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GENERALIZED EXPONENTIAL-TYPE RATIO-CUM-RATIO AND PRODUCT-CUM-PRODUCT ESTIMATORS FOR POPULATION MEAN IN THE PRESENCE OF NON-RESPONSE UNDER STRATIFIED TWO-PHASE RANDOM SAMPLING

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ABSTRACT

In this paper, generalized exponential-type estimators have been proposed for estimating the finite population mean of study variable using information on two auxiliary variables in the presence of non-response under stratified two-phase random sampling. The expressions for the bias and mean square error (MSE) of proposed estimators have been derived in two different situations of non-response. Theoretical comparisons of proposed estimators have been made with modified forms of Hansen and Hurwitz (1946), ratio and product estimators to the stratified two-phase sampling method. An empirical study has also been carried out to demonstrate the performances of proposed estimators.

KEYWORDS

Auxiliary variables; Bias; Mean Square error; Non-response; Stratified Two-phase Random Sampling; Exponential-type estimator.

AMS (1991) *subject classification.* 62D05

1. INTRODUCTION

Estimation of the population mean in sample surveys when some observations are missing due to non-response has been considered by Hansen and Hurwitz (1946). In the presence of non-response, the problem of estimating population mean of the study variable (y) has been discussed by Cochran (1977), Khare and Srivastava (1997) and Singh and Kumar (2008). Singh and Kumar (2009) proposed a general family of estimators under two-phase sampling in the presence of non-response. Singh et al. (2009) have proposed a family of combined-ratio-type estimators for estimating the population mean under non-response by adapting the estimator of Khoshnevisan et al. (2007). Khare and Sinha (2009) proposed some estimators using multi-auxiliary characteristics with known population means in the presence of non-response. Singh et al. (2010) proposed some exponential-type ratio and product type estimators and their generalized version for estimating the population mean of the study variable (y) in the presence of non-response. Singh et al. (2010) suggested some exponential-type ratio type estimators using single auxiliary variable under two-phase sampling in the presence of non-response.

Kadilar and Cingi (2003) extended Upadhyaya and Singh (1999) estimator of simple random sampling in stratified random sampling. Singh et al. (2008, 2009) proposed some exponential estimators in simple random sampling to estimate the population mean of the study variable y, following Bhal and Tuteja (1991). Singh et al. (2008) have considered exponential-ratio-type estimator in stratified random sampling. Koyuncu and Kadilar (2010) have suggested a family of estimators in stratified random sampling following Diana (1993) and Kadilar and Cingi (2003). Upadhyaya et al. (2011) suggested some improved ratio and product exponential type estimators. Malik and Singh (2012) proposed some modified ratio type estimators using geometric mean and harmonic mean for stratified random sampling. Motivated by Singh et al. (2009), Singh and Kumar (2012) proposed some exponential estimators in stratified sampling.

Consider a finite population of size N which is stratified into L homogenous strata. Let N_h be the size of h^{th} stratum $(h=1,2,...,L)$ such that $\sum_{h=1}^{L} N_h = N$ and $\left(y_{hi}, x_{hi}\right)$ be the observations of the study variable (y) and the auxiliary variable $\left(x\right)$, on the i^{th} unit of h^{th} stratum, respectively. Let, $\bar{y}_h$ and $\bar{x}_h$ be the sample means of h^{th} stratum corresponding to the population means $\bar{Y}_h$ and $\bar{X}_h$ respectively. In order to obtain approximations to the bias and mean square error *(MSE)* for the proposed estimators under stratified two-phase sampling, let us define notations and expectations for situation-I and situation-II separately as,

Situation-I: When non-response is observed on all the variables taken at second-phase.

Situation-II: When the study variable is observed with non-response at second-phase whereas the two auxiliary variable(s) are observed with complete response.

i) Notations for Situation-I

$$\left.\begin{aligned}
& e_{oh}^{*} = \frac{\bar{y}_h^{*} - \bar{Y}_h}{\bar{Y}_h}, \quad e_{1h}' = \frac{\bar{x}_h' - \bar{X}_h}{\bar{X}_h}, \quad e_{2h}^{*} = \frac{\bar{z}_h^{*} - \bar{Z}_h}{\bar{Z}_h}, \\
& e_o^{*} = \frac{\sum_{h=1}^{L} P_h \bar{Y}_h e_{oh}^{*}}{\bar{Y}}, \quad e_1' = \frac{\sum_{h=1}^{L} P_h \bar{X}_h e_{1h}'}{\bar{X}}, \quad e_2^{*} = \frac{\sum_{h=1}^{L} P_h \bar{Z}_h e_{2h}^{*}}{\bar{Z}} \\
& \bar{y}_h^{*} = \frac{n_{h(1)} \bar{y}_{n_{h(1)}} + n_{h(2)} \bar{y}_{r_h}}{n_h} \quad \bar{x}_h' = \frac{\sum_{i=1}^{n_h'} x_{hi}'}{n_h'} \text{ and } \bar{z}_h^{*} = \frac{n_{h(1)} \bar{z}_{n_{h(1)}} + n_{h(2)} \bar{z}_{r_h}}{n_h} \\
& \lambda_h = \left(\frac{1}{n_h} - \frac{1}{n_h'}\right), \; \lambda_h' = \left(\frac{1}{n_h'} - \frac{1}{N_h}\right) \; \lambda_h^{*} = \left(\frac{k_h - 1}{n_h}\right) W_{h(2)} \quad \Lambda_h = \lambda_h - \lambda_h' \\
& P_h = \frac{N_h}{N} W_{h(2)} = \frac{N_{h(2)}}{N_h} \; S_{hy}^2 = \sum_{i=1}^{N_h} \frac{\left(y_i - \bar{Y}\right)^2}{N_h - 1} \text{ and } S_{hy_{(2)}}^2 = \sum_{i=1}^{N_{h(2)}} \frac{\left(y_i - \bar{Y}_{h(2)}\right)^2}{N_{h(2)} - 1}
\end{aligned}\right\} \quad (1.1)$$

ii) Notations for Situation-II

$$e_{2h} = \frac{\bar{z}_h - \bar{Z}_h}{\bar{Z}_h},\; e_2 = \frac{\sum_{h=1}^{L} P_h \bar{Z}_h e_{2h}}{\bar{Z}} \text{ and } \bar{z}_h = \frac{\sum_{i=1}^{n_h} z_{hi}}{n_h} \quad (1.2)$$

where e_i in (1.1) and (1.2) for $i=\bar{x}, \bar{y}$, and $\bar{z}$ is the sampling error. Further we assume that

$$E(e_o^*) = E(e_1') = E(e_2^*) = 0,$$
and
$$E(e_1') = E(e_2) = 0 \quad (1.3)$$

respectively for Situation-I and Situation-II.

In stratified random sampling without replacement, we may obtain the following expectations respectively for Situation-I and Situation-II of non-response as,

iii) Expectations for Situation-I

$$V_{r,s,t}^{\prime *} = \sum_{h=1}^{L} P_h^{r+s+t} \frac{E\left(\left(\bar{x}_h' - \bar{X}_h\right)^r \left(\bar{y}_h^* - \bar{Y}_h\right)^s \left(\bar{z}_h^* - \bar{Z}_h\right)^t\right)}{\bar{X}^r \bar{Y}^s \bar{Z}^t}$$

$$E(e_o^*)^2 = \frac{1}{\bar{Y}^2} \sum_{h=1}^{L} P_h^2 \left(\lambda_h S_{yh}^2 + \lambda_h^* S_{yh2}^2\right) = V_{020}^* \quad E(e_1')^2 = \frac{1}{\bar{X}^2} \sum_{h=1}^{L} P_h^2 \left(\lambda_h' S_{xh}^2\right) = V_{200}'$$

$$E(e_2^*)^2 = \frac{1}{\bar{Z}^2} \sum_{h=1}^{L} P_h^2 \left(\lambda_h S_{zh}^2 + \lambda_h^* S_{zh2}^2\right) = V_{002}^* \quad E(e_o^*.e_2^*) = \frac{1}{\bar{Y}\bar{Z}} \sum_{h=1}^{L} P_h^2 \left(\lambda_h S_{yzh} + \lambda_h^* S_{yzh2}\right) = V_{011}^*$$

$$E(e_o^*.e_1') = \frac{1}{\bar{Y}\bar{X}} \sum_{h=1}^{L} P_h^2 \left(\lambda_h' S_{yxh}\right) = V_{110}' \quad E(e_1'.e_2^*) = \frac{1}{\bar{Z}\bar{X}} \sum_{h=1}^{L} P_h^2 \left(\lambda_h' S_{xzh}\right) = V_{101}'$$

$$\vartheta_{200}^* = V_{200}^* - V_{200}' \quad \vartheta_{110}^* = V_{110}^* - V_{110}' \quad (1.4)$$

iv) Expectations for Situation-II

$$E(e_o^*)^2 = \frac{1}{\bar{Y}^2} \sum_{h=1}^{l} P_h^2 \left(\lambda_h S_{yh}^2 + \lambda_h^* S_{yh2}^2\right) = V_{020}^* \quad E(e_1')^2 = \frac{1}{\bar{X}^2} \sum_{h=1}^{l} P_h^2 \lambda_h' S_{xh}^2 = V_{200}'$$

$$E(e_2)^2 = \frac{1}{\bar{Z}^2} \sum_{h=1}^{l} P_h^2 \lambda_h S_{zh}^2 = V_{002} \quad E(e_o^*.e_2) = \frac{1}{\bar{Y}\bar{Z}} \sum_{h=1}^{l} P_h^2 \left(\lambda_h S_{yzh} + \lambda_h^* S_{yzh2}\right) = V_{011}^*$$

$$E(e_o^*.e_1') = \frac{1}{\bar{Y}\bar{X}} \sum_{h=1}^{l} P_h^2 \lambda_h' S_{yxh} = V_{110}' \quad E(e_1'.e_2) = \frac{1}{\bar{Z}\bar{X}} \sum_{h=1}^{l} P_h^2 \lambda_h' S_{xzh} = V_{101}'$$

$$\vartheta_{200} = V_{200} - V_{200}' \quad \vartheta_{110} = V_{110} - V_{110}' \quad (1.5)$$

