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OPTIMAL PROGRESSIVE GROUP-CENSORING PLANS FOR CHEN DISTRIBUTION UNDER COST CONSTRAINT*

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ABSTRACT

In this paper, the optimal design of a progressively group-censored life test with the restriction of experimental budget is developed for the Chen distribution is considered. The maximum likelihood estimates, approximate confidence intervals for the parameters based on progressively group-censored sample are obtained. Wu et al.'s (2008a) approach is used to determine the number of test units, the number of inspections and the length of inspection interval of a life test under a pre-determined budget of experiment such that the determinant of the asymptotic variances-covariance matrix of estimators of parameters is minimum. A numerical example is presented and the sensitivity analysis is also studied.

KEYWORDS

Bathtub shape; D-Optimality; Grouped data; Maximum likelihood method; Progressive Censoring.

1. INTRODUCTION

In reliability analysis, an important issue is how to collect the lifetime data of products in a limited experimental time and a restricted experimental budget. For saving experimental time, censoring is a very common and useful tool. It usually applies when the exact lifetimes are known for only a portion of the products and the remainder of the lifetimes have only partial information. There are several types of censored life tests. Type I and type II are two of the most common censoring schemes. Suppose that n units are placed on a life test. Type I censoring is run over a fixed time period such that beyond this time no failures will be observed. This means that the number of exact lifetimes observed is random. On the other hand, type II censoring is terminated at the time of the m-th ($m \le n$) failure observed such that $n-m$ partially observed lifetimes are known only to exceed certain value. Hence, the termination time is random. Note that, in both

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type I and type II censoring schemes, surviving units can only be removed at the end of the life tests. However, in some practical situations, one has to remove surviving units at the points other than the final termination point. This leads us to the area of progressive censoring. A recent account on progressive censoring can be found in the monograph by Balakrishnan and Aggarwala (2000) or in the review article by Balakrishnan (2007).

In some situations, it is often impossible continuously to observe the testing process and hence, the failure times of test units cannot be recorded exactly. That is, one can only record whether a test unit fails in a time interval instead of measuring lifetime exactly. Thus, the test units are inspected intermittently. This type of inspection is called group-censoring. In the literature, group-censored data have been studied by many researchers such as Cheng and Chen (1988), Chen and Mi (1996), Qian and Correa (2003) and Lu and Tsai (2009). In recent years, progressive group-censoring scheme has received the attention of many researchers. Some important literature can be found, for example, Aggarwala (2001), Xiang and Tse (2005), Yang and Tse (2005) and Wu et al. (2008b).

Recently, there has been a heightened interest in improving reliability of products. Increasingly intense global competition and higher consumer expectations for reliable products are driving this interest. To remain profitable, manufacturers are challenged to design and produce high quality and long life products. Therefore, they must have sound knowledge about product lifetime distributions. To obtain this knowledge, life testing experiments are performed before products are put on the market.

There are some important questions about how to design an appropriate progressively group-censored life test that would results in the optimal estimation of life parameter. Important questions include how to determine the number of test units, the number of inspections and the length of the inspection intervals. One practical problem arising from designing a life test is the limited budget of the experiment. The size of the budget always affects the decisions of the number of test units, number of inspections and the length of inspection intervals and, hence, affects the precision of estimation. Therefore, the problem of obtaining a precise estimation of model parameter under a limited budget of experiment becomes an important issue for the engineers. In the literature, some researchers took cost considerations into account when reliability plans were designed. Some of them are Lui et al. (1993), Tse et al. (2002), Kuş et al. (2011a), Kuş et al. (2011b), Akdoğan et al. (2011), Kuş and Akdoğan (2011), Akdoğan and Kuş (2011) and Akdoğan (2011).

