ON PREDICTIVE INFERENCE FROM THE COMPOUND
RAYLEIGH MODEL BASED ON CENSORED SAMPLES

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ABSTRACT

The purpose of this paper is to derive the predictive inference for future responses from the compound Rayleigh model based on censored samples. The predictive density for a single future response, ith future response, and m future responses are derived on the basis of a type II as well as doubly censored sample. Two numerical examples are considered to illustrate the results. First example deals with a real data set that represents the survival times of 46 patients given chemotherapy treatment. The second example is the doubly censored sample of the heart transplantation surgery patients. Both examples are utilized to illustrate the predictive results. The highest predictive density intervals are determined for a single future response.

KEY WORDS

Compound Rayleigh model; Censored sample; Statistical inference; Bayesian approach; Predictive inference.

2000 Mathematics Subject Classifications:
62F15; 62N01; 62N02; 62N05

1. INTRODUCTION

Predictive inference has been playing an important role in biomedical data analysis. Biomedical researchers rely on past observations to extract information for future outcomes. Incomplete observations are inevitable; however, ignoring or failing to account for such observations can be costly, leading to misinterpretation. Nonetheless, the challenge of incomplete data remains to be resolved. Therefore, appropriate methods need to be used to analyze such type of data. In clinical studies, various types of censored data have been observed.

In doubly censored sample, consider the patient survival times can be ordered with \( x_1 \leq \cdots \leq x_n \); see Ipsen (1949), Sarhan (1955), Khan et al. (2011), Kambo (1978), Hui (1980), Raqab (1995), among others. Most studies based on doubly censored data from the two-parameter exponential model involve inference about the parameters. Sarhan (1955) obtained the mean and standard deviation of certain populations from singly and doubly censored samples. Kambo (1978) derived the maximum likelihood estimators of
the location and scale parameters by using a doubly censored sample. Khan et al. (2011) derived the predictive distributions of future responses on the basis of a doubly censored sample from the Weibull life testing model.


The objective of this paper is to derive the predictive densities for future responses from the compound Rayleigh model and to obtain predictive inference by making use of the Bayesian approach given a type II as well as doubly censored sample. The observed censored data may be available from the clinical records of patients. The motivation for the present paper is to derive predictive densities for responses in the case of censored samples, where data follow a compound Rayleigh model. In our study, we consider Jeffreys prior for both parameters $\eta$ and $\gamma$. We extend the derivation for more than a single future response in the case of a type II censored sample. Furthermore, in real life, one may observe doubly censored data and be interested in predicting future survival days from such types of samples. For this, we derive predictive densities for a single future response, $i$th future response, and $m$ future responses based on a doubly censored sample. Two numerical examples, which constitute of doubly censored samples, are used to illustrate the predictive results.

2. COMPOUND RAYLEIGH MODEL

Suppose $X$ is a random variable that may be defined as the number of survival days for patients with a specific disease. Then the probability density function of $X$ is given by

$$p(x|\eta, \gamma) = \int_{\delta=0}^{\infty} p(x|\delta) p(\delta|\eta, \gamma) \, d\delta, \eta > 0, \gamma > 0,$$

where $p(x|\delta)$ is the Rayleigh model. The parameter $\delta$ is assumed to be a random variable with pdf $p(\delta|\eta, \gamma)$. The model (1) is known as the compound Rayleigh model. For more about compound model, see Thabane (1998), Thabane and Haq (2000), Raiffa and Schlaifer (1961), Zellner (1976), and Haq and Rahman (1992), the pdf of $p(x|\delta)$
where $\delta = 1/2\beta^2$ is a scale parameter. Following Ghitany (2001), $\delta$ has a gamma model i.e., $\delta \sim G(\gamma/2,1/\eta^2)$.

As pointed out by Bain and Engelhardt (1991), $\delta$ may be viewed as an intensity parameter. It has been emphasized by Bain and Engelhardt (1991, p.450) that when the intensity parameter $\delta$ is not fixed, a compound distribution is more suitable for analyzing data. Therefore, it is assumed that the distribution of $\delta$ has a gamma distribution.