2. NON-RESPONSE IN STRATIFIED TWO-PHASE SAMPLING

In stratified two-phase sampling, a first phase sample of size n'_h from the h^{th} stratum using simple random sampling without replacement (SRSWOR) is selected such that $\sum_{h=1}^{L} n'_h = n'$ and observe auxiliary characteristic(s) for these units. A second phase sample of size n_h $(n_h < n'_h)$ is selected SRSWOR such that $\sum_{h=1}^{L} n_h = n$ and collect information on the study variable say y. From the available second-phase sample n_h, only $n_{h(1)}$ units respond to the survey and $n_{h(2)}$ do not respond. From $n_{h(2)}$ non-respondents, a sub-sample of $r_h \left(r_h = \frac{n_{h(2)}}{k_h}, \;\; k_h > 1 \right)$ units is selected and information are obtained from these r_h units.

A modified form of Hansen and Hurwitz (1946) estimator to the stratified two-phase sampling is given as

$$\bar{y}^*_{st} = \sum_{h=1}^{L} P_h \bar{y}^*_h \tag{2.1}$$

The estimator (2.1) is unbiased with variance,

$$Var(y^*_{st}) = \sum_{h=1}^{L} \lambda_h P_h^2 S_{hy}^2 + \sum_{h=1}^{L} \lambda^*_h P_h^2 S_{hy_2}^2 = \bar{Y}^2 V^*_{020} \tag{2.2}$$

Following Khare and Srivastava (1993) and Tabasum Khan (2004), ratio and product estimators are modified to the stratified two-phase sampling in two different situations of non-response separately as,

i) Situation-I

$$\hat{\bar{Y}}_{Rd} = \bar{y}^*_{st} \frac{\bar{x}'_{st}}{\bar{x}^*_{st}} \quad \text{and} \quad \hat{\bar{Y}}_{Pd} = \bar{y}^*_{st} \frac{\bar{x}^*_{st}}{\bar{x}'_{st}} \tag{2.3}$$

The mean square errors of $\hat{\bar{Y}}_{Rd}$ and $\hat{\bar{Y}}_{Pd}$ in (2.3), up to the first order approximation are,

$$MSE\left(\hat{\bar{Y}}_{Rd}\right) = \bar{Y}^2 \left(V^*_{020} + \vartheta^*_{200} - 2\vartheta^*_{110} \right) \tag{2.4}$$

$$MSE\left(\hat{\bar{Y}}_{Pd}\right) = \bar{Y}^2 \left(V^*_{020} + \vartheta^*_{200} + 2\vartheta^*_{110} \right) \tag{2.5}$$

ii) Situation-II

$$\hat{\bar{Y}}^{\circ}_{Rd} = \bar{y}^*_{st} \frac{\bar{x}'_{st}}{\bar{x}_{st}} \quad \text{and} \quad \hat{\bar{Y}}^{\circ}_{Pd} = \bar{y}^*_{st} \frac{\bar{x}_{st}}{\bar{x}'_{st}} \tag{2.6}$$

The mean square errors of $\hat{\bar{Y}}^{\circ}_{Rd}$ and $\hat{\bar{Y}}^{\circ}_{Pd}$ in (2.6), up to the first order approximation are,

$$MSE\left(\hat{\bar{Y}}^{\circ}_{Rd}\right) = \bar{Y}^2\left(V^*_{020} + \vartheta_{200} - 2\vartheta_{110}\right) \tag{2.7}$$

$$MSE\left(\hat{\bar{Y}}^{\circ}_{Pd}\right) = \bar{Y}^2\left(V^*_{020} + \vartheta_{200} + 2\vartheta_{110}\right) \tag{2.8}$$

In the following section, we propose some generalized estimators of population mean for stratified two-phase sampling with non-response.

3. PROPOSED GENERALIZED EXPONENTIAL-TYPE ESTIMATORS

In this section, motivated by Singh and Kumar (2012), and Upadhyaya et al. (2011), some generalized exponential-type ratio-cum-ratio and product-cum-product estimators have been suggested for two different situations of non-response using two auxiliary variables with known means.

3.1 First Proposed Generalized Estimator for Situation-I

We propose a generalized exponential-type ratio-cum-ratio estimator of population mean for situation-I under stratified two-phase sampling in the presence of non-response as,

$$\hat{\bar{Y}}^G_{RR} = \sum_{h=1}^{l} P_h \bar{y}^*_h \exp\left(\frac{\sum_{h=1}^{l} P_h\left(\bar{X}_h - \bar{x}'_h\right)}{\sum_{h=1}^{l} P_h\left(\bar{X}_h + (a-1)\bar{x}'_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{l} P_h\left(\bar{Z}_h - \bar{z}^*_h\right)}{\sum_{h=1}^{l} P_h\left(\bar{Z}_h + (b-1)\bar{z}^*_h\right)}\right) \tag{3.1}$$

where a and b are some suitably chosen scalars whose values are to be estimated so that MSE of $\hat{\bar{Y}}^G_{RR}$ is minimized.

It is remarked that for various values of a and b in (3.1), we get various exponential-type ratio estimators as deduced family of $\hat{\bar{Y}}^G_{RR}$. From this family some are given in below as,

i) For $a=1$ and $b=1$, $\hat{\bar{Y}}^1_{RR}$ is deduced as the family of $\hat{\bar{Y}}^G_{RR}$ as,

$$\hat{\bar{Y}}^1_{RR} = \sum_{h=1}^{L} P_h \bar{y}^*_h \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{X}_h - \bar{x}'_h\right)}{\sum_{h=1}^{L} P_h \bar{X}_h}\right)\exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{Z}_h - \bar{z}^*_h\right)}{\sum_{h=1}^{L} P_h \bar{Z}_h}\right) \tag{3.2}$$

ii) For $a=2$ and $b=1$, $\hat{\bar{Y}}^2_{RR}$ is deduced as the family of $\hat{\bar{Y}}^G_{RR}$ as,

$$\hat{\bar{Y}}^2_{RR} = \sum_{h=1}^{L} P_h \bar{y}^*_h \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{X}_h - \bar{x}'_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{X}_h + \bar{x}'\right)}\right)\exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{Z}_h - \bar{z}^*_h\right)}{\sum_{h=1}^{L} P_h \bar{Z}_h}\right) \tag{3.3}$$

iii) For $a=2$ and $b=2$, $\hat{\bar{Y}}_{RR}^{A}$ is deduced as Singh and Kumar (2012) type estimator as the family of $\hat{\bar{Y}}_{RR}^{G}$ as,

$$\hat{\bar{Y}}_{RR}^{A} = \sum_{h=1}^{L} P_h \bar{y}_h^{*} \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{X}_h - \bar{x}'_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{X}_h + \bar{x}'_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{Z}_h - \bar{z}_h^{*}\right)}{\sum_{h=1}^{L} P_h\left(\bar{Z}_h + \bar{z}_h^{*}\right)}\right) \quad (3.4)$$

Using (1.1) the estimator in (3.1) may be expressed as,

$$\hat{\bar{Y}}_{RR}^{G} \approx \bar{Y}(1+e_0^{*}) \exp\left[-e'_{1G} + (a-1)e'^{2}_{1G}\right] \exp\left[-e_{2G}^{*} + (b-1)e_{2G}^{*2}\right] \quad (3.5)$$

Expanding the right hand side of (3.5) up to the second order of approximation, we have

$$\hat{\bar{Y}}_{RR}^{G} - \bar{Y} \approx \bar{Y}\left[e_0^{*} - e'_{1G} - e_{2G}^{*} + (a-1)e'^{2}_{1G} + (b-1)e_{2G}^{*2} - e_0^{*}e'_{1G} - e_0^{*}e_{2G}^{*} + e'_1 e_{2G}^{*}\right] \quad (3.6)$$

