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REPRODUCTIVITY AND AGE-SPECIFIC FERTILITY RATES IN PAKISTAN AFTER 1981

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ABSTRACT

The purpose of this study is to observe the trends and the patterns of reproductivity in Pakistan using the secondary data on age-specific fertility rates after 1981. Also, the fertility pattern in Pakistan has been described since 1960s. To experience the modest change in reproductivity, the estimated total fertility rate, gross reproduction rate and mean age of childbearing are computed for the above-mentioned period corresponding to the data available for the different years. Further, models have been fitted to the data on age-specific fertility rate and its forward and backward cumulative distributions. Finally, the cross validity prediction power technique has been used to check the validity of these models.

KEY WORDS

Age specific fertility rate; cross validity prediction power; statistical modeling; reproductivity.

1. INTRODUCTION

The study of reproductivity is primarily concerned to model fertility curves. Therefore, for many years, modeling fertility curves has attracted the interest of demographers and remains the area of research. The controversy of actual reproductivity experience by natural process for a population over some period of time requires the calculation of some major reproductivity measures and fertility models. In literature, a variety of reproductivity measures and mathematical models have been proposed which have described the reproductivity pattern; see Pollard et al. (1990), Islam and Ali (2004), Peristera and Kostaki (2007) and Nasir et al. (2007). Some of these mathematical models have been shown to provide excellent fits to age-specific fertility rate distributions of human population described in Hoem et al. (1981). Some useful references to assess the fertility pattern in Pakistan are Sathar et al. (1988), Sathar and Kazi (1990), Sathar (1993), Sathar and Casterline (1998), Hussain and Bittles (1999) and Hagan et al. (1999). The existing literature to assess the fertility in Pakistan is lacking the modeling approach to study the reproductivity. This provides a rationale for the present study. In short, to assess the fertility behavior of a population, the different measures of reproductivity can

be used. The present piece of work mainly concentrates on the reproductivity pattern in Pakistan after 1981.

Thus, the main objectives of this paper are:

- To study the pattern of the age specific fertility schedules;
- To compute the various measures of reproductivity;
- To observe the trends of various measures of reproductivity; and
- To fit some mathematical models to the Age-specific fertility rates to the Forward and Backward cumulative distribution of age-specific fertility rates only for year 2005.

This paper is divided into five sections. Section 1 starts with introduction to the issue whereas in section 2, we have reviewed the fertility pattern of Pakistan from various multiple and independent studies presented in the literature since 1960s. In section 3, we have briefly described sources of data that have been used in reproductivity measures. Model for the Age-Specific Fertility Rates (ASFRs) and cross validity prediction power (CVPP) has been presented in section 4. Section 5 includes discussion on the results and lastly, in section 6 final remarks are given.

2. THE FERTILITY PATTERN IN PAKISTAN

Pakistan is the world's seventh most populous country with a population of 130.5 million in 1998 (Source: Census report of Pakistan). To assess the fertility pattern in Pakistan, we have used estimates of total fertility rate (TFR), which is the most widely used index of fertility (Hinde, 1998). Numerous fertility surveys indicate that the TFR in Pakistan remained above 6.0 births per women throughout the 1980s (for details see Table 1). From Table 1, all demographic analysis points to decline in fertility in the 1990s, but there is controversy about decline in fertility in Pakistan before 1990s, as evident from the results presented in Table 1 from the first demographic survey carried out in the 1960s up to the 1990s, demographers could not point out with confidence about the decline in fertility in Pakistan. But after 1990s up to the present time, the demographic research indicate a modest decline in fertility, for example, during the four year period from 1991 through 1994, the estimates imply a decline of 0.30 births. The Pakistan fertility and family planning survey (PFFPS) of 1996-97 provides an estimate of TFR to be 5.30, which suggest a slightly more rapid decline during the 1990s than implied by the previous surveys. Collectively these estimates from different sources imply decline in fertility in Pakistan particularly after 1990s.

