Pak. J. Statist.
2008 Vol. 24(2) pp 87-98

ESTIMATION OF CURRENT POPULATION PRODUCT IN SUCCESSIVE SAMPLING

Amelia V. García Luengo
Department of Statistics and Applied Mathematics, University of Almería,
La Cañada de San Urbano 04120 Almería, Spain
Email: amgarcia@ual.es

ABSTRACT

The problem of estimation of the population product and the change, for the current occasion, based on the samples selected over two occasions has been considered. Expressions for optimum estimator and its variance have been derived. The values of optimum matched proportion has been tabulated. The gain in efficiency of the proposed estimate over the direct estimate using no information gathered on the first occasion is computed. An empirical study is made to study the performance of the proposed strategy.

KEY WORDS

Successive sampling, population product estimator, gain in efficiency, matching fraction, change estimator.

AMS Classification (62D05)

1. INTRODUCTION

If the composition and characteristics of the units remained unchanged, a single occasion would be enough to perform a sampling, as the results would always be valid. In practice, the aforementioned changes prevent us from that simplification and, at the same time, give rise to a set of targets -such as cross estimation of population parameters and net changes, estimations of average values of parameters through time, etc. that can be analyzed by means of continuous surveys.

If a population unit value in a occasion can be related to the same unit in the next occasion, then we are enabled to use the information obtained in the preceding occasion to improve the current estimation of the population parameter. To this effect, the sample must be obtained in such a way that the sample units in the two successive occasion have some common units, in order to use the preceding sampling information.

These are some of the reasons which explain that partial replacement of sample units should be used:

1. It reduces costs (using totally new samples at each time can be unduly expensive).
2. It increases the estimators' accuracy.
3. The indefinite presence of the same units in the sample can result in failures and efficiency reduction of the estimators.

The theory on successive sampling, has been discussed extensively by several authors in the case of estimating the population mean (total) (Rao and Mudholkar, 1967; Artés and García, 2001, 2002).

In many practical situations the estimate of the population ratio and product of two characters for the most recent occasion may be of considerable interest. The theory of estimation the population ratio of two characters over two occasions has been considered by Rao (1957), Rao y Pereira (1968), Thipathi and Sinha (1976), Okafor and Arnab (1987), Okafor (1992), Artés and García (2001), García and Artés (2002) among others.

In this paper we have presented some sampling strategies for estimating, by a linear estimate, the population product of two characters over two occasions.

2. SELECTION OF THE SAMPLE

Suppose that the samples are of size n on both occasions, we use a simple random sampling and the size of the population N is sufficiently large for the factor of correction be ignored.

Let a simple random sample of size n be selected on the first occasion from a universe of size N. The measurements are taken on two characteristics y and x in each of two occasions. When selecting the second sample, we assume that $m = pn\left(0 < p < 1\right)$. The units of the second sample is selected from the sampling units of the first occasion sample.

3. NOTATION

$x_i\left(y_i\right)$, the variable $x\left(y\right)$ on *i*th occasion, i = 1, 2,

$P_1 = \overline{Y}_1\overline{X}_1\left(P_2 = \overline{Y}_2\overline{X}_2\right)$, the population product on the first (second) occasion,

$\hat{P}_1 = \overline{y}_1\overline{x}_1\left(\hat{P}_2 = \overline{y}_2\overline{x}_2\right)$, the estimator of the population product on the first (second) occasion,

$R_1 = \dfrac{\overline{Y}_1}{\overline{X}_1}\left(R_2 = \dfrac{\overline{Y}_2}{\overline{X}_2}\right)$, the population ratio on the first (second) occasion,

$\hat{R}_1 = \dfrac{\overline{y}_1}{\overline{x}_1}\left(\hat{R}_2 = \dfrac{\overline{y}_2}{\overline{x}_2}\right)$, the estimator of the population ratio on the first (second) occasion,

$\rho_1\left(\rho_2\right)$, the correlation coefficients between the variables y_1 and x_1 (y_2 and x_2),

$\rho_3\left(\rho_4\right)$, the correlation coefficients between the variables y_2 and x_1 (y_1 and x_2),

$\rho_5\left(\rho_6\right)$, the correlation coefficients between the variables x_1 and x_2 (y_1 and y_2),

$\hat{P}_{1m}\left(\hat{P}_{2m}\right)$, the estimator of the population product on the first (second) occasion based on the matched sample of m units,

$\hat{P}_{1u}\left(\hat{P}_{2u}\right)$, the estimator of the population product on the first (second) occasion based on the unmatched sample of u units.

We construct the optimum estimate of the population product for the second occasion, P_2, by combining the estimates of the populations products for the matched and unmatched sample on the first and on the second occasion. (Hansen, Hurwitz and Madow, 1953).