Using (3.6) the bias and the *MSE* equations of $\hat{\bar{Y}}_{RR}^{G}$ are obtained as,

$$\text{Bias}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{RR}}^{\mathrm{G}}\right) \approx \bar{Y}\left[\left(\frac{a-1}{a^2}\right)V'_{200} + \left(\frac{b-1}{b^2}\right)V_{002}^{*} - \frac{1}{a}V'_{110} - \frac{1}{b}V_{011}^{*} + \frac{1}{ab}V'_{101}\right] \quad (3.7)$$

$$MSE\left(\hat{\bar{Y}}_{RR}^{G}\right) \approx \bar{Y}^2\left[V_{020}^{*} + \frac{1}{a^2}V'_{200} + \frac{1}{b^2}V_{002}^{*} - 2\left(\frac{1}{a}V'_{110} + \frac{1}{b}V_{011}^{*} - \frac{1}{ab}V'_{101}\right)\right] \quad (3.8)$$

In order to obtain minimum MSE of $\hat{\bar{Y}}_{RR}^{G}$, we differentiate MSE ($\hat{\bar{Y}}_{RR}^{G}$) in (3.8) with respect to a and b respectively. The optimum value of a and b are obtained as

$$a = \frac{\left(V'_{200}V_{002}^{*} - V'^{2}_{101}\right)}{\left(V'_{110}V_{002}^{*} - V'_{101}V_{011}^{*}\right)} \quad \text{and} \quad b = \frac{\left(V'_{200}V_{002}^{*} - V'^{2}_{101}\right)}{\left(V'_{200}V_{011}^{*} - V'_{101}V'_{110}\right)} \quad (3.9)$$

Substitution for the optimal values of a and b in (3.9) yields the minimum value of MSE($\hat{\bar{Y}}_{RR}^{G}$) as

$$\min.MSE\left(\hat{\bar{Y}}_{RR}^{G}\right) \approx \bar{Y}^2\left[V_{020}^{*} - \left(\frac{V_{002}^{*}V'^{2}_{110} + V'_{200}V_{011}^{*2} - 2V'_{101}V_{011}^{*}V'_{110}}{V'_{200}V_{002}^{*} - V'^{2}_{101}}\right)\right] \quad (3.10)$$

The bias and *MSE* expressions for deduced family $\hat{\bar{Y}}_{RR}^{G}$ may be obtained by putting the values of a and b in (3.7) and (3.8) respectively as

i) For $a=1$ and $b=1$, the bias and MSE of $\hat{\bar{Y}}_{RR}^{1}$ is obtained as

$$\mathrm{Bias}\left(\hat{\bar{Y}}_{RR}^{1}\right) \approx \bar{Y}\left(-V'_{110} - V^{*}_{011} + V'_{101}\right) \quad (3.11)$$

$$MSE\left(\hat{\bar{Y}}_{RR}^{1}\right) \approx \bar{Y}^{2}\left[V^{*}_{020} + V'_{200} + V^{*}_{002} - 2\left(V'_{110} + V^{*}_{011} - V'_{101}\right)\right] \quad (3.12)$$

ii) For $a=2$ and $b=1$, the bias and MSE of $\hat{\bar{Y}}_{RR}^{2}$ is obtained as

$$\mathrm{Bias}\left(\hat{\bar{Y}}_{RR}^{2}\right) \approx \bar{Y}\left(\frac{1}{4}V'_{200} - \frac{1}{2}V'_{110} - V^{*}_{011} + \frac{1}{2}V'_{101}\right) \quad (3.13)$$

$$MSE\left(\hat{\bar{Y}}_{RR}^{2}\right) \approx \bar{Y}^{2}\left[V^{*}_{020} + \frac{1}{4}V'_{200} + V^{*}_{002} - 2\left(\frac{1}{2}V'_{110} + V^{*}_{011} - \frac{1}{2}V'_{101}\right)\right] \quad (3.14)$$

iii) For $a=2$ and $b=2$, the bias and MSE of

$$\mathrm{Bias}\left(\hat{\bar{Y}}_{RR}^{A}\right) \approx \bar{Y}\left[\frac{1}{4}\left(V'_{200} + V^{*}_{002}\right) - \frac{1}{2}\left(V'_{110} + V^{*}_{011} - \frac{V'_{101}}{2}\right)\right] \quad (3.15)$$

$$MSE\left(\hat{\bar{Y}}_{RR}^{A}\right) \approx \bar{Y}^{2}\left[V^{*}_{020} + \frac{1}{4}\left(V'_{200} + V^{*}_{002}\right) - V'_{110} - V^{*}_{011} + \frac{1}{2}V'_{101}\right] \quad (3.16)$$

3.2 Second Proposed Generalized Estimator for Situation-I

We propose another generalized exponential-type product-cum-product estimator of population mean for situation-I under stratified two-phase sampling as,

$$\hat{\bar{Y}}_{PP}^{G} = \sum_{h=1}^{L} P_h \bar{y}_h^{*} \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h - \bar{X}_h\right)}{\sum_{h=1}^{L} P_h\left((c-1)\bar{x}'_h + \bar{X}_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h^{*} - \bar{Z}_h\right)}{\sum_{h=1}^{L} P_h\left((d-1)\bar{z}_h^{*} + \bar{Z}_h\right)}\right) \quad (3.17)$$

where c and d are some suitably chosen scalars whose values are to be estimated so that MSE of $\hat{\bar{Y}}_{PP}^{G}$ is minimized.

It is remarked that for various values of c and d in (3.17), we get various exponential-type ratio estimators as deduced family of $\hat{\bar{Y}}_{PP}^{G}$. From this family some are given in below as,

i) For $c=1$ and $d=1$, $\hat{\bar{Y}}_{PP}^{1}$ is deduced as the family of $\hat{\bar{Y}}_{PP}^{G}$ as

$$\hat{\bar{Y}}_{PP}^{1}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'-\bar{X}_h\right)}{\sum_{h=1}^{L}P_h\bar{X}_h}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}-\bar{Z}_h\right)}{\sum_{h=1}^{L}P_h\bar{Z}_h}\right) \tag{3.18}$$

ii) For $c=2$ and $d=1$, $\hat{\bar{Y}}_{PP}^{2}$ is deduced as the family of $\hat{\bar{Y}}_{PP}^{G}$ as

$$\hat{\bar{Y}}_{PP}^{2}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'-\bar{X}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{X}_h+\bar{x}'\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}+\bar{Z}_h\right)}{\sum_{h=1}^{L}P_h\bar{Z}_h}\right) \tag{3.19}$$

iii) For $c=2$ and $d=2$, $\hat{\bar{Y}}_{PP}^{3}$ is deduced as the family of $\hat{\bar{Y}}_{PP}^{G}$ as,

$$\hat{\bar{Y}}_{PP}^{A}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'-\bar{X}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h'+\bar{X}_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}-\bar{Z}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}+\bar{Z}_h\right)}\right) \tag{3.20}$$

Using (1.1) the estimator (3.17) may be expressed as

$$\hat{\bar{Y}}_{PP}^{G}\approx\bar{Y}(1+e_0^{*})\exp\left[e_{1G}'+(c-1)e_{1G}'^{2}\right]\exp\left[e_{2G}^{*}+(d-1)e_{2G}^{*2}\right] \tag{3.21}$$

Expanding the right hand side of (3.21) up to the second order of approximation, we have

$$\hat{\bar{Y}}_{PP}^{G}-\bar{Y}\approx\bar{Y}\left[e_0^{*}+e_{1G}'+e_{2G}-(c-1)e_{1G}'^{2}-(d-1)e_{2G}^{2}+e_0^{*}e_{1G}'+e_0^{*}e_{2G}^{*}+e_1'e_{2G}^{*}\right] \tag{3.22}$$

Using (3.22) the bias and *MSE* equations of $\hat{\bar{Y}}_{PP}^{G}$ are obtained as,

$$\text{Bias}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{PP}}^{\mathrm{G}}\right)\approx-\bar{Y}\left[\left(\frac{c-1}{c}\right)V_{200}'+\left(\frac{d-1}{d^{2}}\right)V_{002}^{*}-\frac{1}{c}V_{110}'-\frac{1}{d}V_{011}^{*}+\frac{1}{cd}V_{101}'\right] \tag{3.23}$$

$$MSE\left(\hat{\bar{Y}}_{PP}^{G}\right)\approx\bar{Y}^{2}\left[V_{020}^{*}+\frac{1}{c^{2}}V_{200}'+\frac{1}{d^{2}}V_{002}^{*}+2\left(\frac{1}{c}V_{110}'+\frac{1}{d}V_{011}^{*}+\frac{1}{cd}V_{101}'\right)\right] \tag{3.24}$$

