } X'=\begin{pmatrix} 0.18 & 5.12 \\ -2.88 & 7.86 \end{pmatrix} X+\begin{pmatrix} 3.321 \\ 3.242 \end{pmatrix},$$

$$f_2' \text{ is } X'=\begin{pmatrix} 0.18 & 5.12 \\ -2.88 & 7.86 \end{pmatrix} X+\begin{pmatrix} 3.327 \\ 3.246 \end{pmatrix},$$

$$f_3' \text{ and } f_4' \text{ are } X'=\begin{pmatrix} \frac{6550}{13467} & -\frac{12800}{40401} \\ \frac{800}{4489} & \frac{50}{4489} \end{pmatrix} X+\begin{pmatrix} -\frac{5399544109772040}{2^{53}} \\ -\frac{5762442175356192}{2^{53}} \end{pmatrix},$$

$$f_5' \text{ Is } X'=\begin{pmatrix} 0.14 & 5.12 \\ -2.88 & 7.82 \end{pmatrix} X+\begin{pmatrix} 3.31 \\ 3.23 \end{pmatrix},\quad f_6' \text{ Is } X'=\begin{pmatrix} 0.14 & 5.12 \\ -2.88 & 7.82 \end{pmatrix} X+\begin{pmatrix} 3.315 \\ 3.235 \end{pmatrix},$$

$$f_7' \text{ and } f_8' \text{ are } X'=\begin{pmatrix} \frac{19550}{39601} & -\frac{12800}{39601} \\ \frac{7200}{39601} & \frac{350}{39601} \end{pmatrix} X+\begin{pmatrix} -\frac{5217827348157208}{2^{53}} \\ -\frac{5576060568782298}{2^{53}} \end{pmatrix}$$

lab_1' and lab_6' are $X'=X$, lab_2', lab_3' and lab_4' are $X'=1.01X$, lab_5' is $X'=\frac{1}{1.01^2}X$, lab_7', lab_8' and lab_9' are $X'=0.99X$, lab_{10}' is $X'=\frac{1}{0.99^2}X$. The state transition model of real traffic lights control vehicle flow system as shown in Figure 21. We can get the ready trace set of model system H_1 is

$$\begin{aligned} RT(H_1)=\{\pi_1',\pi_2',\pi_3',\pi_4' \mid \pi_1' &= s_0' lab_1' s_1' f_1' s_2' lab_2' s_3' f_2' s_4' f_2' s_5' lab_3' s_6' f_3' s_7' f_3' s_8', \\ \pi_2' &= s_0' lab_1' s_1' f_1' s_2' lab_2' s_3' f_2' s_4' f_2' s_5' lab_4' s_9' f_4' s_{10}' f_4' s_{11}' f_4' s_{12}', \\ \pi_3' &= s_0' s_{13}' lab_6' f_5' s_{14}' lab_7' s_{15}' f_6' s_{16}' f_6' s_{17}' lab_8' s_{18}' f_7' s_{19}' f_7' s_{20}', \\ \pi_4' &= s_0' s_{13}' lab_6' f_5' s_{14}' lab_7' s_{15}' f_6' s_{16}' f_6' s_{17}' lab_9' s_{24}' f_8' s_{23}' f_8' s_{22}' \} \end{aligned}$$

The state transition model of approximate traffic lights control vehicle flow system as shown in Figure 22. The ready trace set of model system H_2 is

$$RT\left(H_1\right) = \{\pi_1, \pi_2, \pi_3, \pi_4 \mid \pi_1$$
$$= s_0 lab_1 s_1 f_1 s_2 lab_2 s_3 f_2 s_4 f_2 s_5 lab_3 s_6 f s_4 f_2 s_5 lab_4 s_9 f_4 s_{10} f_4 s_{11} f_4 s_{12},$$
$$\pi_2 = s_0 lab_1 s_1 f_1 s_2 lab_2 s_3 f_2$$
$$\pi_3 = s_0 s_{13} lab_6 f_5 s_{14} lab_7 s_{15} f_6 s_{16} f_6 s_{17} lab_8 s_{18} f_7 s_{19} f_7 s_{20},$$
$$\pi_4 = s_0 s_{13} lab_6 f_5 s_{14} lab_7 s_{15} f_6 s_{16} f_6 s_{17} lab_9 s_{24} f_8 s_{23} f_8 s_{22},_3 s_7 f_3 s_8\}$$