Table 1:
Estimates of Total Fertility Rates in Pakistan during 1960s-2000s

Decade	Source	TFR
1960s	Pakistan Growth Estimation Experiment (1962-65)	8.0
	National Impact Survey (1968-69)	5.0
	Pakistan Growth Survey (1968-71)	6.0
	Pakistan Fertility Survey (1975)	7.1
1970s	Pakistan Fertility Survey (1975)	6.3
	Pakistan Labor force and Migration Survey (1979)	7.1
	Pakistan Labor force and Migration Survey (1979)	6.5
	Pakistan Growth Survey (1976-81)	6.9
1980s	Pakistan Contraceptive prevalence Survey (1984-85)	6.0
	Pakistan Demographic Survey (1984-88)	6.9
	Pakistan Demographic and Health Survey (1990-91)	5.4
1990s	Pakistan Demographic Survey (1992)	5.8
	Pakistan Contraceptive prevalence Survey (1994-95)	5.6
	Pakistan Fertility and Family Planning Survey (1996-97)	5.3
2000s	Pakistan Reproductive Health and Family Planning Survey (2000-01)	4.8
	Pakistan Demographic Survey (2001)	4.1
	Pakistan Demographic Survey (2003)	3.9
	Pakistan Demographic Survey (2005)	3.8

3. DATA AND METHODOLOGY

3.1 Sources of Data

A Secondary data on age specific fertility rates (ASFR) of Pakistan during 1980s-2000s have been taken from the Pakistan Demographic Surveys (1984-1986, 1988-1992, 1995-1997, 1999-2001, 2003, 2005), which are shown in Table 2. This data has been used to estimate the different measures of reproductivity. The relevant data for the years 1987 and 1993-1994 are not available in Federal Bureau of Statistics (FBS) and that is why we have dealt with the best available data to infer on fertility pattern in Pakistan. Moreover, in this paper the data of ASFR for 2005 has been used to fit mathematical models.

Table 2:
Age Specific Fertility Rates (ASFR) per 1000 woman of Pakistan during 1980s-2000s

Age group (z-z+5)	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1984	65.76	268.33	367.57	314.42	226.07	109.56	37.88
1985	59.15	272.98	350.79	326.98	235.29	108.57	47.88
1986	54.31	265.75	360.26	303.12	226.22	125.98	52.16
1988	66.00	263.60	333.00	278.30	203.30	111.20	41.80
1989	75.70	265.80	323.40	274.30	197.10	102.00	41.60
1990	75.50	274.80	313.20	276.00	175.90	97.00	30.50
1991	69.00	258.20	315.40	259.00	186.50	82.30	27.40
1992	73.30	261.40	312.90	254.50	162.60	74.50	27.80
1995	59.10	243.40	305.10	241.90	148.10	90.10	29.60
1996	54.7 0	258.2 0	295.90	255.40	143.00	65.50	23.20
1997	52.3 0	231 00	273.20	211.20	142.90	68.40	30.70
1999	36.20	205.60	256.90	203.60	118.30	61.70	25.80
2000	32.90	195.10	244.20	203.80	114.50	54.40	22.90
2001	24.20	162.00	242.90	197.20	118.50	57.90	21.90
2003	23.70	163.10	229.60	190.00	112.70	49.00	18.80
2005	20.30	157.60	225.50	179.90	106.60	50.10	18.10

Source: Pakistan Demographic Surveys 1984-2005

3.2 Reproductivity Measures

Before presenting the reproductivity measures, we considered a fertility index, TFR due to its close connection with the reproductivity measures.

i) The TFR has been estimated using the formula of

$$TFR = 5\sum_{\alpha}^{\beta} ASFR \tag{3.1}$$

and $$TFR = \int_{\alpha}^{\beta} f_y \, dy \,, \tag{3.2}$$

where ‘ α ’ and ‘ β ’ are the minimum and maximum years of reproductive life span of a woman. Generally ‘ α ’ shows the age of woman at minarchi which is taken to be 15 years and ‘ β ’ is taken to be 49 years.