In order to obtain minimum MSE of $\hat{\bar{Y}}_{PP}^{G}$, we differentiate *MSE* ($\hat{\bar{Y}}_{PP}^{G}$) in (3.24) with respect to c and d respectively. The optimum value of c and d are obtained as

$$c = -\frac{R_x\left(V'_{200}V^*_{002} - V'^2_{101}\right)}{\left(V'_{110}V^*_{002} - V'_{101}V^*_{011}\right)} \quad \text{and} \qquad d = -\frac{R_z\left(V'_{200}V^*_{002} - V'^2_{101}\right)}{\left(V'_{200}V^*_{011} - V'_{101}V'_{110}\right)} \tag{3.25}$$

Substitution for the optimal values of c and d in (3.24) yields the minimum value of $MSE(\hat{\bar{Y}}^G_{PP})$ as

$$\min.MSE\left(\hat{\bar{Y}}^G_{PP}\right) \approx \bar{Y}^2\left[V^*_{020} - \left(\frac{V^*_{002}V'^2_{110} + V'_{200}V^{*2}_{011} - 2V'_{101}V^*_{011}V'_{110}}{V'_{200}V^*_{002} - V'^2_{101}}\right)\right] \tag{3.26}$$

The bias and *MSE* expressions for deduced family $\hat{\bar{Y}}^G_{PP}$ may be obtained by putting the values of c and d in (3.23) and (3.24) respectively as

i) For $c = 1$ and $d = 1$, the bias and *MSE* of $\hat{\bar{Y}}^1_{PP}$ is obtained as,

$$\text{Bias}\left(\hat{\bar{Y}}^1_{PP}\right) \approx \bar{Y}\left(-V'_{110} - V^*_{011} + V'_{101}\right) \tag{3.27}$$

$$MSE\left(\hat{\bar{Y}}^1_{PP}\right) \approx \bar{Y}^2\left(V^*_{020} + V'_{200} + V^*_{002} + 2\left(V'_{110} + V^*_{011} + V'_{101}\right)\right) \tag{3.28}$$

ii) For $c = 2$ and $d = 1$, the bias and *MSE* of $\hat{\bar{Y}}^2_{PP}$ is obtained as,

$$\text{Bias}\left(\hat{\bar{Y}}^2_{PP}\right) \approx \bar{Y}\left(\frac{1}{4}V'_{200} - \frac{1}{2}V'_{110} - V^*_{011} + \frac{1}{2}V'_{101}\right) \tag{3.29}$$

$$MSE\left(\hat{\bar{Y}}^2_{PP}\right) \approx \bar{Y}^2\left(V^*_{020} + \frac{1}{4}V'_{200} + V^*_{002} + 2\left(\frac{1}{2}V'_{110} + V^*_{011} + \frac{1}{2}V'_{101}\right)\right) \tag{3.30}$$

iii) For $c = 2$ and $d = 2$, the bias and *MSE* of $\hat{\bar{Y}}^A_{PP}$ is obtained as,

$$\text{Bias}\left(\hat{\bar{Y}}^A_{PP}\right) \approx -\bar{Y}\left(\frac{1}{4}\left(V'_{200} + V^*_{002}\right) - \frac{1}{2}\left(V'_{110} + V^*_{011} + \frac{V'_{101}}{2}\right)\right) \tag{3.31}$$

$$MSE\left(\hat{\bar{Y}}^A_{PP}\right) \approx \bar{Y}^2\left(V^*_{020} + \frac{1}{4}\left(V'_{200} + V^*_{002}\right) + V'_{110} + V^*_{011} + \frac{1}{2}V'_{101}\right) \tag{3.32}$$

3.3 First Proposed Generalized Estimator for Situation-II

We propose a generalized exponential-type estimator of population mean for situation-II under stratified two-phase sampling as,

$$\hat{\bar{Y}}_{RR}^{G\circ} = \sum_{h=1}^{l} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{l} P_h \left(\bar{X}_h - \bar{x}_h'\right)}{\sum_{h=1}^{l} P_h \left(\bar{X}_h + \left(a^\circ - 1\right)\bar{x}_h'\right)} \right) \exp\left(\frac{\sum_{h=1}^{l} P_h \left(\bar{Z}_h - \bar{z}_h\right)}{\sum_{h=1}^{l} P_h \left(\bar{Z}_h + (b^\circ - 1)\bar{z}_h\right)} \right) \quad (3.33)$$

where a° and b° are some suitably chosen scalars whose values are to be estimated so that *MSE* of $\hat{\bar{Y}}_{RR}^{G\circ}$ is minimized.

It is remarked that for various values of a° and b° in (3.33), we get various exponential-type ratio estimators as deduced family of $\hat{\bar{Y}}_{RR}^{G\circ}$. From this family some are given in below as,

i) For $a^\circ = 1$ and $b^\circ = 1$, $\hat{\bar{Y}}_{RR}^{1\circ}$ is deduced as the family of $\hat{\bar{Y}}_{RR}^{G\circ}$ as

$$\hat{\bar{Y}}_{RR}^{1\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{X}_h - \bar{x}_h'\right)}{\sum_{h=1}^{L} P_h \bar{X}_h} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{Z}_h - \bar{z}_h^*\right)}{\sum_{h=1}^{L} P_h \bar{Z}_h} \right) \quad (3.34)$$

ii) For $a^\circ = 2$ and $b^\circ = 1$, $\hat{\bar{Y}}_{RR}^{2\circ}$ is deduced as the family of $\hat{\bar{Y}}_{RR}^{G\circ}$ as

$$\hat{\bar{Y}}_{RR}^{2\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{X}_h - \bar{x}_h'\right)}{\sum_{h=1}^{L} P_h \left(\bar{X}_h + \bar{x}'\right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{Z}_h - \bar{z}_h^*\right)}{\sum_{h=1}^{L} P_h \bar{Z}_h} \right) \quad (3.35)$$

iii) For $a^\circ = 2$ and $b^\circ = 2$, $\hat{\bar{Y}}_{RR}^{A\circ}$ is deduced as Singh and Kumar (2012) as the family of $\hat{\bar{Y}}_{RR}^{G\circ}$ for situation-II.

$$\hat{\bar{Y}}_{RR}^{A\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{X}_h - \bar{x}_h'\right)}{\sum_{h=1}^{L} P_h \left(\bar{X}_h + \bar{x}_h'\right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{Z}_h - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h \left(\bar{Z}_h + \bar{z}_h\right)} \right) \quad (3.36)$$

The proposed estimator in (3.33) follows naturally in exactly the same fashion as that for Situation-I in Section 3.1. In addition, the relation between a° and b° is the same as that for Situation-I in Section 3.1. Finally, the same is true for the MSE and the bias. It is therefore directly from Section 3.1 we may have the bias and the MSE equations of $\hat{\bar{Y}}_{RR}^{G\circ}$ are obtained as,

$$\text{Bias}\left(\hat{\bar{Y}}_{RR}^{G\circ}\right) \approx \bar{Y}\left[\left(\frac{a^\circ - 1}{a^{\circ 2}}\right) V_{200}' + \left(\frac{b^\circ - 1}{b^{\circ 2}}\right) V_{002} - \frac{1}{a^\circ} V_{110}' - \frac{1}{b^\circ} V_{011} + \frac{1}{a^\circ b^\circ} V_{101}' \right] \quad (3.37)$$

$$MSE\left(\hat{\bar{Y}}_{RR}^{G\circ}\right) \approx \bar{Y}^2\left[V_{020}^* + \frac{1}{a^{\circ 2}}V'_{200} + \frac{1}{b^{\circ 2}}V_{002} - 2\left(\frac{1}{a^\circ}V'_{110} + \frac{1}{b^\circ}V_{011} - \frac{1}{a^\circ b^\circ}V'_{101}\right)\right] \tag{3.38}$$

$MSE\left(\hat{\bar{Y}}_{RR}^{G\circ}\right)$ is minimized for the optimal values of a° and b° given as

$$a^\circ = \frac{\left(V'_{200}V_{002} - V'^2_{101}\right)}{\left(V'_{110}V_{002} - V'_{101}V_{011}\right)} \quad \text{and} \quad b^\circ = \frac{\left(V'_{200}V_{002} - V'^2_{101}\right)}{\left(V'_{200}V_{011} - V'_{101}V'_{110}\right)} \tag{3.39}$$

Substitution for the optimal values of a° and b° in (3.38) yields the minimum value of $MSE\left(\hat{\bar{Y}}_{RR}^{G\circ}\right)$ as

$$\min .MSE\left(\hat{\bar{Y}}_{RR}^{G\circ}\right) \approx V_{020}^* - \left(\frac{V_{002}V'^2_{110} + V'_{200}V^2_{011} - 2V'_{101}V_{011}V'_{110}}{V'_{200}V_{002} - V'^2_{101}}\right) \tag{3.40}$$

The bias and *MSE* expressions for deduced family $\hat{\bar{Y}}_{RR}^{G\circ}$ may be obtained by putting the values of a° and b° in (3.37) and (3.38) respectively.