4. MEANS AND VARIANCES OF POPULATION PRODUCTS

In this section, we find means and variances of population products. Following Hansen et al. (1953) methods, we use

$$\hat{P}_2' = a\hat{P}_{1u} + b\hat{P}_{1m} + c\hat{P}_{2m} + d\hat{P}_{2u},$$

where $\hat{P}_{1u}, \hat{P}_{1m}, \hat{P}_{2m}$ and $\hat{P}_{2u}$ are defined in section 3.

$$E\left(\hat{P}_{1u}\right) = E\left(\hat{P}_{1m}\right) = P_1 \text{ and } E\left(\hat{P}_{2u}\right) = E\left(\hat{P}_{2m}\right) = P_2$$

We find that

$$\hat{P}_2' = (a+b)P_1 + (c+d)P_2.$$

If we now require that $\hat{P}_2'$ be an unbiased estimate of P_2, we must have

$$a+b=0 \text{ and } c+d=1$$

so that

$$\hat{P}_2' = a\left(\hat{P}_{1u} - \hat{P}_{1m}\right) + c\hat{P}_{2m} + (1-c)\hat{P}_{2u}.$$

The variance of $\hat{P}_2'$ is

$$V\left(\hat{P}_2'\right) = a^2\left(\frac{1}{q} + \frac{1}{p}\right)\frac{1}{n\bar{X}_1^2}A + c^2\frac{1}{pn\bar{X}_2^2}B + (1-c)^2\frac{1}{qn\bar{X}_2^2}B - 2ac\,Cov\left(\hat{P}_{1m}, \hat{P}_{2m}\right),$$

where

$$A = S_{y_1}^2 + \hat{R}_1^2 S_{x_1}^2 + 2\hat{R}_1 Cov(y_1, x_1),$$

$$B = S_{y_2}^2 + \hat{R}_2^2 S_{x_2}^2 + 2\hat{R}_2 Cov(y_2, x_2),$$

$$Cov\left(\hat{P}_{1m}, \hat{P}_{2m}\right) = \frac{1}{pn\bar{X}_1\bar{X}_2}\Big[Cov(y_1, y_2) + \hat{R}_1 Cov(y_2, x_1) + \hat{R}_2 Cov(y_1, x_2) + \hat{R}_1\hat{R}_2 Cov(x_1, x_2)\Big].$$

We wish to choose values of a and c that minimize $V\left(\hat{P}_2'\right)$. Equating to zero the derivatives of $V\left(\hat{P}_2'\right)$ with respect to a and c, it follows that the optimum values are

$$a_{opt} = \frac{pq\bar{X}_1 ABC}{A^2 B\bar{X}_2 - q^2 \bar{X}_2 AC^2} = \frac{pqBC\bar{X}_1}{\left(AB - q^2C^2\right)\bar{X}_2} \quad c_{opt} = \frac{pAB}{AB - q^2C^2}$$

where

$$C = \left[Cov\left(y_1, y_2\right) + \hat{R}_1 Cov\left(y_2, x_1\right) + \hat{R}_2 Cov\left(y_1, x_2\right) + \hat{R}_1\hat{R}_2 Cov\left(x_1, x_2\right)\right].$$

Thus, the estimate with optimum values for a and c may be written

$$\hat{P}_2' = \frac{pq\bar{x}_1 ABC}{\left(AB - q^2C^2\right)A\bar{x}_2}\left(\hat{P}_{1u} - \hat{P}_{1m}\right) = \frac{pAB}{AB - q^2C^2}\hat{P}_{2m}\left(1 - \frac{pAB}{AB - q^2C^2}\right)\hat{P}_{2u}, \tag{4.1}$$

and its variance is

$$V\left(\hat{P}_2'\right) = \frac{B}{\bar{X}_2^2 n}\frac{AB - qC^2}{q^2C^2}. \tag{4.2}$$

Note that if $q = 0, p = 1$, complete matching or $q = 0, p = 1$, no matching this variance (4.2) has the same value,

$$V\left(\hat{P}_2'\right) = \frac{1}{\bar{X}_2^2 n}\left(S_{y_2}^2 + \hat{R}_2^2 S_{x_2}^2 + \hat{R}_2 Cov\left(y_2, x_2\right)\right).$$

Thus, for current estimates, equal precision is obtained either by keeping the same sample or by changing it on every occasion.