ii) Gross reproduction rate (GRR) is estimated using the formula

$$GRR = \frac{B^F}{B^T} \int_{\alpha}^{\beta} f_y d_y \,. \tag{3.3}$$

This can be approximated as

$$GRR = 5\frac{B^F}{B^T}\sum_{\alpha}^{\beta} f_y \,, \tag{3.4}$$

where $\frac{B^F}{B^T}$ is the proportion of all births which are female and f_y is the age specific fertility rate at the age group 'z to z+5' in the reproductive ages. Also, another approximation of GRR is given by Hinde (1998)

$$GRR = \left(\frac{SR.}{100 + SR.}\right) TFR \,,$$

where SR stands for sex-ratio at birth.

iii) The mean childbearing age (MCA) can be expressed as

$$MCA = \frac{\int_{\alpha}^{\beta} y\, f(y)\, dy}{\int_{\alpha}^{\beta} f(y)\, dy} \,, \tag{3.5}$$

which can approximately estimated by

$$\bar{M} = \frac{\sum_{\alpha}^{\beta} y\, f_y}{\sum_{\alpha}^{\beta} f_y} \,. \tag{3.6}$$

4. MODELING OF AGE SPECIFIC FERTILITY RATES

It is widely known that the distribution of ASFR has a typical shape. The standard fertility shape depicting the age pattern of fertility with zero values before age 10, low but increasing positive values between 15-19, rising to a maximum between 20-29, and then decreasing slowly to reach zero by age 50. This non-linear pattern was the reciprocal of the V-shape. In the literature non-linear regression models come in all shape and size. Some well-known families of models are polynomial models, exponential family, power family and yield-density models. To best capture the fertility pattern of Pakistan, we have used polynomial models because other models cannot adequately describe the fertility pattern of Pakistan. The polynomial models have also been preferred in some instance Brown and Newman (2002), Newman et al. (2004) and Markovic and Sekalic (2006). Using the scattered plot of age specific fertility rates of Pakistan for the year 2005 only shown in Figure 1, it is observed that age specific fertility rates can be fitted by polynomial model with respect to different ages in year. Therefore, an nth degree polynomial model was considered and the form of the *n*th degree polynomial model is

$$y = b_o + \sum_{j=1}^{p} b_j z^j + \varepsilon \,, \tag{4.1}$$

where 'z' is the mid value of age group in years; y is age specific fertility rates; b_o is the constant; b_j is the coefficient of z^j (j= 1, 2,3…,p) and ε is the stochastic error term of the model.

Using the plot of observed forward cumulative ASFR in Pakistan for 2005 shown in Figure 2, it has been observed that forward cumulative ASFR follows an nth degree polynomial model with respect to different ages in years, therefore the form of the model is

$$y^f = b_o + \sum_{j=1}^{p} b_j z^j + \varepsilon , \qquad (4.2)$$

where 'z' is the mid value of age group in years; y^f is forward cumulative age specific fertility rates in 2005 of Pakistan. Where, b_o is the constant; b_j is the coefficient of z^j (j = 1, 2, 3,…, p) and ε is the chance error term of the model.

Using the plot of backward cumulative age specific fertility rates of Pakistan in 2005 shown in Figure 3, it has been observed that backward cumulative age specific fertility rates follows an nth polynomial model with respect to different ages in year. Therefore, the model is

$$y^b = b_o + \sum_{j=1}^{p} b_j z^j + \varepsilon , \qquad (4.3)$$

where 'z' is the mid value of age group in years; y^b is backward cumulative age specific fertility rates; b_o is the constant; b_j is the coefficient of z^j (j = 1, 2, 3…, p) and ε is the disturbance term of the model.

In all these models, we have to select suitable P for which the error sum of square is minimum.