The pdf of $\delta$ is given by

$$
p(\delta|\eta, \gamma) = \begin{cases} \frac{\eta^\gamma \delta^{\gamma-1}}{\Gamma(\gamma)} \exp\{-\eta^2 \delta\}, & \delta > 0; \gamma, \eta > 0, \\ 0 & \text{elsewhere.} \end{cases}
$$

The final form of the probability density function of the compound Rayleigh model is given by

$$
p(x|\eta, \gamma) = \frac{x^\gamma \eta^\gamma}{(\eta^2 + x^2)^{\gamma/2 + 1}} \exp\{-\eta^2 x\}, \quad x > 0; \gamma, \eta > 0,
$$

and

$$
F_X(x) = 1 - \frac{\eta^\gamma}{(\eta^2 + x^2)^{\gamma/2}}, x > 0; \gamma, \eta > 0.
$$

The following properties are drawn from model (3).

The $r$th moment about the origin is given by

$$
E(x^r) = \frac{\eta^r \Gamma(\frac{\gamma-r}{2}) \Gamma(\frac{\gamma+1}{2})}{\Gamma(\frac{\gamma}{2})}, \gamma > r, \eta > 0.
$$

The mean and the variance of $x$ are obtained as

$$
E(x) = \frac{\eta^\gamma \Gamma(\frac{\gamma-1}{2})}{\Gamma(\frac{\gamma}{2})}, \gamma > 1, \eta > 0,
$$

and

$$
Var(x) = \eta^2 \left[ 4 \Gamma(\frac{\gamma-2}{2}) \Gamma(\frac{\gamma}{2}) - \pi \left( \Gamma(\frac{\gamma-1}{2}) \right)^2 \right]/4 \left( \Gamma(\frac{\gamma}{2}) \right)^2, \gamma > 2, \eta > 0.
$$

Similarly, for the choice of $r$, one can obtain the corresponding higher order expectation of $x$.

Ghitany (2001) considered a compound Rayleigh model with application to randomly censored data. He showed the goodness of fit of the compound Rayleigh model by using a medical data set (chemotherapy alone versus a combination of chemotherapy and radiation therapy in the treatment of locally advanced nonresectable gastric carcinoma).
On Predictive Inference from the Compound Rayleigh Model

He obtained the maximum likelihood estimates and 95% confidence intervals for the parameters. Bekker et al. (2000) discussed the generalization of the compound Rayleigh distribution. They derived the Bayes estimators for the survival time parameters, namely the hazard function, the survival distribution function, and the mean survival time. They estimated a single patient’s survival time by using a type II censored sample.

3. THE PREDICTIVE DENSITIES BASED ON A TYPE II CENSORED SAMPLE

Let \( n \) patients be followed up after diagnosis of a specific disease and assume that the experiment is terminated after \( k \) patients’ survival days are recorded. Let \( x_1 \) be the first patient’s survival days, \( x_2 \) be the next patient’s survival days and so on. Thus, \( x_1 \leq x_2 \leq \cdots \leq x_k \); and \( \mathbf{x} = (x_1, ..., x_k) \) forms an observed type II censored sample. For a given set of data \( \mathbf{x} = (x_1, ..., x_k) \) from (3), following Khan et al. (2003, 2009) and Khan and Albatineh (2013), the likelihood function is given by

\[
G(\eta, \gamma|\mathbf{x}) = \frac{n!}{(n-k)!} \prod_{i=1}^{k} p(x_i|\eta, \gamma)[1 - F(x_i)]^{n-k}, k = 1, 2, ..., n.
\]

\[
\propto \gamma^k \eta^k \left( \prod_{i=1}^{k} x_i \right) \left( \eta^2 + x_i^2 \right)^{-\left(\frac{\gamma+1}{2}\right)} \left( \eta^\gamma (\eta^2 + x_i^2)^{-\frac{\gamma}{2}} \right)^{n-k}.
\]

We consider Jeffreys (1961) prior for the parameters, \( \gamma \) and \( \eta \). It is assumed that \( \gamma \) and \( \eta \) are non-negative and they are independently distributed. The parameter, \( \gamma \) has the following prior density

\[
p(\gamma) \propto \frac{1}{\gamma}, \gamma > 0.
\]

Similarly, \( \eta \) has the following prior density

\[
p(\eta) \propto \frac{1}{\eta}, \eta > 0.
\]

Combining equations (4) and (5), one would have

\[
p(\eta, \gamma) \propto \frac{1}{\eta \gamma}, \gamma, \eta > 0.
\]