If $\bar{x}_1 = \bar{x}_2$, the estimate given by (1) is somewhat simplified

$$\hat{P}_2' = \frac{pqABC}{\left(AB - q^2C^2\right)A}\left(\hat{P}_{1u} - \hat{P}_{1m}\right) + \frac{pAB}{AB - q^2C^2}\hat{P}_{2m} + \left(1 - \frac{pAB}{AB - q^2C^2}\right)\hat{P}_{1u},$$

but its variance is unchanged, that is,

$$V\left(\hat{P}_2'\right) = \frac{B}{\bar{X}_2^2 n}\frac{AB - qC^2}{AB - q^2C^2}.$$

Note, also, that an estimate for the first occasion is given by (4.1) simply by interchanging P_1's and P_2's if the estimate for the first occasion can await a time until data for both occasions are available.

$$\hat{P}_1' = \frac{pq\bar{x}_2 ABC}{\left(AB - q^2C^2\right)\bar{x}_1 B}\left(\hat{P}_{2u} - \hat{P}_{2m}\right) + \frac{pAB}{AB - q^2C^2}\hat{P}_{1m} + \left(1 - \frac{pAB}{AB - q^2C^2}\right)\hat{P}_{1u} \,. \tag{4.3}$$

Its variance is

$$V\left(\hat{P}_1'\right) = \frac{B}{\bar{X}_1^2 n}\frac{AB - qC^2}{AB - q^2C^2}$$

Equating to zero the derivative of $V\left(\hat{P}_2'\right)$ with respect to q, we find that the variance of $\hat{P}_2'$ will have its minimum value if we choose

$$q_{opt} = \frac{AB - \sqrt{A^2B^2 - C^2AB}}{C^2} = \frac{AB}{C^2}\left[1 - \sqrt{1 - C^2/AB}\right], \;\; AB > C^2 \,, \tag{4.4}$$

and

$$V_{\min}\left(\hat{P}_2'\right) = \frac{B}{\bar{X}_2^2 n}\frac{AB + \sqrt{A^2B^2 - C^2AB}}{2AB} = \frac{B}{\bar{X}_2^2 n}\frac{AB}{C^2}\frac{1 + \sqrt{1 - C^2/AB}}{2} \tag{4.5}$$

However, if only the estimate using information gathered on the second occasion is considered, the estimator of the population product is

$$\hat{P} = p\hat{P}_{2m} + q\hat{P}_{2u} \,,$$

and its variance is

$$V\left(\hat{P}\right) = \frac{1}{\bar{X}_2^2 n}\left(S_{y_2}^2 + \hat{R}_2^2 S_{x_2}^2 + 2\hat{R}_2 Cov\left(y_2, x_2\right)\right) = \frac{B}{\bar{X}_2^2 n} \,. \tag{4.6}$$

We find

$$\frac{B}{\bar{X}_2^2 n}\frac{1 + \sqrt{1 - C^2/AB}}{2} \le \frac{B}{\bar{X}_2^2 n} \,. \tag{4.7}$$

We can compute the gain in precision G of the estimate obtained by using a linear estimate over the direct estimate using no information gathered on the first occasion

$$G = \frac{V\left(\hat{P}\right)}{V\left(\hat{P}_2'\right)} = \frac{AB - q^2C^2}{AB - qC^2} \,, \tag{4.8}$$

or

$$G_{opt} = \frac{V\left(\hat{P}\right)}{V_{\min}\left(\hat{P}_2'\right)} = \frac{2}{1 + \sqrt{1 - C^2/AB}} \,, \tag{4.9}$$

If now A, B and C are rewritten in terms of the correlation coefficients and the coefficients of variation

$$A = \overline{Y}_1^2 \left(C_{y_1}^2 + C_{x_1}^2 + 2\rho_1 C_{y_1} C_{x_1} \right);$$

$$B = \overline{Y}_2^2 \left(C_{y_2}^2 + C_{x_2}^2 + 2\rho_2 C_{y_2} C_{x_2} \right);$$

$$C = \overline{Y}_1 \overline{Y}_2 \left(\rho_6 C_{y_1} C_{y_2} + \rho_3 C_{x_1} C_{x_2} + \rho_4 C_{x_2} C_{y_1} + \rho_5 C_{x_1} C_{x_2} \right)$$

and assuming that

$$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0$$

$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho; \ \ \rho_5 = \rho_6 = \rho_0 .$$

The expressions (4.4), (4.8) and (4.9), become

$$q_{opt} = \frac{(1+\rho)^2 - \sqrt{(1+\rho)^4 - (\rho_0+\rho)^2 (1+\rho)^2}}{(\rho_0+\rho)^2} ,$$

$$G = \frac{\left(1+\rho^2+2\rho\right) - q^2\left(\rho_0^2+\rho^2+2\rho_0\rho\right)}{\left(1+\rho^2+2\rho\right) - q\left(\rho_0^2+\rho^2+2\rho_0\rho\right)} ,$$

$$G_{opt} = \frac{1(1+\rho)}{(1+\rho) + \sqrt{(1+\rho)^2 - (\rho_0+\rho)^2}} ,$$

Table 1:
Gain in precision, G, of the estimate proposed over the direct estimate.