In this paper, we will focus on the designing problem of a progressively group-censored life test under a two-parameter lifetime distribution with bathtub shape or increasing failure rate function which was investigated by Chen (2000). In the past decade, The Chen distribution attracted many researchers to study on it. Several works of statistical inference have been done, for example, Ali Selim (2012), Lee et al. (2007), Rastogi et al. (2012), Sahran et al. (2010), Wu et al. (2005), Wu (2008), Wu et al. (2011) and Wu et al. (2009). The purpose of this study is to explore the optimum number of test units, the number of inspections and the length of inspection intervals with the restriction of experimental budget when the lifetimes of test units follow the Chen distribution. A mathematical model with decision variables and the cost of the experiment used by Wu

et al. (2008a) and Wu and Huang (2010) is considered. The method of nonlinear mixed integer programming is used to obtain the optimal plans.

The rest of the paper is organized as follows: The problem is formulated in Section 2 and the maximum likelihood estimates and interval estimates of the parameters are obtained. An algorithm for obtaining the optimal plans is discussed in Section 3. In Section 4, a numerical example is discussed. Results of sensitivity analysis are presented in Section 5. Finally, concluding remarks are provided to close the paper in Section 6.

2. MODEL DESCRIPTION AND PARAMETER ESTIMATION

Let the lifetime random variable X of a test unit have a Chen distribution. The probability density function and cumulative distribution function are given, respectively, by

$$f(x) = \lambda\beta x^{\beta-1} \exp\left\{\lambda\left(1-\exp\left(x^{\beta}\right)\right)+x^{\beta}\right\},\ x>0,$$

and

$$F(x) = 1-\exp\left\{\lambda\left(1-\exp\left(x^{\beta}\right)\right)\right\},\quad x>0, \tag{1}$$

where $\beta>0$ and $\lambda>0$ are parameters.

Suppose that n independent units are simultaneously placed on a life test at time 0 and run until time τ_1, at which point the number of failed units n_1 are counted and r_1 surviving units are removed from the test; starting from time τ_1, the $n-n_1-r_1$ non-removed surviving units are run until time τ_2, at which point the number of failures n_2 are counted and r_2 surviving units are removed from the test and so on. At time τ_k, the number of failed units n_k are counted and the remaining surviving $r_k = n-\sum_{i=1}^{k} n_i - \sum_{j=1}^{k-1} r_j$ units are all removed, thereby terminating the test. This scheme is called progressive type-I group-censoring and may be depicted pictorially in Figure 1.

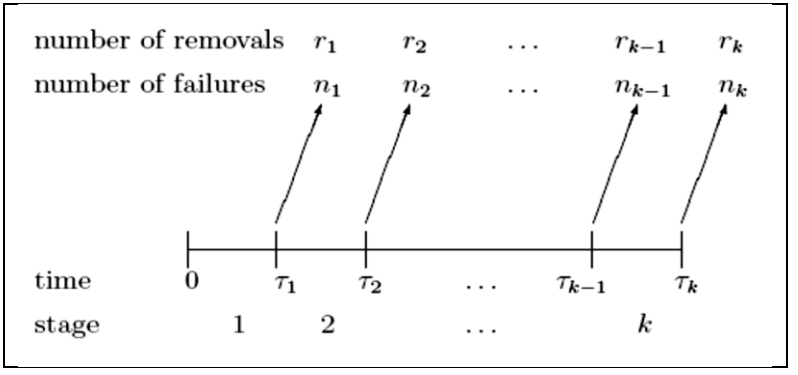

Fig. 1: k -level Progressive Type-I Group-Censoring

The values of $r_1, r_2, \ldots, r_k$ are determined by the pre-specified percentages of removals of the remaining live units $p_1, p_2, \ldots, p_k$ (with $p_k = 1$); that is, $r_i = (m_i - n_i)\, p_i$, where $m_i = n - \sum_{j=1}^{i-1} n_j - \sum_{j=1}^{i-1} r_j$ is the number of non-removed surviving units at the beginning of the i-th stage, for $i = 1, 2, \ldots, k$. Then, we have

$$N_i \mid n_{i-1}, \ldots, n_1, r_{i-1}, \ldots, r_1 \sim \textit{Binomial}\,(m_i, q_i), \tag{2}$$