Thus, the posterior density of \( \eta \) and \( \gamma \) given \( \mathbf{x} \) is given by

\[
p(\eta, \gamma|\mathbf{x}) \propto g(\eta, \gamma|\mathbf{x}) p(\gamma, \eta)
\]

\[
= \psi_1(\mathbf{x}) \left( \prod_{i=1}^{k} x_i \right) \gamma^{\gamma} (\eta^2 + x_i^2)^{-\left(\frac{\gamma+1}{2}\right)} \left( \eta^\gamma (\eta^2 + x_i^2)^{-\frac{\gamma}{2}} \right)^{n-k} / \eta \gamma,
\]

where \( \psi_1(\mathbf{x}) \) is a normalizing constant and it may be evaluated given a type II censored sample.
3.1 Predictive Density for $i$th Ordered Future Response

One may be interested in a clinical experiment to obtain $i$th patient’s future survival time. Consider the model (3) and let $y_i$ be the $i$th ordered future response in a set of $\omega$ future responses. Following Khan et al. (2006) the predictive density of $y_i$ given $\gamma$ and $\eta$ is given by

$$p(y_i|\gamma, \eta) = \frac{y_i \gamma \eta \omega!}{(i-1)! (\omega-i)!} (\eta^2 + y_i^2)^{-\frac{\gamma}{2} + 1} \left(1 - \eta^\gamma (\eta^2 + y_i^2)^{-\frac{\gamma}{2}}\right)^{i-1} \times \left(\eta^\gamma (\eta^2 + y_i^2)^{-\frac{\gamma}{2}}\right)^{\omega-i}.$$

Thus, the predictive density for $y_i$ given a type II censored sample is given by

$$p(y_i|x) = \int_{y=0}^{+\infty} \int_{\eta=0}^{+\infty} p(y_i|\gamma, \eta) \ p(\gamma, \eta|x) \ d\eta d\gamma$$

$$\propto \int_{y=0}^{+\infty} \int_{\eta=0}^{+\infty} \frac{y_i \gamma \eta \omega!}{(i-1)! (\omega-i)!} (\eta^2 + y_i^2)^{-\frac{\gamma}{2} + 1} \left(1 - \eta^\gamma (\eta^2 + y_i^2)^{-\frac{\gamma}{2}}\right)^{i-1} \times \left(\eta^\gamma (\eta^2 + y_i^2)^{-\frac{\gamma}{2}}\right)^{\omega-i} \times \left(\frac{\prod_{i=1}^{k} x_i \gamma \eta^\gamma (\eta^2 + x_i^2)^{-\frac{\gamma}{2} + 1}}{\eta^\gamma d\eta d\gamma}\right)$$

When $i = 1$, we have single future response density function and for $i = m$, we have $m$ future responses density function.

4. THE PREDICTIVE DENSITIES GIVEN A DOUBLY CENSORED SAMPLE

Let $x_1, ..., x_n$ be an ordered random sample of size $n$ from model (3), where $x_i \leq ... \leq x_k$ is the $k$ smallest ordered observations and $x_{q+1} \leq ... \leq x_n$ is the $(n-q)$ largest ordered observations from the sample. Only the remaining ordered observations $x = (x_{k+1}, ..., x_q)$ are used for statistical analysis. It is assumed that the sample data are modeled by the compound Rayleigh distribution. Following Khan et al. (2006) the likelihood function of $\gamma$ and $\eta$ for a given doubly censored sample $x = (x_{k+1}, ..., x_q)$ is given by

$$L(\eta, \gamma|x) = [F(x_k + 1)]^k [1 - F(x_q)]^{n-q} [\prod_{i=k+1}^{q} p(x_i)]$$

$$\propto \left(1 - \eta^\gamma (\eta^2 + x_{k+1}^2)^{-\frac{\gamma}{2}}\right)^k \left(\eta^\gamma (\eta^2 + x_q^2)^{-\frac{\gamma}{2}}\right)^{n-q}$$

$$\times \left(\prod_{i=k+1}^{q} x_i \gamma \eta^\gamma (\eta^2 + x_i^2)^{-\frac{\gamma}{2} + 1}\right).$$
Considering the prior distribution from model (6), the posterior distribution of \( \gamma \) and \( \eta \) given \( x \) is

\[
p(\eta, \gamma | x) \propto L(\eta, \gamma | x) \, p(\gamma, \eta) \\
\propto \left( 1 - \eta^\gamma (\eta^2 + x_{k+1}^2)^{-\gamma} \right)^k \left( \eta^\gamma (\eta^2 + x_{n}^2)^{-\gamma} \right)^{n-q} \\
\times \left( \prod_{i=k+1}^{n} x_{i} \gamma \eta^\gamma (\eta^2 + x_{i}^2)^{-(\gamma / 2)} \right) / \eta^\gamma.
\]