3.4 Second Proposed Generalized Estimator for Situation-II

We propose another generalized exponential-type product-cum-product estimator of population mean for situation-II as

$$\hat{\bar{Y}}_{PP}^{G\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h - \bar{X}_h\right)}{\sum_{h=1}^{L} P_h\left(\left(c^\circ - 1\right)\bar{x}'_h + \bar{X}_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h - \bar{Z}_h\right)}{\sum_{h=1}^{L} P_h\left((d^\circ - 1)\bar{z}_h + \bar{Z}_h\right)}\right) \tag{3.41}$$

It is remarked that for various values of c° and d° in (3.41), we get various exponential-type ratio estimators as deduced family of $\hat{\bar{Y}}_{PP}^{G\circ}$. From this family some are given in below as,

i) For $c^\circ = 1$ and $d^\circ = 1$, $\hat{\bar{Y}}_{PP}^{1\circ}$ is deduced as the family of $\hat{\bar{Y}}_{PP}^{G\circ}$ as

$$\hat{\bar{Y}}_{PP}^{1\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h - \bar{X}_h\right)}{\sum_{h=1}^{L} P_h \bar{X}_h}\right)\exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h^* - \bar{Z}_h\right)}{\sum_{h=1}^{L} P_h \bar{Z}_h}\right) \tag{3.42}$$

ii) For $c^{\circ}=2$ and $d^{\circ}=1$, $\hat{\bar{Y}}_{PP}^{2\circ}$ is deduced as the family of $\hat{\bar{Y}}_{PP}^{G\circ}$ as

$$\hat{\bar{Y}}_{PP}^{2\circ}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'-\bar{X}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{X}_h+\bar{x}'\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}+\bar{Z}_h\right)}{\sum_{h=1}^{L}P_h\bar{Z}_h}\right) \tag{3.43}$$

iii) For $c^{\circ}=2$ and $d^{\circ}=2$, $\hat{\bar{Y}}_{PP}^{A\circ}$ is deduced as Singh and Kumar (2012) as the family of $\hat{\bar{Y}}_{RR}^{G\circ}$ for situation-II.

$$\hat{\bar{Y}}_{PP}^{A\circ}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'-\bar{X}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h'+\bar{X}_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h-\bar{Z}_h\right)}{\sum_{h=1}^{L}P_h\left(\bar{z}_h+\bar{Z}_h\right)}\right) \tag{3.44}$$

The proposed estimator in (3.41) follows naturally in exactly the same fashion as that for Situation-I in Section 3.2. In addition, the relation between c° and d° is the same as that for Situation-I in Section 3.2. Finally, the same is true for the MSE and the bias. It is therefore directly from Section 3.2, we may have the bias and the MSE equations of $\hat{\bar{Y}}_{PP}^{G\circ}$ are obtained as,

$$\text{Bias}\left(\hat{\bar{\text{Y}}}_{\text{PP}}^{\text{G}\circ}\right)\approx-\bar{Y}\left[\left(\frac{c^{\circ}-1}{c^{\circ 2}}\right)V_{200}'+\left(\frac{d^{\circ}-1}{d^{\circ 2}}\right)V_{002}-\frac{1}{c^{\circ}}V_{110}'-\frac{1}{d^{\circ}}V_{011}+\frac{1}{c^{\circ}d^{\circ}}V_{101}'\right] \tag{3.45}$$

$$MSE\left(\hat{\bar{Y}}_{PP}^{G\circ}\right)\approx\bar{Y}^2\left[V_{020}^{*}+\frac{1}{c^{\circ 2}}V_{200}'+\frac{1}{d^{\circ 2}}V_{002}-2\left(\frac{1}{c^{\circ}}V_{110}'+\frac{1}{d^{\circ}}V_{011}-\frac{1}{c^{\circ}d^{\circ}}V_{101}'\right)\right] \tag{3.46}$$

In order to obtain minimum MSE of $\hat{\bar{Y}}_{PP}^{G\circ}$, we differentiate $MSE\left(\hat{\bar{Y}}_{PP}^{G\circ}\right)$ in (3.46) with respect to c° and d° respectively. The optimum value of c° and d° are obtained as

$$c^{\circ}=-\frac{R_x\left(V_{200}'V_{002}-V_{101}'^{2}\right)}{\left(V_{110}'V_{002}-V_{101}'V_{011}\right)}\text{ and}\qquad\text{and } \mathrm{d}^{\circ}=-\frac{R_z\left(V_{200}'V_{002}-V_{101}'^{2}\right)}{\left(V_{200}'V_{011}-V_{101}'V_{110}'\right)} \tag{3.47}$$

Substitution for the optimal values of c° and d° in (3.45) yields the minimum value of $MSE\left(\hat{\bar{Y}}_{PP}^{G\circ}\right)$ as

$$\min.MSE(\hat{\bar{Y}}_{PP}^{G\circ})\approx\bar{Y}^2\left[V_{020}^{*}-\left(\frac{V_{002}V_{110}'^{2}+V_{200}'V_{011}^{2}-2V_{101}'V_{011}V_{110}'}{V_{200}'V_{002}-V_{101}'^{2}}\right)\right] \tag{3.48}$$

The bias and *MSE* expressions for deduced family $\hat{\bar{Y}}_{PP}^{G\circ}$ may be obtained by putting the values of c° and d° in (3.45) and (3.46) respectively.

4. EFFICIENCY COMPARISONS

Now we compare the proposed generalized estimators with usual Hansen and Hurwitz's (1946) unbiased estimator $\bar{y}_{st}^{*}$ and some other considered estimators under two different cases as

i) Ratio-cum-ratio Estimators under Situation-I

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{1}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{V'_{200}+V_{200}^{*}}{2\left(V'_{110}+V_{011}^{*}-V'_{101}\right)}<1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{1}) < MSE(\bar{Y}_{Rd}^{*}) \\ \text{if} & \dfrac{\left(V'_{200}+V_{002}^{*}\right)-\left(\vartheta_{200}^{*}-2\vartheta_{110}^{*}\right)}{2\left(V'_{110}+V_{011}^{*}-V'_{101}\right)} \end{array} \right\rangle \tag{4.1}$$

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{A}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{V'_{200}+V_{200}^{*}}{2\left(2V'_{110}+2V_{011}^{*}-V'_{101}\right)}<1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{A}) < MSE(\bar{Y}_{Rd}^{*}) \\ \text{if} & \dfrac{\left(V'_{200}+V_{002}^{*}\right)-4\left(\vartheta_{200}^{*}-2\vartheta_{110}^{*}\right)}{2\left(2V'_{110}+2V_{011}^{*}-V'_{101}\right)}<1 \end{array} \right\rangle \tag{4.2}$$

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{2}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{V'_{200}+4V_{200}^{*}}{4\left(V'_{110}+2V_{011}^{*}-V'_{101}\right)}<1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{2}) < MSE(\bar{Y}_{Rd}^{*}) \\ \text{if} & \dfrac{\left(V'_{200}+4V_{002}^{*}\right)-4\left(\vartheta_{200}^{*}-2\vartheta_{110}^{*}\right)}{4\left(V'_{110}+2V_{011}^{*}-V'_{101}\right)}<1 \end{array} \right\rangle \tag{4.3}$$

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{G}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{2V'_{101}V^{*}_{011}V'_{110}}{4\left(V'^{2}_{110}V^{*}_{002}+V'_{200}V^{*2}_{011}\right)} < 1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{RR}^{G}) < MSE(\bar{Y}_{Rd}^{*}) \\ \text{if} & \dfrac{\left(\vartheta^{*}_{200}-2\vartheta^{*}_{110}\right)\left(V'^{2}_{101}-V'_{200}V^{*}_{002}\right)}{\left(V^{*}_{002}V'^{2}_{110}+V'_{200}V^{*2}_{011}-2V'_{101}V'_{110}V^{*}_{011}\right)} < 1 \end{array} \right\rangle \qquad (4.4)$$

ii) Product-cum-product Estimators under Situation-I

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{1}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{-\left(V'_{200}+V^{*}_{200}\right)}{2\left(V'_{110}+V^{*}_{011}+V'_{101}\right)} > 1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{1}) < MSE(\bar{Y}_{Pd}^{*}) \\ \text{if} & \dfrac{\left(\vartheta^{*}_{200}+2\vartheta^{*}_{110}\right)-\left(V'_{200}+V^{*}_{002}\right)}{2\left(V'_{110}+V^{*}_{011}+V'_{101}\right)} > 1 \end{array} \right\rangle \qquad (4.5)$$