where $q_i = \left(F(\tau_i) - F(\tau_{i-1})\right) / \left(1 - F(\tau_{i-1})\right) = 1 - \exp\left\{-\lambda\left[\exp\left(\tau_i^{\beta}\right) - \exp\left(\tau_{i-1}^{\beta}\right)\right]\right\}$ is the probability that a unit survives at time τ_{i-1} and will fail before time τ_i, for $i = 1, 2, \ldots, k$, where $\tau_0 = 0$ and the function $F(\cdot)$ is defined in Equation (1). When the lengths of inspection intervals are all equal; that is, $\tau_i - \tau_{i-1} = \tau$, $i = 1, 2, \ldots, k$, one has

$$q_i = 1 - \exp\left\{-\lambda\left[\exp\left((i\tau)^{\beta}\right) - \exp\left(\left((i-1)\tau\right)^{\beta}\right)\right]\right\}. \tag{3}$$

In this paper, we assume that the lengths of inspection intervals are all equal. This assumption is also convenient for practitioners. Based on the observed data and Equation (2), the likelihood function is

$$L(\beta, \lambda) \propto \prod_{i=1}^{k} q_i^{n_i} (1 - q_i)^{m_i - n_i}$$

so that log-likelihood function can be written as

$$\log\left(L(\beta, \lambda)\right) \propto \sum_{i=1}^{k} \left[n_i \log(q_i) + (m_i - n_i) \log(1 - q_i)\right],$$

where q_i is defined as in Equation (2).

Hence we can employ Newton–Raphson method for finding the maximum likelihood estimates (MLEs) numerically. That is, we can solve the equations

$$\frac{\partial \log\left(L(\beta, \lambda)\right)}{\partial \beta} = \sum_{i=1}^{k} \frac{n_i}{q_i} \frac{\partial q_i}{\partial \beta} - \frac{(m_i - n_i)}{(1 - q_i)} \frac{\partial q_i}{\partial \beta}$$

and

$$\frac{\partial \log\left(L(\beta, \lambda)\right)}{\partial \lambda} = \sum_{i=1}^{k} \frac{n_i}{q_i} \frac{\partial q_i}{\partial \lambda} - \frac{(m_i - n_i)}{(1 - q_i)} \frac{\partial q_i}{\partial \lambda}$$

by using the second-order derivative forms

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\beta^2} = \sum_{i=1}^{k} \frac{n_i}{q_i^2}\left(\frac{\partial^2 q_i}{\partial\beta^2} q_i - \left(\frac{\partial q_i}{\partial\beta}\right)^2\right) - \frac{(m_i - n_i)}{(1-q_i)^2}\left(\frac{\partial^2 q_i}{\partial\beta^2}(1-q_i) + \left(\frac{\partial q_i}{\partial\beta}\right)^2\right),$$

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\lambda^2} = \sum_{i=1}^{k} \frac{n_i}{q_i^2}\left(\frac{\partial^2 q_i}{\partial\lambda^2} q_i - \left(\frac{\partial q_i}{\partial\lambda}\right)^2\right) - \frac{(m_i - n_i)}{(1-q_i)^2}\left(\frac{\partial^2 q_i}{\partial\lambda^2}(1-q_i) + \left(\frac{\partial q_i}{\partial\lambda}\right)^2\right),$$

and

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\beta\partial\lambda} = \sum_{i=1}^{k} -\frac{n_i}{q_i^2}\frac{\partial q_i}{\partial\lambda}\frac{\partial q_i}{\partial\beta} + \frac{\partial^2 q_i}{\partial\beta\partial\lambda}\frac{n_i}{q_i} - \frac{(m_i - n_i)}{(1-q_i)^2}\frac{\partial q_i}{\partial\lambda}\frac{\partial q_i}{\partial\beta} - \frac{\partial^2 q_i}{\partial\beta\partial\lambda}\frac{(m_i - n_i)}{(1-q_i)},$$

where

$$\frac{\partial q_i}{\partial\lambda} = -\frac{(1-q_i)}{\lambda}\log(1-q_i),$$

$$\frac{\partial q_i}{\partial\beta} = \lambda(1-q_i)\left\{\left[\exp\left((i\tau)^\beta\right)(i\tau)^\beta \log(i\tau)\right] - \left[\exp\left(((i-1)\tau)^\beta\right)((i-1)\tau)^\beta \log((i-1)\tau)\right]\right\},$$