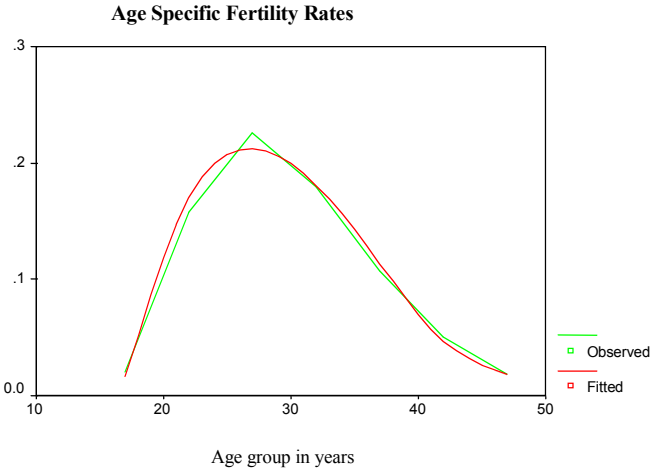

Figure 1: The graph of observed and fitted model of distribution of Age Specific Fertility Rates (ASFR) of Pakistan in 2005.

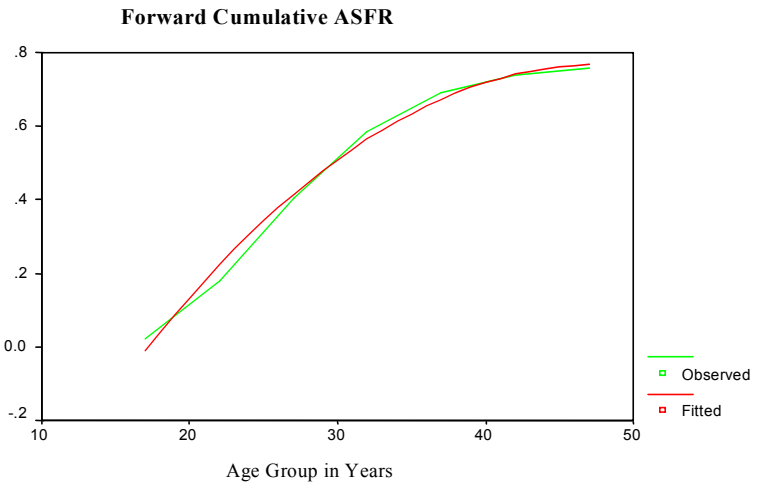

Figure 2: The graph of observed and fitted model of forward cumulative distribution of Age Specific Fertility Rates (ASFR) of Pakistan in 2005.

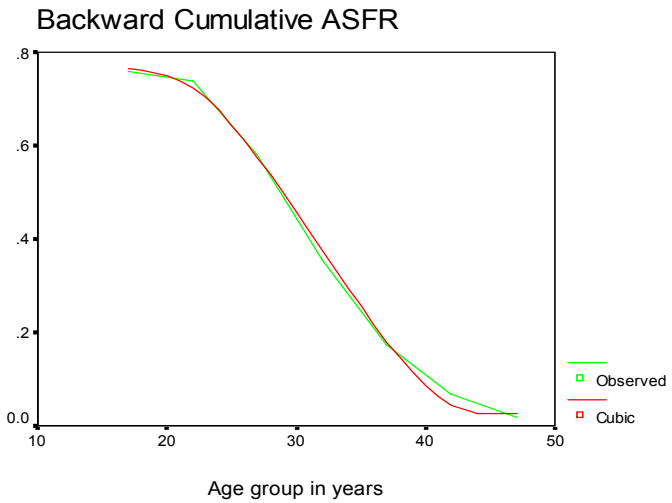

Figure 3: The graph of observed and fitted model of backward cumulative distribution of Age Specific Fertility Rates (ASFR) of Pakistan in 2005.

4.2 Model Validation

To check how much these models are stable over the population, a measure of effectiveness, the cross validity prediction power (CVPP), ρ_c^2 is applied which was presented by Herzberg (1969). The formula for CVPP is

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-p-1)(n-p-2)}(1-R^2)\,, \qquad (4.4)$$

where n is the number of cases, P is the number of explanatory variables in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable. Using the above statistics, it can be concluded that if the prediction equation is applied to many other samples from the same population, then (ρ_{cv}^2) 100% of the variance on the predicted variable would be explained by the model (See Stevens, 1996).