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{A}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{-\left(V'_{200}+V^{*}_{200}\right)}{2\left(2V'_{110}+2V^{*}_{011}+V'_{101}\right)} > 1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{A}) < MSE(\bar{Y}_{Pd}^{*}) \\ \text{if} & \dfrac{4\left(\vartheta^{*}_{200}+2\vartheta^{*}_{110}\right)-\left(V'_{200}+V^{*}_{002}\right)}{2\left(2V'_{110}+2V^{*}_{011}+V'_{101}\right)} > 1 \end{array} \right\rangle \qquad (4.6)$$

$$\left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{2}) < MSE(\bar{y}_{st}^{*}) \\ \text{if} & \dfrac{-\left(V'_{200}+4V^{*}_{200}\right)}{4\left(V'_{110}+2V^{*}_{011}-V'_{101}\right)} > 1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \begin{array}{ll} & MSE(\hat{\bar{Y}}_{PP}^{2}) < MSE(\bar{Y}_{Pd}^{*}) \\ \text{if} & \dfrac{4\left(\vartheta^{*}_{200}-2\vartheta^{*}_{110}\right)-\left(V'_{200}+4V^{*}_{002}\right)}{4\left(V'_{110}+2V^{*}_{011}+V'_{101}\right)} > 1 \end{array} \right\rangle \qquad (4.7)$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{G}_{PP}) < MSE(\bar{y}^{*}_{st}) \\ \dfrac{V'_{101}V^{*}_{011}V'_{110}}{2\left(V'^{2}_{110}V^{*}_{002}+V'_{200}V^{*2}_{011}\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{G}_{PP}) < MSE(\bar{Y}^{*}_{Pd}) \\ \dfrac{\left(\vartheta^{*}_{200}+2\vartheta^{*}_{110}\right)\left(V'^{2}_{101}-V'_{200}V^{*}_{002}\right)}{\left(V^{*}_{002}V'^{2}_{110}+V'_{200}V^{*2}_{011}-2V'_{101}V'_{110}V^{*}_{011}\right)} < 1 \end{array} \right\rangle \quad (4.8)$$

iii) Ratio-cum-ratio Estimators under Situation-II

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{1}_{RR}) < MSE(\bar{y}^{*}_{st}) \\ \dfrac{V'_{200}+V_{200}}{2\left(V'_{110}+V_{011}-V'_{101}\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{1}_{RR}) < MSE(\bar{Y}^{*}_{Rd}) \\ \dfrac{\left(V'_{200}+V_{002}\right)-\left(\vartheta_{200}-2\vartheta_{110}\right)}{2\left(V'_{110}+V_{011}-V'_{101}\right)} \end{array} \right\rangle \quad (4.9)$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{A}_{RR}) < MSE(\bar{y}^{*}_{st}) \\ \dfrac{V'_{200}+V_{200}}{2\left(2V'_{110}+2V_{011}-V'_{101}\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{A}_{RR}) < MSE(\bar{Y}^{*}_{Rd}) \\ \dfrac{\left(V'_{200}+V_{002}\right)-4\left(\vartheta_{200}-2\vartheta_{110}\right)}{2\left(2V'_{110}+2V_{011}-V'_{101}\right)} < 1 \end{array} \right\rangle \quad (4.10)$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{2}_{RR}) < MSE(\bar{y}^{*}_{st}) \\ \dfrac{V'_{200}+4V_{200}}{4\left(V'_{110}+2V_{011}-V'_{101}\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}^{2}_{RR}) < MSE(\bar{Y}^{*}_{Rd}) \\ \dfrac{\left(V'_{200}+4V_{002}\right)-4\left(\vartheta_{200}-2\vartheta_{110}\right)}{4\left(V'_{110}+2V_{011}-V'_{101}\right)} < 1 \end{array} \right\rangle \quad (4.11)$$

$$\left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{RR}^{G}) < MSE(\bar{y}_{st}^{*}) \\ \dfrac{V'_{101}V_{011}V'_{110}}{2\left(V'^{2}_{110}V_{002}+V'_{200}V^{2}_{011}\right)}<1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{RR}^{G}) < MSE(\bar{Y}_{Rd}^{*}) \\ \dfrac{\left(\vartheta_{200}-2\vartheta_{110}\right)\left(V'^{2}_{101}\text{-}V'_{200}V_{002}\right)}{\left(V_{002}V'^{2}_{110}+V'_{200}V^{2}_{011}-2V'_{101}V'_{110}V_{011}\right)}<1 \end{array} \right\rangle \tag{4.12}$$

iv) Product-cum-product Estimators under Situation-II

$$\left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{1}) < MSE(\bar{y}_{st}^{*}) \\ \dfrac{-\left(V'_{200}+V_{200}\right)}{2\left(V'_{110}+V_{011}+V'_{101}\right)}>1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{1}) < MSE(\bar{Y}_{Pd}^{*}) \\ \dfrac{\left(\vartheta_{200}+2\vartheta_{110}\right)-\left(V'_{200}+V_{002}\right)}{2\left(V'_{110}+V_{011}+V'_{101}\right)}>1 \end{array} \right\rangle \tag{4.13}$$

$$\left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{A}) < MSE(\bar{y}_{st}^{*}) \\ \dfrac{-\left(V'_{200}+V_{200}\right)}{2\left(2V'_{110}+2V_{011}+V'_{101}\right)}>1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{A}) < MSE(\bar{Y}_{Pd}^{*}) \\ \dfrac{4\left(\vartheta_{200}+2\vartheta_{110}\right)-\left(V'_{200}+V_{002}\right)}{2\left(2V'_{110}+2V_{011}+V'_{101}\right)}>1 \end{array} \right\rangle \tag{4.14}$$

$$\left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{2}) < MSE(\bar{y}_{st}^{*}) \\ \dfrac{-\left(V'_{200}+4V_{200}\right)}{4\left(V'_{110}+2V_{011}-V'_{101}\right)}>1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \text{if} \quad \begin{array}{l} MSE(\hat{\bar{Y}}_{PP}^{2}) < MSE(\bar{Y}_{Pd}^{*}) \\ \dfrac{4\left(\vartheta_{200}-2\vartheta_{110}\right)-\left(V'_{200}+4V_{002}\right)}{4\left(V'_{110}+2V_{011}+V'_{101}\right)}>1 \end{array} \right\rangle \tag{4.15}$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}_{PP}^{G}) < MSE(\bar{y}_{st}^{*}) \\ \dfrac{V'_{101}V_{011}V'_{110}}{2\left(V'^{2}_{110}V_{002} + V'_{200}V^{2}_{011}\right)} < 1 \end{array} \right\rangle$$

$$\text{and} \quad \left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\bar{Y}}_{PP}^{G}) < MSE(\bar{Y}_{Pd}^{*}) \\ \dfrac{\left(\vartheta_{200} + 2\vartheta_{110}\right)\left(V'^{2}_{101} - V'_{200}V_{002}\right)}{\left(V_{002}V'^{2}_{110} + V'_{200}V^{2}_{011} - 2V'_{101}V'_{110}V_{011}\right)} < 1 \end{array} \right\rangle \tag{4.16}$$

5. EMPIRICAL STUDY

In order to examine the performance of proposed estimators under stratified two-phase sampling, we have taken two different stratified populations:

Population-I: (source: Koyuncu and Kadilar (2009))

We consider No. of teachers as study variable *(Y)*, No. of students as auxiliary variable *(X)*, and No. of classes in primary and secondary schools as another auxiliary variable *(Z)* for 923 districts at six 6 regions (1: Marmara, 2: Agean, 3: Mediterranean, 4: Central Anatolia, 5: Black Sea, and 6: East and Southeast Anatolia) in Turkey in 2007.

Population-II: (source: detailed livelihood assessment of flood affected districts of Pakistan September 2011, Food Security Cluster, Pakistan)

We consider food expenditure as study variable *(Y)*, household earn as auxiliary variable *(X)*, and total expenditure in May (2011) as another auxiliary variable *(Z)* for (6940) male and (1678) female households in flood affected districts of Pakistan.

The Neyman allocation has been used for allocating the samples to different strata. The comparison of proposed generalized exponential-type ratio-cum-ratio and exponential-type product-product estimators have been made with respect to Hansen and Hurwitz's (1946), stratified two-phase ratio and stratified two-phase product estimators. The information for two populations is given in appendix-A Table-5.

6. CONCLUSION

The MSE values of the estimators are computed in Table 3-4 using (2.2), (2.4), (2.5), (2.7), (2.8), (3.10), (3.12), (3.14), (3.16), (3.26), (3.28), (3.30), (3.32), (3.38), (3.40), (3.46), and (3.48). Table 1-2 indicates that the performance of exponential-type ratio-cum-ratio estimators in both situations is better than modified stratified two-phase due to Hansen and Hurwitz's (1946) estimator $\bar{y}_{st}^{*}$. The generalized exponential-type ratio-cum-ratio estimators are more efficient as compared to ratio estimators $\left(\hat{\bar{Y}}_{Rd}\, , \, \hat{\bar{Y}}_{Rd}^{\circ}\right)$ modified to the stratified two-phase for both situations of non-response. Furthermore, it is observed that proposed estimators are more efficient than ratio estimators for both situations of non-response $\left(\hat{\bar{Y}}_{Rd}\, , \, \hat{\bar{Y}}_{Rd}^{\circ}\right)$.