$$\frac{\partial^2 q_i}{\partial\lambda^2} = \frac{\log(1-q_i)}{\lambda}\frac{\partial q_i}{\partial\lambda},$$

$$\frac{\partial^2 q_i}{\partial\beta^2} = \lambda(1-q_i)\left\{\exp\left((i\tau)^\beta\right)\left((i\tau)^\beta\right)\log(i\tau)\log(i\tau)\left((i\tau)^\beta + 1\right) - \exp\left(((i-1)\tau)^\beta\right)\left(((i-1)\tau)^\beta\right)\left(\log((i-1)\tau)^\beta\right)\left(\log((i-1)\tau)^\beta\right)\left(((i-1)\tau)^\beta + 1\right)\right\} - \left(\frac{\partial q_i}{\partial\beta}\right)^2 \frac{1}{1-q_i},$$

and

$$\frac{\partial^2 q_i}{\partial\beta\partial\lambda} = \frac{\left\{1 + \log(1-q_i)\right\}}{\lambda}\frac{\partial q_i}{\partial\beta}.$$

The asymptotic normality of the MLEs can be derived in the usual way. The Fisher information matrix can be obtained by taking the negative of the expectations of the second partial derivatives of log-likelihood function. Following the same procedure in Wu et al. (2008a), one can obtain the Fisher information matrix based on progressive type-I group-censored sample as

$$\mathbf{I}(\beta,\lambda)=\begin{bmatrix} \sum_{i=1}^{k} E(M_i)\frac{(\partial q_i/\partial\beta)^2}{q_i(1-q_i)} & \sum_{i=1}^{k} E(M_i)\frac{(\partial q_i/\partial\beta)(\partial q_i/\partial\lambda)}{q_i(1-q_i)} \\ \sum_{i=1}^{k} E(M_i)\frac{(\partial q_i/\partial\beta)(\partial q_i/\partial\lambda)}{q_i(1-q_i)} & \sum_{i=1}^{k} E(M_i)\frac{(\partial q_i/\partial\lambda)^2}{q_i(1-q_i)} \end{bmatrix},$$

where

$$E(M_i)=n\prod_{j=1}^{i-1}(1-q_j)(1-p_j),\ i=1,2,\ldots,k.$$

For a large sample size n, the MLEs $(\hat{\beta},\hat{\lambda})'$ have an approximate bivariate normal distribution with mean vector $(\beta,\lambda)'$ and variance-covariance matrix $\mathbf{I}^{-1}(\beta,\lambda)$, where $\mathbf{I}^{-1}(\beta,\lambda)$ is the inverse of Fisher information matrix. In practice, we usually $\mathbf{I}^{-1}(\beta,\lambda)$ by $\mathrm{I}^{-1}(\beta,\lambda)$. Thus, the approximate confidence intervals or confidence region for β and λ can be easily established.

3. OPTIMAL PLANS

To obtain a precise estimation of life parameters, frequently asked questions include 'How many units does the experimenter need to test?', 'How long does the experimenter need to run the life test?' or 'How many times does the experimenter need to inspect the units in the life test?' Simply put, more test units, more test time and more number of inspections will generate more information, which improves the precision of estimates. However, in practice, the budget of an experiment is limited. Therefore, the problem of obtaining a precise estimation of life parameters under a restricted cost of experiment is an important issue for the reliability analyst.

Let n denote the number of units on test, k the number of inspections and τ the length of inspection interval. The cost of experiment includes the following four parts:

1) Installation cost: This is the cost of installing all test units at the beginning of the life experiment, say C_a. It does not depend on the number of test units.

2) Sample cost: This is the cost of test units. Let C_s be the cost of a test unit. Then, the total sample cost is nC_s.

3) Inspection cost: This cost includes the cost of using inspection equipment and material. It depends on the number of inspections. Let C_i denote the cost of one inspection. Then, the total inspection cost is kC_i.