5. RESULTS AND DISCUSSION

Table 2 represents the ASFR in Pakistan during 1980s to 2005. To see the trend of fertility, the data have been presented in graph paper shown in Figure 4. From Figure 4, it is found that all the fertility schedules show the traditional pattern that is the reciprocal of V- shape. Beyond these fertility schedules, it is observed that the highest ASFRs in the age group (25-29) years are 368, 360, 333, 323, 313, 315, 313, 305, 296, 273, 257, 244, 243, 230, 226, per thousand women for the years 1984, 1986, 1988-1992, 1995-97, 1999-2001, 2003, and 2005 respectively. Also, Table 2 and Figure 4 reveal the lowest fertility rates of all fertility schedules in the last age group, 45-49 years. From the above presentation, it is found that decline in fertility is observable and the age- interval 25-29 years is the most fertile period and the age–interval 45-49 years is the least fertile period in the reproductive span of Pakistani woman.

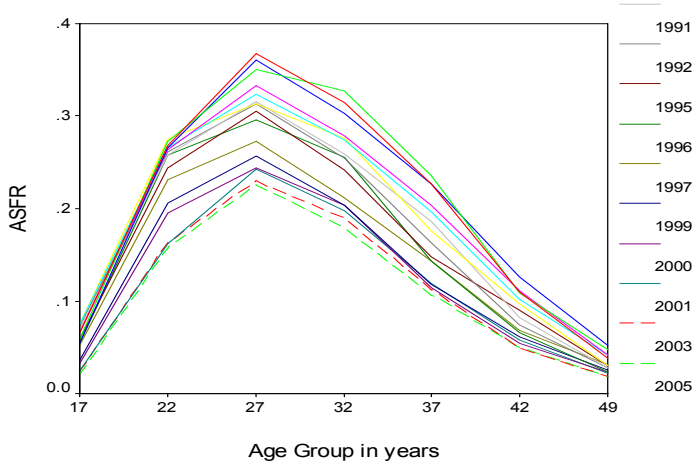

Figure 4: Age Specific Fertility Rates (ASFR) of Pakistan at even point's time during 1980s-2000s. X; Age Group in the Years and Y; Age Specific Fertility Rates.

Table 3 represents the TFR, GRR and MC, for the different years in Pakistan from 1980s to 2000s. To see the trend of TFR, the data have been plotted in graph paper shown in Figure 5. From Figure 5, it is observed that TFR is neither increasing nor decreasing in the interval 1984-86, and then it started to decrease very slowly up to 1996. After 1996 it

shows a modest decline during 1997-99. After year 2000 a very slow decline in TFR is observed up to 2005. To see the trend of GRR, the data have been presented in the same graph shown in Figure 5. From Figure 5 it is observed that GRR is increasing from 1984 to 1985, and started decreasing up to 2001.

Table 3:
Total Fertility Rate (TFR), Gross Reproduction Rate (GRR), Mean age of Childbearing (MCA) of Pakistan during 1980s-2000s

Years	TFR	GRR	MCA
1984	6.95	3.33	30.05
1985	7.01	3.41	30.29
1986	6.94	3.34	30.49
1988	6.49	3.10	30.45
1989	6.40	3.07	29.83
1990	6.21	3.03	29.47
1991	5.99	2.92	29.47
1992	5.84	2.86	29.17
1995	5.59	2.70	29.57
1996	5.48	2.46	29.11
1997	5.05	2.43	29.42
1999	4.54	2.17	29.48
2000	4.34	2.09	29.46
2001	4.12	1.97	29.94
2003	3.93	2.03	29.71
2005	3.79	2.00	29.75