4.1 Predictive Density for a Single Future Response

Let \( z \) be a future response from the model specified by (3). Naturally, \( z \) is independent of the observed data. The predictive density for \( z \) given a set of doubly censored data \( x = (x_{q+1}, \ldots, x_{n}) \) is given by

\[
p(z | x) = \int_{\gamma=0}^{\gamma=\infty} \int_{\eta=0}^{\eta=\infty} p(z | \gamma, \eta) \, p(\gamma, \eta | x) \, d\eta \, d\gamma
\]

\[
\propto \int_{\gamma=0}^{\gamma=\infty} \int_{\eta=0}^{\eta=\infty} z \gamma \eta^\gamma \eta^\gamma (\eta^2 + z^2)^{-(\gamma / 2)} \left( 1 - \eta^\gamma (\eta^2 + x_{k+1}^2)^{-\gamma} \right)^k \\
\times \left( \eta^\gamma (\eta^2 + x_{n}^2)^{-\gamma} \right)^{n-q} \\
\times \left( \prod_{i=k+1}^{n} x_{i} \gamma \eta^\gamma (\eta^2 + x_{i}^2)^{-(\gamma / 2)} \right) / \eta^\gamma \, d\eta \, d\gamma.
\]

4.2 Predictive Density for \( i \)th Ordered Future Response

One may be interested in a clinical experiment to obtain \( i \)th patient’s future survival time. Consider the model (3) and let \( z_{i} \) be the \( i \)th ordered future response in a set of \( \omega \) future responses. Following Khan et al. (2006) the predictive density of \( z_{i} \) given \( \gamma \) and \( \eta \) is given by

\[
p(z_{i} | \gamma, \eta) = \frac{z_{i} \gamma \eta^\gamma \omega^i}{(i-1)! \omega!} \left( \frac{\eta^2 + z_{i}^2}{(\gamma / 2)} \right)^{i-1} \left( 1 - \eta^\gamma (\eta^2 + z_{i}^2)^{-\gamma} \right) \\
\times \left( \eta^\gamma (\eta^2 + z_{n}^2)^{-\gamma} \right)^{\omega-i}.
\]

Thus, the predictive density for \( z_{i} \) given a doubly censored sample is given by

\[
p(z_{i} | x) = \int_{\gamma=0}^{\gamma=\infty} \int_{\eta=0}^{\eta=\infty} p(z_{i} | \gamma, \eta) \, p(\gamma, \eta | x) \, d\eta \, d\gamma
\]

\[
\propto \int_{\gamma=0}^{\gamma=\infty} \int_{\eta=0}^{\eta=\infty} \frac{z_{i} \gamma \eta^\gamma \omega^i}{(i-1)! \omega!} \left( \frac{\eta^2 + z_{i}^2}{(\gamma / 2)} \right)^{i-1} \\
\times \left( 1 - \eta^\gamma (\eta^2 + z_{i}^2)^{-\gamma} \right) \left( \eta^\gamma (\eta^2 + x_{k+1}^2)^{-(\gamma / 2)} \right)^k \\
\times \left( \eta^\gamma (\eta^2 + x_{n}^2)^{-(\gamma / 2)} \right)^{n-q} \\
\times \left( \prod_{i=k+1}^{n} x_{i} \gamma \eta^\gamma (\eta^2 + x_{i}^2)^{-(\gamma / 2)} \right) / \eta^\gamma \, d\eta \, d\gamma.
\]