4) Operation cost: This includes salaries of operators, utilities and depreciation of test equipment, etc. It is proportional to the testing time. Let C_o be the operation cost in the time interval between two inspections. Then, the total operation cost is $k\tau C_o$.

Therefore, the total cost of experiment is:

$$C_T = C_a + nC_s + kC_i + k\tau C_o.$$

Note that the asymptotic variance-covariance matrix $\mathbf{I}^{-1}(\beta,\lambda)$ of the MLEs $\hat{\beta}$ and $\hat{\lambda}$ is a function of n, k and τ. For a specific plan (n,k,τ), we can compute the asymptotic variance-covariance matrix of the MLEs. We want to determine the optimal plan (n,k,τ) under cost considerations. Since the parameter (β,λ) is two-dimensional, optimality can be defined in terms of the following criterion: D-optimality: Minimizing the determinant of the asymptotic variance-covariance matrix. D-optimality provides an overall measure of variability of the estimates. For simplicity, let $G(n,k,\tau)$ be the function to be minimized in this criterion. Then, the optimal design problem consists in finding n, k and τ, which minimize $G(n,k,\tau)$. However, the determination of n, k and τ, is restricted to the budget of experiment, say, C_r. Hence, the optimal design problem can be expressed as follows:

$$\begin{aligned}&\text{minimize } G(n,k,\tau)\\ &\text{subject to } C_a + nC_s + kC_i + k\tau C_o \le C_r\\ &k,n \in N \text{ and } \tau > 0,\end{aligned}$$

where N is the set of positive integers. Since the objective function and constraint are both nonlinear functions of decision variables n, k and τ, it is difficult to obtain a closed form of the solution. Therefore, in order to find the optimal solution for the problem of nonlinear mixed integer programming, a modified algorithm proposed by Kuş et al. (2011b) for finding the optimal solution is as follows:

Algorithm

Step 1. Set the values of cost parameters C_a, C_s, C_i, C_o and C_r and give the values parameters (β,λ).

Step 2. Calculate the upper bound of the number of test units. Under the constraint of total experimental cost, the upper bound is

$$\tilde{n} = \left[\frac{C_r - C_a - C_i}{C_s}\right],$$

where $[x]$ is greatest integer that is less than or equal to x. Set $n = 2$.

Step 3. Compute the upper bound of the number of inspections for a given n. Using the constraint of total experimental cost and a given value of n, compute the upper bound

$$\tilde{k}_n = \left[\frac{C_r - C_a - nC_s}{C_i} \right]$$

Step 4. Compute the upper bound of the length of inspection interval. Using the constraint of total experimental cost, for all $k \in N \;\; 1 \le k \le \tilde{k}_n$, compute the upper bound of the length of inspection interval $\tilde{\tau}_{kn} = \frac{C_r - C_a - nC_s - kC_i}{kC_o}$ and obtain $\tau' = \arg \min_{\tau} G(n,k,\tau)$.

Step 5. Calculate the corresponding value of objective function $G\left(n,k,\tilde{\tau}_{kn}\right)$ and $G\left(n,k,\tau'\right)$.

Step 6. If $G\left(n,k,\tilde{\tau}_{kn}\right) > G\left(n,k,\tau'\right)$ and $\tau' < \tilde{\tau}_{kn}$, set $\tau_{kn} = \tau'$, else $\tau_{kn} = \tilde{\tau}_{kn}$.

Step 7. Let function $\phi(n) = G(n,k_n,\tau_{kn}) = \min_{1 \le k \le \tilde{k}_n} G(n,k,\tau_{kn})$.

Step 8. Set $n = n+1$. If $n \le \tilde{n}$ go to Step 3, else go to Step 9.

Step 9. Compute the optimal value of objective function $\phi(n^*) = \min_{2 \le n \le \tilde{n}} \phi(n) = G(n^*,k^*,\tau^*) = \min_{2 \le n \le \tilde{n}} G(n,k_n,\tau_{kn})$.

Step 10. The optimal design $\left(n^*,k^*,\tau^*\right)$ is obtained.