The *PREs* for the proposed family exponential-type estimators increases if inverse sampling rate k_h increases from 2.0 to 3.5 at $W_h = 20\%$ and $W_h = 30\%$ but this is not true for $\hat{\bar{Y}}_{RR}^{A}$. For $W_h = 10\%$, *PREs* of the estimators decreases in situation-I of non-response. In situation-II of non-response, the *PREs* for Singh and Kumar (2012) type estimators including stratified two-phase ratio estimator decreases if the inverse sampling rate k_h increases from 2.0 to 3.5 at each of higher non-response.

It is found from Table 1-2 that generalized exponential-type product-cum-product estimators are more efficient than modified stratified two-phase Hansen and Hurwitz's (1946) estimator $\bar{y}_{st}^{*}$ and stratified two-phase ratio estimators $\left(\hat{\bar{Y}}_{Pd}, \hat{\bar{Y}}_{Pd}^{\circ}\right), \hat{\bar{Y}}_{Pd}^{\circ}$ in both situations of non-response. When we further observe we can infer on the basis of *PREs* that performance of generalized exponential-type estimators in section 3 is better in both situations of non-response than the performance of the Singh and Kumar (2012) type estimators. Furthermore we can see that generalized exponential-type estimators are more efficient in situation-I than the generalized exponential-type estimators in situation-II. Finally we conclude that the class of generalized exponential-type estimators can be proposed for their practical application for both of the situations of non-response.

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APPENDIX

Table 1:

Percent Relative Efficiencies (*PRE*s) of Estimators with respect to $\bar{y}_{st}^{*}$
for Different Values of k_h each at Different Rate of Non-Response
under Situation-I using Two Different Populations

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\bar{Y}}_{Rd}$	$\hat{\bar{Y}}_{RR}^{A}$	$\hat{\bar{Y}}_{RR}^{1}$	$\hat{\bar{Y}}_{RR}^{2}$	$\hat{\bar{Y}}_{Pd}$	$\hat{\bar{Y}}_{PP}^{A}$	$\hat{\bar{Y}}_{PP}^{1}$	$\hat{\bar{Y}}_{PP}^{2}$	$\hat{\bar{Y}}_{RR}^{G}$	$\hat{\bar{Y}}_{PP}^{G}$
10%	2.0	1	100	366.24	302.78	437.25	992.41	***	***	***	***	2581.70	***
		2	100	***	***	***	***	39.24	49.81	13.25	17.45	***	122.03
	2.5	1	100	390.45	295.79	456.06	987.43	***	***	***	***	2670.70	***
		2	100	***	***	***	***	38.28	49.99	13.34	17.35	***	121.75
	3.0	1	100	413.87	290.04	473.48	983.20	***	***	***	***	2759.89	***
		2	100	***	***	***	***	37.45	50.15	13.42	17.26	***	121.49
	3.5	1	100	436.53	285.25	489.68	979.58	***	***	***	***	2847.59	***
		2	100	***	***	***	***	36.71	50.30	13.49	17.18	***	121.26
20%	2.0	1	100	409.22	293.83	473.46	1003.22	***	***	***	***	2676.38	***
		2	100	***	***	***	***	39.72	52.17	14.05	18.29	***	122.26
	2.5	1	100	452.16	285.69	505.56	1002.44	***	***	***	***	2787.51	***
		2	100	***	***	***	***	39.04	53.29	14.47	18.53	***	122.08
	3.0	1	100	492.71	279.61	533.95	1001.84	***	***	***	***	2886.71	***
		2	100	***	***	***	***	38.47	54.29	14.84	18.74	***	121.93
	3.5	1	100	531.06	274.90	559.23	1001.35	***	***	***	***	2975.23	***
		2	100	***	***	***	***	38.00	55.17	15.18	18.92	***	121.80
30%	2.0	1	100	425.00	291.75	487.81	1009.61	***	***	***	***	2601.63	***
		2	100	***	***	***	***	38.68	53.49	14.55	18.60	***	122.01
	2.5	1	100	474.18	283.65	524.63	1010.80	***	***	***	***	2671.29	***
		2	100	***	***	***	***	37.79	54.99	15.12	18.92	***	121.76
	3.0	1	100	520.08	277.76	556.68	1011.70	***	***	***	***	2730.02	***
		2	100	***	***	***	***	37.11	56.25	15.61	19.19	***	121.57
	3.5	1	100	563.00	273.30	584.81	1012.41	***	***	***	***	2780.02	***
		2	100	***	***	***	***	36.57	57.33	16.04	19.41	***	121.42

(***) shows population is not applicable.

Table 2:
Percent relative efficiencies (*PRE*s) of estimators with respect to $\bar{y}_{st}^{*}$ for different values of k_h each at different rate of non-response under situation-II using two different populations

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\bar{Y}}_{Rd}$	$\hat{\bar{Y}}_{RR}^{A}$	$\hat{\bar{Y}}_{RR}^{1}$	$\hat{\bar{Y}}_{RR}^{2}$	$\hat{\bar{Y}}_{Pd}$	$\hat{\bar{Y}}_{PP}^{A}$	$\hat{\bar{Y}}_{PP}^{1}$	$\hat{\bar{Y}}_{PP}^{2}$	$\hat{\bar{Y}}_{RR}^{G}$	$\hat{\bar{Y}}_{PP}^{G}$
10%	2.0	1	100	231.36	234.67	263.80	398.18	***	***	***	***	493.14	***
		2	100	***	***	***	***	22.96	51.87	14.22	19.17	***	120.17
	2.5	1	100	209.94	212.44	234.02	323.29	***	***	***	***	377.77	***
		2	100	***	***	***	***	38.28	53.01	14.79	19.89	***	119.09
	3.0	1	100	194.52	196.51	213.40	278.47	***	***	***	***	314.75	***
		2	100	***	***	***	***	37.45	54.10	15.35	20.60	***	118.12
	3.5	1	100	182.90	184.54	198.28	248.63	***	***	***	***	275.03	***
		2	100	***	***	***	***	36.71	55.15	15.91	21.29	***	117.25
20%	2.0	1	100	198.01	200.11	218.00	288.04	***	***	***	***	327.85	***
		2	100	***	***	***	***	39.72	54.27	15.43	20.71	***	117.99
	2.5	1	100	177.03	178.50	190.78	234.68	***	***	***	***	257.03	***
		2	100	***	***	***	***	39.04	56.36	16.58	22.13	***	116.29
	3.0	1	100	163.45	164.57	173.77	204.91	***	***	***	***	219.80	***
		2	100	***	***	***	***	38.47	58.27	17.68	23.51	***	114.88
	3.5	1	100	153.94	154.83	162.12	185.92	***	***	***	***	196.84	***
		2	100	***	***	***	***	38.00	60.02	18.76	24.84	***	113.70
30%	2.0	1	100	188.75	190.56	205.85	263.27	***	***	***	***	294.30	***
		2	100	***	***	***	***	38.68	56.67	16.75	22.35	***	116.05
	2.5	1	100	168.59	169.84	180.16	215.81	***	***	***	***	233.25	***
		2	100	***	***	***	***	37.79	59.57	18.48	24.49	***	113.99
	3.0	1	100	155.90	156.84	164.51	189.72	***	***	***	***	201.39	***
		2	100	***	***	***	***	37.11	62.11	20.14	26.51	***	112.40
	3.5	1	100	147.17	147.92	153.97	173.23	***	***	***	***	181.83	***
		2	100	***	***	***	***	36.57	64.35	21.73	28.43	***	111.14

(***) shows population is not applicable.