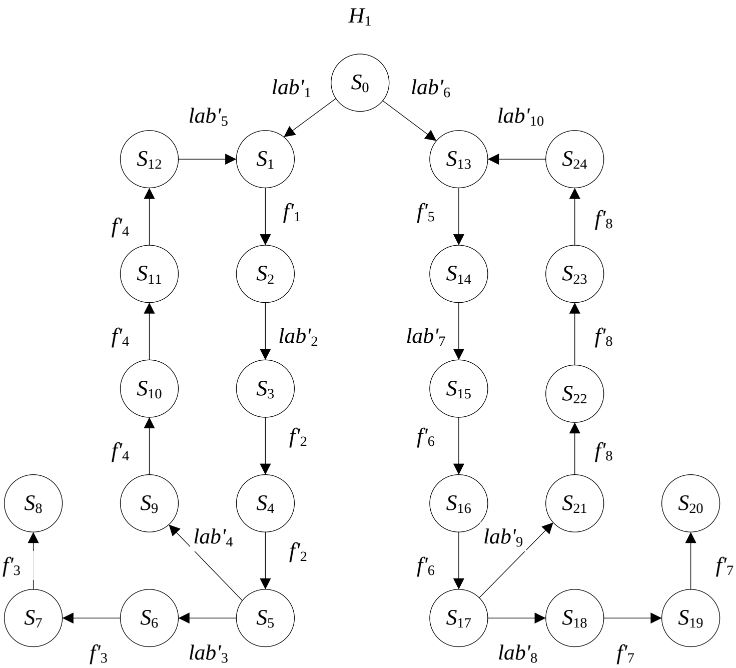


Fig. 21: The State Transition Model of Real Traffic Lights Control Vehicle Flow System

f_1, f_2, f_5 and f_6 are $X' = \begin{pmatrix} 0.16 & 5.12 \\ -2.88 & 7.84 \end{pmatrix} X + \begin{pmatrix} 3.32 \\ 3.24 \end{pmatrix}$, f_3, f_4, f_7 and f_8 are $X' = \begin{pmatrix} 0.49 & -0.32 \\ 0.18 & 0.01 \end{pmatrix} X + \begin{pmatrix} -0.59 \\ -0.63 \end{pmatrix}$, $lab_i (i = 1, \cdots, 10)$ is $X' = X$. The state transition model of approximate traffic lights control vehicle flow system as shown in Figure 22.

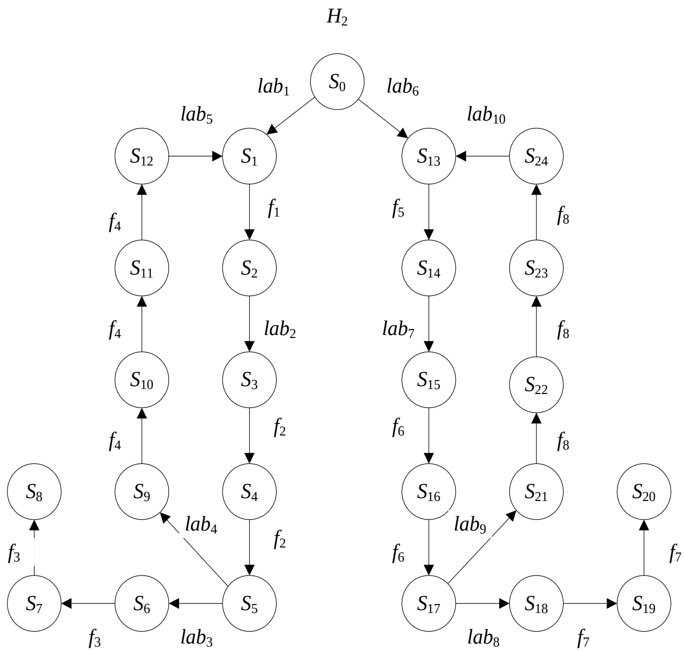


Fig. 22: State Transition Model of Approximate Traffic Lights Control Vehicle Flow System

$W_{\max 1} \approx 0.0407 < \varepsilon$. From numerical calculation results $\pi_1 ' =_{(1,0.045,\|\cdot\|)} \pi_1$,

$\pi_2 ' =_{(1,0.045,\|\cdot\|)} \pi_2$, $\pi_3 ' =_{(1,0.045,\|\cdot\|)} \pi_3, \pi_4 ' =_{(1,0.045,\|\cdot\|)} \pi_4$, All traces of H1 and H2 are approximate ready trace equivalence. The approximate ready trace equivalence model of real traffic lights control vehicle flow system was got through the ready trace equivalence definition, i.e.