Note that, in Step 1, the values of β and λ are usually unknown. One needs to conduct a pilot study or search history results to get the possibly adequate values of them.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

We apply the proposed methods to a numerical example and use the algorithm from Aggarwala (2001) to generate data with $n = 60$, $k = 10$, $\tau = 0.1390$, $\beta = 2$ and $\lambda = 3$. The pre-specified percentages of removals are $\left(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\right) = \left(0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 1\right)$. The data are presented in Table 1.

Table 1
Progressively Type-I Group-Censored Sample

i	1	2	3	4	5	6	7	8	9	10
n_i	0	0	0	2	2	5	4	10	3	4
r_i	3	2	2	2	2	2	1	1	0	15
p_i	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	1

We obtain the MLEs of λ and β to be $\hat{\lambda} = 3.6778$ and $\hat{\beta} = 1.6048$, respectively. We use these estimates in the design of our new experiment. Suppose further the values of cost parameters are as follows: $C_a = 6, C_s = 65, C_i = 3.25, C_o = 10$ and $C_r = 4000$. Thus, the optimal design problem is:

$$\begin{aligned} &\min && G(n, k, \tau) \\ &\textit{subject to} && 6 + 65n + 3.25k + 10k\tau \le 4000 \end{aligned}$$

Using the algorithm proposed in Section 4, we can obtain the optimal design as follows:

D-Optimality: $n^* = 60,\ k^* = 15,\ \tau^* = 0.1122$.

In the algorithm proposed for searching the optimal solutions, the computation is involved with the unknown parameters β and λ. As we mentioned in Section 3, prior experience or historical data can be used to pre-estimate the parameters. However, these estimates, accurately speaking, are not guaranteed to be equal to the unknown parameters. In addition, the optimal solution may also be influenced by the values of cost parameters. Thus, it is important to study the influence of various estimates and different cost values on searching the optimal solutions.

We now study sensitivity of the optimal solution to changes in the values of the different parameters associated with the life experiment. These parameters can be divided into two parts: (1) the parameters in the failure time distribution, i.e., β and λ; and (2) the parameters in the cost of experiment, i.e., C_a, C_s, C_i, C_o and C_r. We will study how the optimal solution is influenced by the estimates of distribution parameters and by the cost parameters of the experiment in Table 2 and Table 3, respectively.

As we mentioned in Section 3, in practice, the values of distribution parameters are usually unknown. We have to use prior information or data from a pilot test to get their estimates. However, none can guarantee that the estimates are exactly equal to the

unknown parameters. Thus, we will discuss the influence of changing values of estimated parameters on the optimal solutions.

The MLEs of β and λ obtained in Section 4 are $\hat{\beta}=1.6048$ and $\hat{\lambda}=3.6778$, respectively. Thus, the 95% approximate confidence intervals for β and λ are $(1.1279, 2.0816)$ and $(2.5271, 4.8285)$, respectively. We choose various values of β and λ in their 95% approximate confidence intervals for sensitivity analysis. Set $(C_a, C_s, C_i, C_o, C_r)=(6, 65, 3.25, 10, 4000)$ which are the values of cost parameters used in Section 4. We assume that the pre-specified percentages of removals in each stage are all equal. That is, $p_1 = p_2 = \cdots = p_{k-1} = p$ and $p_k = 1$. Now, the optimal solutions of n, k and τ for $p=0.05$ are given in Table 2.

Table 2
Optimal values of n, k and τ for fixed
$C_a=6, C_s=65, C_i=3.25, C_o=10$ and $C_r=4000$

λ	β	n	k	τ	$k\tau$	D-optimality
2.5271	1.1279	60	21	0.1061	2.2281	0.0029
	1.6048	60	12	0.1363	1.6356	0.0055
	2.0816	60	19	0.1490	2.8310	0.0093
3.6778	1.1279	60	23	0.0778	1.7894	0.0063
	1.6048	60	15	0.1122	1.6830	0.0116
	2.0816	61	6	0.1583	0.9498	0.0193
4.8285	1.1279	60	24	0.0621	1.4904	0.0108
	1.6048	60	15	0.0969	1.4535	0.0198
	2.0816	61	6	0.1456	0.8736	0.0328

Table 2 shows that n is not sensitive to the changes in different parameter values. The length of inspection interval τ is increasing function of β and decreasing function of λ. The termination time of experiment $k\tau$ is sensitive to both parameters values. The value of D-optimality is also increasing functions of β and λ. All comments on Table 2 are given under D-optimality criterion.