4.3 Predictive Density for \( m \) Future Responses

One may be interested in a clinical experiment to obtain \( m \) future patients survival time. Let \( z_{1}, \ldots, z_{m} \) be the a set of ordered future responses from model (3). Then
following Khan et al. (2006) the predictive density of \( z = (z_1, \ldots, z_m) \) given a set of doubly censored sample \( x = (x_{k+1}, \ldots, x_q) \) is given by

\[
p(z | x) = \int_{y=0}^{+\infty} \int_{\eta=0}^{+\infty} m! \prod_{i=1}^{m} p(z_i | y, \eta) p(y, \eta) | x) d\eta dy \\
\propto \int_{y=0}^{+\infty} \int_{\eta=0}^{+\infty} \prod_{i=1}^{m} z_i y^\eta m! (\eta^2 + z_i^2)^{-(\frac{\eta}{y}+1)} (1 - \eta^y (\eta^2 + x_i^2)^{\frac{y}{2}})^k \\
\times \left( \eta^y (\eta^2 + x_0^2)^{\frac{y}{2}} \right)^{n-q} \left( \prod_{i=k+1}^{q} x_i \eta^y (\eta^2 + x_i^2)^{\frac{y}{2}+1} \right) / \eta y d\eta dy.
\]

For \( m = 1 \), we have single future response density function.

5. THE LOG-LIKELIHOOD AND REPARAMETERIZATION

Let \( x_1, \ldots, x_n \) be a random sample of size \( n \) from model (3). Then the log likelihood function from the compound Rayleigh model specified in equation (3) is given by

\[
\text{LogL}(\eta, y|x) = n \log (y) + n \log (\eta) + \sum_{i=1}^{n} \log (x_i) - (y/2 + 1) \sum_{i=1}^{n} \log (\eta^2 + x_i^2).
\]

The posterior model for \( \rho_1 \) and \( \rho_2 \) is given by

\[
p(\rho_1, \rho_2 | x) \propto p(\rho_1, \rho_2) \exp \left( n \rho_1 + n \rho_2 \exp(\rho_1) + \sum_{i=1}^{n} \log (x_i) \\
- \left( 1 + \frac{\exp(\rho_1)}{2} \right) \sum_{i=1}^{n} \log \left( \left( \exp(\rho_2) \right)^2 + x_i^2 \right) \right).
\]

One may use a reparameterization method considering the log-likelihood function by setting \( \rho_1 = \log (y) \) and \( \rho_2 = \log (\eta) \). The parameters \( \rho_1 \) and \( \rho_2 \) are independently distributed. Furthermore, one may assume Jeffreys (1961) non-informative prior for both \( \rho_1 \) and \( \rho_2 \), where \(-\infty < \rho_1 < +\infty\) and \(-\infty < \rho_2 < +\infty\). To obtain non-informative prior for \( \rho_1 \) and \( \rho_2 \), it may be assumed a uniform prior distribution for \( \rho_i \) over the interval \( U(-d_i, d_i) \), for \( i = 1, 2 \). By using the reparameterization one may obtain better performance of the Gibbs sampling algorithm. Ahmed et al. (2008) considered a parametric estimation for the Birnbaum-Saunders lifetime distribution based on a new parameterization.

6. REAL LIFE EXAMPLES

We consider two numerical examples. The first example describes a real data sample that follows the compound Rayleigh model, and the second example presents another real data sample which deals with survival days of the patients who had heart transplantation surgery. The predictive density for a single future response obtained in Section 4.1 is utilized for both data sets.
Example 1:

In this section a real data set is used to obtain predictive inference. Bekker et al. (2000) discussed in detail the maximum likelihood estimates for the parameters. Ghitany (2001) considered random censored data from Stablein et al. (1981), where data represent two arms of a clinical trial comparing chemotherapy alone versus a combination of chemotherapy and radiation therapy in the treatment of locally advanced, nonresectable gastric carcinoma. Ghitany (2001) showed that the data set follows the compound Rayleigh survival model. In this example, data set represents the survival times in years of a group of patients given chemotherapy treatment alone. Abushal (2011) used this data set and showed that the data set follows the compound Rayleigh model. The data are arranged already in order and form a sample of size \( n = 46 \). To make a doubly censored sample, it is assumed that original sample had size of \( n = 60 \), and the first five and the last nine observations are unavailable due to lost to follow up and are discarded from the ordered sample. This sample is utilized to obtain the predictive inference for a future response.