Table 3:

MSEs of $\bar{y}_{st}^{*}$, $\hat{\bar{Y}}_{Rd}$, $\hat{\bar{Y}}_{Pd}$ and Proposed Estimators for Different Values of k_h each at Different Rate of Non-Response under Situation-I Using Two Different Populations

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\bar{Y}}_{Rd}$	$\hat{\bar{Y}}_{RR}^{A}$	$\hat{\bar{Y}}_{RR}^{1}$	$\hat{\bar{Y}}_{RR}^{2}$	$\hat{\bar{Y}}_{Pd}$	$\hat{\bar{Y}}_{PP}^{A}$	$\hat{\bar{Y}}_{PP}^{1}$	$\hat{\bar{Y}}_{PP}^{2}$	$\hat{\bar{Y}}_{RR}^{G}$	$\hat{\bar{Y}}_{PP}^{G}$
10%	2.0	1	2694.00	735.65	889.84	616.18	271.48	***	***	***	***	104.36	***
		2	0.49	***	***	***	***	1.25	0.99	3.71	2.82	***	0.40
	2.5	1	2921.20	748.15	987.60	640.53	295.84	***	***	***	***	109.38	***
		2	0.51	***	***	***	***	1.34	1.03	3.86	2.97	***	0.42
	3.0	1	3148.10	760.65	1085.41	664.88	320.19	***	***	***	***	114.07	***
		2	0.54	***	***	***	***	1.44	1.07	4.01	3.12	***	0.44
	3.5	1	3375.00	773.14	1183.20	689.24	344.54	***	***	***	***	118.52	***
		2	0.56	***	***	***	***	1.53	1.12	4.16	3.27	***	0.46
20%	2.0	1	3090.60	755.24	1051.82	652.76	308.07	***	***	***	***	115.48	***
		2	0.54	***	***	***	***	1.36	1.04	3.85	2.96	***	0.44
	2.5	1	3515.70	777.53	1230.60	695.41	350.71	***	***	***	***	126.12	***
		2	0.59	***	***	***	***	1.51	1.11	4.07	3.18	***	0.48
	3.0	1	3940.80	799.82	1409.38	738.05	393.36	***	***	***	***	136.52	***
		2	0.64	***	***	***	***	1.66	1.17	4.29	3.40	***	0.52
	3.5	1	4365.90	822.11	1588.16	780.70	436.00	***	***	***	***	146.74	***
		2	0.69	***	***	***	***	1.80	1.24	4.51	3.62	***	0.56
30%	2.0	1	3253.40	765.49	1115.11	666.93	322.24	***	***	***	***	125.05	***
		2	0.60	***	***	***	***	1.54	1.12	4.10	3.21	***	0.49
	2.5	1	3759.80	792.91	1325.54	716.66	371.97	***	***	***	***	140.75	***
		2	0.67	***	***	***	***	1.78	1.22	4.45	3.55	***	0.55
	3.0	1	4266.30	820.33	1535.96	766.39	421.70	***	***	***	***	156.28	***
		2	0.75	***	***	***	***	2.02	1.33	4.79	3.90	***	0.62
	3.5	1	4772.80	847.74	1746.38	816.12	471.43	***	***	***	***	171.68	***
		2	0.82	***	***	***	***	2.25	1.44	5.14	4.24	***	0.68

(***) shows population is not applicable.

Table 4:

MSEs of $\bar{y}_{st}^{*}$, $\hat{\bar{Y}}_{Rd}^{\circ}$, $\hat{\bar{Y}}_{Pd}^{\circ}$ and Proposed Estimators for Different Values of k_h each at Different Rate of Non-Response under Situation-II using Two Different Populations

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\bar{Y}}_{Rd}$	$\hat{\bar{Y}}_{RR}^{A}$	$\hat{\bar{Y}}_{RR}^{1}$	$\hat{\bar{Y}}_{RR}^{2}$	$\hat{\bar{Y}}_{Pd}$	$\hat{\bar{Y}}_{PP}^{A}$	$\hat{\bar{Y}}_{PP}^{1}$	$\hat{\bar{Y}}_{PP}^{2}$	$\hat{\bar{Y}}_{RR}^{G}$	$\hat{\bar{Y}}_{PP}^{G}$
10%	2.0	1	2694.00	1164.52	1148.12	1021.33	676.64	***	***	***	***	546.34	***
		2	0.49	***	***	***	***	2.14	0.95	3.46	2.57	***	0.41
	2.5	1	2921.20	1391.44	1375.05	1248.26	903.57	***	***	***	***	773.27	***
		2	0.51	***	***	***	***	1.34	0.97	3.48	2.59	***	0.43
	3.0	1	3148.10	1618.37	1601.98	1475.19	1130.49	***	***	***	***	1000.20	***
		2	0.54	***	***	***	***	1.44	0.99	3.50	2.61	***	0.46
	3.5	1	3375.00	1845.30	1828.91	1702.12	1357.42	***	***	***	***	1227.13	***
		2	0.56	***	***	***	***	1.53	1.02	3.53	2.63	***	0.48
20%	2.0	1	3090.60	1560.86	1544.47	1417.68	1072.98	***	***	***	***	942.69	***
		2	0.54	***	***	***	***	1.36	1.00	3.51	2.61	***	0.46
	2.5	1	3515.70	1985.96	1969.57	1842.78	1498.09	***	***	***	***	1367.79	***
		2	0.59	***	***	***	***	1.51	1.05	3.56	2.66	***	0.51
	3.0	1	3940.80	2411.06	2394.67	2267.88	1923.19	***	***	***	***	1792.89	***
		2	0.64	***	***	***	***	1.66	1.09	3.60	2.71	***	0.55
	3.5	1	4365.90	2836.17	2819.77	2692.98	2348.29	***	***	***	***	2217.99	***
		2	0.69	***	***	***	***	1.80	1.14	3.65	2.76	***	0.60
30%	2.0	1	3253.40	1723.62	1707.23	1580.44	1235.75	***	***	***	***	1105.45	***
		2	0.60	***	***	***	***	1.54	1.05	3.56	2.67	***	0.51
	2.5	1	3759.80	2230.11	2213.71	2086.93	1742.23	***	***	***	***	1611.94	***
		2	0.67	***	***	***	***	1.78	1.13	3.64	2.75	***	0.59
	3.0	1	4266.30	2736.59	2720.20	2593.41	2248.72	***	***	***	***	2118.42	***
		2	0.75	***	***	***	***	2.02	1.20	3.71	2.82	***	0.67
	3.5	1	4772.80	3243.06	3226.68	3099.90	2755.20	***	***	***	***	2624.90	***
		2	0.82	***	***	***	***	2.25	1.28	3.79	2.90	***	0.74

(***) shows population is not applicable.

Table 5: Data Sets

		Population-I						Population-II	
Stratum (h)		**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**
Stratified Mean, S.Ds and Correlation Coefficients	N_h	127	117	103	*170*	*205*	*201*	6940	*1678*
	n_h	31	21	29	38	22	39	750	181
	n'_h	70	50	75	95	70	90	1874	453
	S_{yh}	883.84	644.92	1033.40	810.58	403.65	711.72	21.43	22.13
	S_{xh}	30486.70	15180.77	27549.69	18218.93	8497.77	23094.14	16625.33	12861.40
	S_{zh}	555.58	365.46	612.95	458.03	260.85	397.05	19394.09	16143.74
	$\bar{Y}_h$	703.74	413.00	573.17	424.66	267.03	393.84	47.98	48.06
	$\bar{X}_h$	20804.59	9211.79	14309.30	9478.85	5569.95	12997.59	18746.55	14303.98
	$\bar{Z}_h$	498.28	318.33	431.36	311.32	227.20	313.71	19124.75	14742.47
	ρ_{xyh}	0.94	1.00	0.99	0.98	0.99	0.97	-0.48	-0.44
	ρ_{xzh}	0.94	0.97	0.98	0.96	0.97	1.00	0.91	0.80
	ρ_{yzh}	0.98	0.98	0.98	0.98	0.96	0.98	-0.44	-0.35
W_h=10% Non-response	S_{yh2}	510.57	386.77	1872.88	1603.30	264.19	497.84	20.48	21.74
	S_{xh2}	9446.93	9198.29	52429.99	34794.90	4972.56	12485.10	18121.44	15492.72
	S_{zh2}	303.92	278.51	960.71	821.29	190.85	287.99	22010.50	20204.85
	ρ_{xy2}	1.00	1.00	1.00	0.97	1.00	0.93	-0.48	-0.54
	ρ_{xz2}	0.99	0.99	1.00	0.96	0.99	0.98	0.86	0.77
	ρ_{yz2}	0.99	0.99	1.00	0.99	0.99	0.96	-0.39	-0.32
W_h=20% Non-response	S_{yh2}	396.77	406.15	1654.40	1333.35	335.83	903.91	20.74	22.63
	S_{xh2}	7439.16	8880.46	45784.78	29219.30	6540.43	28411.44	16155.37	13887.44
	S_{zh2}	244.56	274.42	965.42	680.28	214.49	469.86	19251.39	17323.10
	ρ_{xy2}	1.00	0.99	1.00	0.98	1.00	0.99	-0.49	-0.49
	ρ_{xz2}	0.99	0.99	0.98	0.96	0.98	0.98	0.88	0.84
	ρ_{yz2}	0.99	0.98	0.98	0.99	0.98	0.99	-0.43	-0.33
W_h=30% Non-response	S_{yh2}	500.26	356.95	1383.70	1193.47	289.41	825.24	21.47	22.44
	S_{xh2}	14017.99	7812.00	38379.77	26090.60	5611.32	24571.95	16877.33	12852.95
	S_{zh2}	284.44	247.63	811.21	631.28	188.30	437.90	19985.52	16007.36
	ρ_{xy2}	0.96	0.99	1.00	0.98	1.00	0.97	-0.48	-0.44
	ρ_{xz2}	0.91	0.98	0.98	0.97	0.98	0.96	0.89	0.83
	ρ_{yz2}	0.97	0.98	0.98	0.99	0.98	0.98	-0.43	-0.28