Changes in cost parameters of the experiment can affect the determination of the optimal design. We consider the values of distribution parameters to be $\lambda=3.6778$ and $\beta=1.6048$. Using these values of the distribution parameters, the sensitivity of each of the decision variables n, k and τ to changes in the cost parameters of the experiment is

examined. Tables 3 shows that a higher value of C_r causes a higher value of n; the number of test units is insensitive to changes in C_i, C_a and C_o. Larger value of C_s results in a smaller value of n. In addition, inspection interval k are sensitive to all cost parameters. The length of inspection interval τ is almost insensitive.

5. CONCLUSION

The subject of progressive censoring has received attention in the past few years. The Chen distribution can be widely used in reliability applications because of the form of its hazard function which is useful in practice. In this paper, we derived the MLEs of the parameters of Chen distribution under progressive type-I group-censoring. The approximate confidence intervals and region can be easily established. Determining the appropriate number of test units, number of inspections and length of inspection interval under a limited budget of experiment is an important decision problem for experimenters when conducting a life test. Wu et al. (2008a)'s algorithm modified by Kuş et al. (2011b) is used to set up the optimal design in this paper. Using this algorithm, we can obtain the optimal values of decision variables based on D-Optimality. Since the proposed algorithm is needed to provide the values of model parameters before searching the optimal solutions, we also studied the sensitivity analysis of cost parameters and distribution parameters. The results show that, based on D-optimality criterion, the number of test units is not sensitive to the values of distribution parameters, but the number of inspections and the length of inspection intervals are influenced by the values of distribution parameters. The values of cost parameters have effect on the three decision variables. Finally, the proposed approach can lead to better designs for conducting progressive type-I group-censoring life tests. It provides an efficient use of one's resources and to achieve the precision that one can expect to have with such a design. This approach is intuitive and can be useful to engineers.

Table 3
D-Optimal Values of n, k and τ for Different Costs Values under $\lambda = 3.6778$ and $\beta = 1.6048$

C_r	C_s	C_i	C_o	C_a	n	k	τ	D-optimality
1000	65	3.25	10	6	14	18	0.1122	0.2136
2000					30	10	0.1140	0.0465
3000					45	15	0.1122	0.0207
4000					60	15	0.1122	0.0116
5000					76	12	0.1122	0.0072
6000					91	17	0.1122	0.0051
4000	45	3.25	10	6	87	17	0.1122	0.0055
	55				72	7	0.1373	0.0083
	65				60	15	0.1122	0.0116
	75				52	9	0.1185	0.0155
	85				46	8	0.1263	0.0198
	95				41	15	0.1122	0.0249
4000	65	2.25	10	6	61	8	0.1262	0.0114
		2.75			61	7	0.1373	0.0116
		3.25			60	15	0.1122	0.0116
		3.75			60	14	0.1122	0.0116
		4.25			60	14	0.1122	0.0116
		4.75			60	14	0.1122	0.0116
		5.25			60	13	0.1122	0.0116
4000	65	3.25	7	6	60	19	0.1122	0.0116
			8		60	18	0.1122	0.0116
			9		60	14	0.1122	0.0116
			10		60	15	0.1122	0.0116
			11		60	17	0.1122	0.0116
			12		60	15	0.1122	0.0116
			13		60	14	0.1122	0.0116
4000	65	3.25	10	3	61	7	0.1321	0.0116
				4	60	17	0.1122	0.0116
				5	60	17	0.1122	0.0116
				6	60	15	0.1122	0.0116
				7	60	19	0.1122	0.0116
				8	60	20	0.1122	0.0116
				9	60	16	0.1122	0.0116

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