In the case of compound Rayleigh model, \( \rho_1 = \log(\gamma) \) and \( \rho_2 = \log(\eta) \) are used under the reparameterization. The following non-informative priors are used for \( \rho_1 \) and \( \rho_2 : U(-1, 1) \) and \( U(-1, 1) \), respectively. In the case of the simulated data, there are 1,001 burn-in samples excluded, and the results are based on the additional 60,000 iterative samples. The summary results of the parameters are reported in Table 1.

### Table 1

**Summary Results for the Posterior Parameters based on 46 Patients**

<table>
<thead>
<tr>
<th>Node</th>
<th>Mean</th>
<th>SD</th>
<th>MC Error</th>
<th>2.5%</th>
<th>Median</th>
<th>97.5%</th>
<th>Iteration begins</th>
<th>Iteration ends</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>2.672</td>
<td>0.04525</td>
<td>3.068E-4</td>
<td>2.553</td>
<td>2.686</td>
<td>2.717</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.3757</td>
<td>0.007835</td>
<td>5.666E-5</td>
<td>0.3681</td>
<td>0.3734</td>
<td>0.3968</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>0.9827</td>
<td>0.01722</td>
<td>1.670E-4</td>
<td>0.9373</td>
<td>0.988</td>
<td>0.9996</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>-0.9791</td>
<td>0.02044</td>
<td>1.478E-4</td>
<td>-0.9995</td>
<td>-0.9852</td>
<td>-0.9243</td>
<td>1001</td>
<td>60000</td>
</tr>
</tbody>
</table>

Example 2:

A data set related to heart transplantation surgery is taken from a local hospital in Newark, New Jersey. The data is a sample of observations of the survival days of 168 patients after a heart transplantation surgery is performed. The number of days the patient survived after surgery is modeled as a compound Rayleigh model. The first ten and last twenty-two observations were censored; therefore, the data is in the form of a doubly censored sample. The sample mean = 41.70 and variance = 1108.30.

We used ‘Mathematica version 6.0’, Wolfram Research (2008), and EasyFit 5.4 Professional software, MathWave Technologies (2010) to obtain the maximum likelihood
estimates $\hat{\gamma} = 3.4597$ and $\hat{\eta} = 51.6670$ for the parameters $\gamma$ and $\eta$ by making use of a doubly censored sample. The results of goodness of fit tests are reported in Table 2 and the survival function plot of 168 patients’ is given in Figure 1.

<table>
<thead>
<tr>
<th><strong>Table 2</strong></th>
<th>Goodness of Fit Tests for the Compound Rayleigh Model based on Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K-S Test Statistic</strong></td>
<td>0.05839</td>
</tr>
<tr>
<td>Choice of alpha ($\alpha$)</td>
<td>0.05</td>
</tr>
<tr>
<td>Critical Value</td>
<td>0.10477 ($p$-value = 0.59488)</td>
</tr>
<tr>
<td>Decision on compound Rayleigh model</td>
<td>Accept</td>
</tr>
<tr>
<td><strong>A-D Test Statistic</strong></td>
<td>0.44087</td>
</tr>
<tr>
<td>Choice of alpha ($\alpha$)</td>
<td>0.05</td>
</tr>
<tr>
<td>Critical Value</td>
<td>2.5018 ($p$-value = 0.78350)</td>
</tr>
<tr>
<td>Decision on compound Rayleigh model</td>
<td>Accept</td>
</tr>
<tr>
<td><strong>Chi-Squared Test Statistic</strong></td>
<td>9.7746</td>
</tr>
<tr>
<td>Choice of alpha ($\alpha$)</td>
<td>0.05</td>
</tr>
<tr>
<td>Critical Value</td>
<td>14.067 ($p$-value = 0.20171)</td>
</tr>
<tr>
<td>Decision on compound Rayleigh model</td>
<td>Accept</td>
</tr>
</tbody>
</table>

One may interest to know about the highest predictive density (HPD) interval. An HPD interval is the interval which includes the most probable values of a given density at a given significance level, subject to the condition that the density function has the same value at both end points. Due to unimodality of the predictive density for a single future response, the HPD interval $[g_1, g_2]$ for $z$ must simultaneously satisfy the following two conditions:

$$\Pr(g_1 \leq z \leq g_2) = 1 - \alpha \text{ and } p(g_1 | x) = p(g_2 | x),$$

where $g_1$ and $g_2$ are to be arbitrarily chosen so that $p(g_1 | x) = p(g_2 | x)$. For more about HPD intervals, see Sinha (1986), and Box and Tiao (1973).