$H_1 =_{(1,0.045,\|\cdot\|)} H_2$.

$\pi_1 ' =_{(1,0.045,\|\cdot\|)} \pi_1, \pi_1 =_{(1,0.045,\|\cdot\|)} \pi_2, \pi_2 =_{(1,0.045,\|\cdot\|)} \pi_2 '$, from the axiom Ap1-7,

$\pi_1 ' =_{(1,0.045,\|\cdot\|)} \pi_2 '$,

Similarly, $\pi_3 ' =_{(1,0.045,\|\cdot\|)} \pi_4 '$, H2 can be used the following expression to indicate.

$$H2:\ lab_1 f_1 lab_2 f_2 f_2 (lab_3 f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5) \oplus$$
$$lab_6 f_5 lab_7 f_6 f_6 (lab_8 f_7 f_7 \oplus lab_9 f_8 f_8 f_8)$$
$$= lab_1 f_1 lab_2 f_2 f_2 (lab_3 f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5) \oplus lab_1$$
$$f_1 lab_2 f_2 f_2 (lab_8 f_7 f_7 \oplus lab_9 f_8 f_8 f_8) \qquad (AP_1)$$
$$= lab_1 f_1 lab_2 f_2 f_2 (lab_3 f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5) \oplus (lab_8$$
$$f_7 f_7 \oplus lab_9 f_8 f_8 f_8)) \qquad (R1)$$
$$= lab_1 f_1 lab_2 f_2 f_2 ((lab_3 f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5) \oplus (lab_3$$
$$f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5)) \qquad (AP1)$$
$$= lab_1 f_1 lab_2 f_2 f_2 ((lab_3 f_3 f_3 \oplus lab_4 f_4 f_4 f_4 lab_5) \qquad (A3)$$

From the above reasoning by using the axiom system in Table 2, approximate ready trace equivalence model of real traffic lights control vehicle flow system H_3 has been got as shown in Figure 23. And $H_1 =_{(1,0.045,||||)} H_2 =_{(1,0.045,||||)} H_3$. Through the use of axiom system and approximation theory, approximate ready trace equivalence system of real traffic lights control vehicle flow system as shown in Figure 24.

From above computation and analysis, we know that approximate ready trace equivalence of linear algebraic transition systems can optimize linear algebraic programs as reduce the system well states.

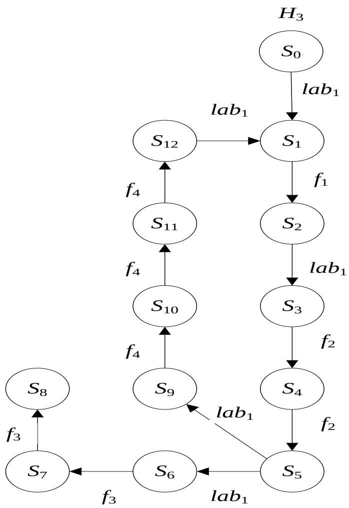


Fig. 23: Approximate Ready Trace Equivalence Model of H_3

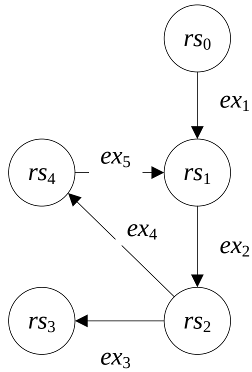


Fig. 24: Approximate Ready Trace Equivalence System

Through the use of axiom system and approximation theory, approximate ready trace equivalence system of real traffic lights control vehicle flow system as shown in Figure 24.

From above computation and analysis, we know that approximate ready trace equivalence of linear algebra transition systems can optimize linear algebra programs as well as reduce the system states.

6. CONCLUSIONS

Approximate ready trace equivalence on the basis of reference 1, 2 for linear algebra transition system is proposed. The approximate ready trace equivalence can be used to model reduction and optimization of linear algebra transition system. What's more, the error is under control. Then axiom system for the approximate ready trace equivalence is given and its completeness is proven. The theory proposed a door for applying from the theory of process algebra model testing and optimization to transition system. Approximate ready trace equivalence for differential algebra transition system will be studied in the following work.

ACKNOWLEDGEMENTS

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