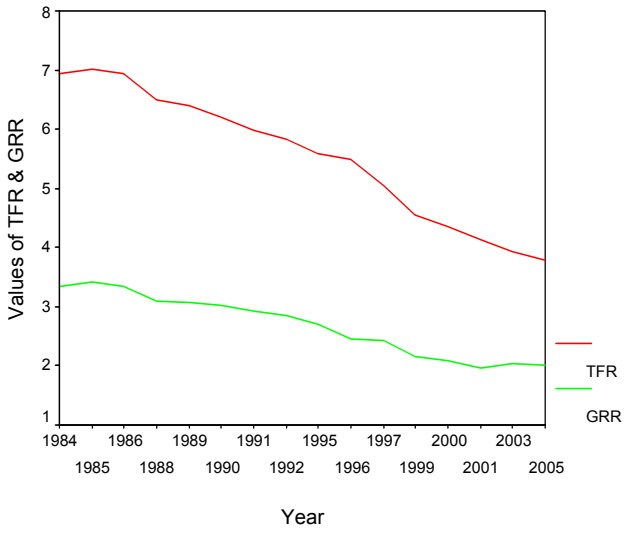

Figure 5: Graph of the estimated TFR and GRR during 1984-2005.

The rate of decrement of GRR were 7.8 %, 5.59 %, and 18.93 % , during 1984-1989, 1992-1995, and 1997-2001 respectively, in which it is found that the rate of decrement is highest during the interval 1997-2001. Column 3 of Table 3 indicates the mean age of childbearing (MC) in Pakistan during 1980s-2000s. To study the trend of MC, we consider the rate of decrement of MC from 1984 to 2005, which is 0.99 %. This indicates the trend of MC is more or less constant.

The polynomial model is assumed for ASFR in Pakistan for the year 2005 and the fitted equation is as follows:

$$y = -1.837958 + (0.188346)\,Z - (0.005509)\,Z^2 + (0.0000498)\,Z^3, \qquad (5.1)$$

And, another polynomial model is assumed for forward cumulative ASFR of Pakistan for the year 2005 and the fitted equation is

$$y^f = -1.103807 + (0.78339)\,Z - (0.000820)\,Z^2, \qquad (5.2)$$

another polynomial model is assumed for backward cumulative ASFR of Pakistan for the year 2005 and the fitted equation is

$$y^b = -0.644987 + (0.181337)\,Z - (0.007053)\,Z^2 + (0.0000744)\,Z^3. \qquad (5.3)$$

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 4. From this Table it is seen that the fitted models in equation (5.1) through (5.3) are highly cross validated and their shrinkage are 0.0469, 0.13366 and 0.01075, these imply that the fitted model for ASFR will be stable more than 94 %, the fitted model for forward cumulative ASFR will be stable more than 79 % and the fitted model for backward cumulative ASFR will be stable more then 98 %.

Table 4:

Estimated Cross Validity Prediction Power (CVPP), ρ^2_{cv} of the Predicted Equations of Age Specific Fertility Rates and its Forward and Backward Cumulative Distribution of Pakistan during 1980s-2000s

Models	n	P	R^2	ρ^2_{cv}
1. Equation 5.1	7	3	0.990043	0.94309
2. Equation 5.2	7	2	0.92803	0.79437
3. Equation 5.3	7	3	0.997716	0.98697

Table 5: Model Fitting

Models	Proportion of Explained Variations	Parameter Estimates	P-value
Model 1	0.990043	b_0	0.0019
		b_1	0.0020
		b_2	0.0028
		b_3	0.0042
Model 2	0.92803	b_0	0.0010
		b_1	0.0008
		b_2	0.0036
Model 3	0.997716	b_1	0.0126
		b_2	0.0077
		b_3	0.0074

6. FINAL REMARKS

In this study, from the various multiple and independent sources ASFRs in Pakistan show the declining trend and the traditional reciprocal of V-shape pattern. It is also concluded that the age interval 25-29 years is the most fertile period and the age group 45-49 years is the least fertile period in the reproductive life of Pakistani woman, MC has been approximately constant during 1980s-2000s. The parameter TFR and GRR of reproductivity started to decrease modestly after 1996 up to 2000. ASFR and its forward and backward cumulative distribution in Pakistan for the year 2005 have followed polynomial models.

However, this study is based on the available ASFRs during 1980s-2000s from various sources, but for the estimation of missing years ASFRs, special statistical formulation are required, which remains to be explored.

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