We estimated the predictive inference for a future response and their results are given in Table 3. We determined certain levels of HPD intervals for a single future response given a doubly censored sample which are specified by $g_1$ and $g_2$, and their results are reported in Table 4. For the posterior parameters, the following non-informative priors are used for $\rho_1$ and $\rho_2$: $U(-1, 1)$ and $U(-1, 1)$, respectively. The summary results of the posterior parameters are given in Table 5.
Fig. 1: Survival Function based on 168 Patients’ Survival Days after Heart Transplantation Surgery.

Table 3
Summary Results of the Predictive Inference for a Single Future Response

<table>
<thead>
<tr>
<th></th>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 60, q = 46, k = 5 )</td>
<td>( n = 190, q = 168, k = 10 )</td>
<td></td>
</tr>
<tr>
<td>( \mu_1' = 1.41261 )</td>
<td>( \mu_1' = 49.2395 )</td>
<td></td>
</tr>
<tr>
<td>( \mu_2 = 3.38503 )</td>
<td>( \mu_2 = 297.87 )</td>
<td></td>
</tr>
<tr>
<td>( \mu_3 = 14.5306 )</td>
<td>( \mu_3 = -68691.50 )</td>
<td></td>
</tr>
<tr>
<td>( \mu_4 = 96.9569 )</td>
<td>( \mu_4 = 4.97921 \times 10^7 )</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 = 5.44354 )</td>
<td>( \beta_1 = 0.178865 )</td>
<td></td>
</tr>
<tr>
<td>( \beta_2 = 8.46165 )</td>
<td>( \beta_2 = 5.61876 )</td>
<td></td>
</tr>
</tbody>
</table>
Table 4
Summary Results of the Highest Predictive Density Intervals (HPD) for a Single Future Response

<table>
<thead>
<tr>
<th>Example 1: HPD Intervals</th>
<th>Example 2: HPD Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>(0.0032, 4.47)</td>
</tr>
<tr>
<td>95%</td>
<td>(0.0014, 7.39)</td>
</tr>
<tr>
<td>98%</td>
<td>(0.0008, 10.30)</td>
</tr>
<tr>
<td>99%</td>
<td>(0.0006, 12.68)</td>
</tr>
</tbody>
</table>

Table 5
Summary Results for the Posterior Parameters based on 168 Patients Survival Data by Making Use of the Software ‘WinBUGS’

<table>
<thead>
<tr>
<th>Node</th>
<th>Mean</th>
<th>SD</th>
<th>MC Error</th>
<th>2.5%</th>
<th>Median</th>
<th>97.5%</th>
<th>Iteration begins</th>
<th>Iteration ends</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.379</td>
<td>0.009809</td>
<td>6.194E−5</td>
<td>0.3682</td>
<td>0.3763</td>
<td>0.4043</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3739</td>
<td>0.005951</td>
<td>3.886E−5</td>
<td>0.368</td>
<td>0.3721</td>
<td>0.3899</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>-0.9707</td>
<td>0.02543</td>
<td>1.605E−4</td>
<td>-0.9991</td>
<td>-0.9774</td>
<td>-0.9055</td>
<td>1001</td>
<td>60000</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.984</td>
<td>0.01568</td>
<td>1.024E−4</td>
<td>-0.9996</td>
<td>-0.9886</td>
<td>-0.9419</td>
<td>1001</td>
<td>60000</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS
We derived the predictive densities for future responses from the compound Rayleigh model given a type II as well as doubly censored sample. We derived predictive densities for a single future response, $i$th future response, and $m$ future responses based on a type II as well as doubly censored sample. Compared with the previous results for a single future response from the compound Rayleigh model given a type II censored sample by Bekker et al. (2000), our results are general. In addition, we extended the predictive results in the case of a doubly censored sample.

The predictive inference for a single future response given the doubly censored samples is obtained. The highest predictive density intervals are determined for a single future response. We used two numerical examples to illustrate the predictive results. An advanced computational software package ‘Mathematica 6.0’ was used to perform all necessary calculations related to the predictive survival days. By using the MCMC method we showed the summary results for the posterior parameters.

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REFERENCES