	ρ = 0.3			ρ = 0.6			ρ = 0.9		
q / ρ_0	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
0.3	1.04	1.05	1.05	1.07	1.09	1.08	1.09	1.12	1.18
0.6	1.12	1.15	1.15	1.14	1.19	1.19	1.16	1.22	1.22
0.9	1.24	1.37	1.44	1.25	1.39	1.48	1.25	1.40	1.51

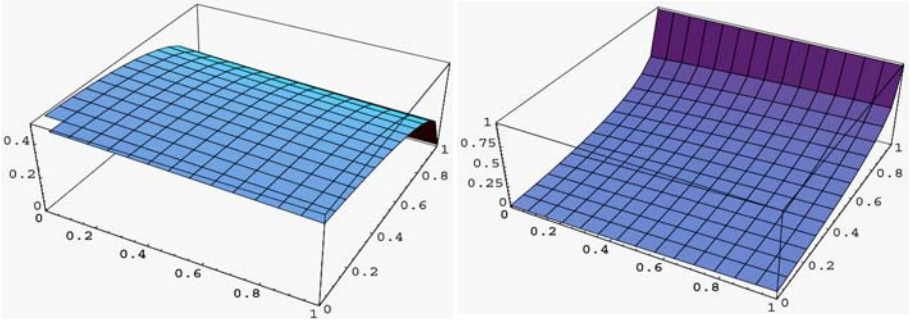

Figure 1: Optimum matching fraction $1 = q_{opt}$ and Gain in precision, $G_{opt} - 1$

If, also we assume that

$$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0$$

$$\rho_0 = \rho$$

the expressions (4.4) and (4.9), become

$$q_{opt} = \frac{(1+\rho)^2 - \sqrt{(1+\rho)^4 - 4\rho^2 (1+\rho)^2}}{4\rho^2},$$

$$G_{opt} = \frac{2(1+\rho)}{(1+\rho) + \sqrt{(1+\rho)^2 - 4\rho^2}},$$

The figure (2) shows for a series of values of ρ the optimum that should be matched and the gain in precision compared with no matching. The best percentage to match never exceeds 50% and decrease steadily as ρ increases. The greatest attainable gain in precision is 100% when $\rho = 1$. Unless ρ is high, the gain are modest.

Although the optimum percentage to match varies with ρ, only a single percentage can be used in practice for all items in a survey.

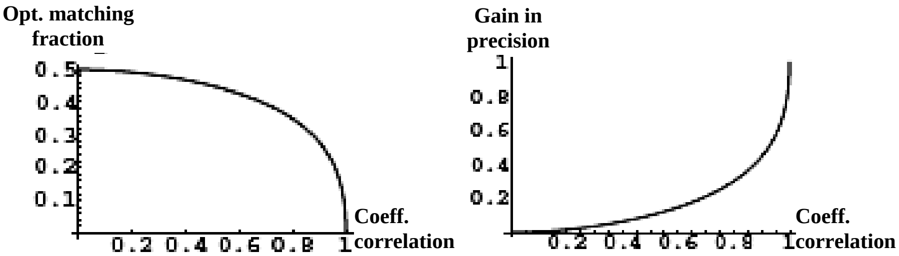

Figure 2: Optimum matching fraction, when $\rho_0 = \rho$ and Gain in precision, G_{opt}, when $\rho_0 = \rho$

5. ESTIMATES OF THE CHANGE

One possible obvious estimate of the change Δ, can be made by using simple averages on both occasions

$$\Delta = p\hat{P}_{2m} + q\hat{P}_{2u} - \left(p\hat{P}_{1m} + q\hat{P}_{1u} \right)$$
$$= p\left(\hat{P}_{2m} - \hat{P}_{1m} \right) + q\left(\hat{P}_{2u} + \hat{P}_{1u} \right).$$

This estimator is unbiased

$$E\left(\Delta\right) = p\left[E\left(\hat{P}_{2m}\right) - E\left(\hat{P}_{1m}\right) \right] + q\left[\left(\hat{P}_{2u}\right) - E\left(\hat{P}_{1u}\right) \right]$$
$$= p\left[P_2 - P_1\right] + q\left[P_2 - P_1\right] = P_2\left(p+q\right) = P_2 = P_1 ,$$

and its variance is given by

$$V\left(\Delta\right) = \frac{1}{n\bar{X}_1^2} A + \frac{1}{n\bar{X}_2^2} B - 2p\frac{1}{n\bar{X}_1\,\bar{X}_2} C . \qquad (5.1)$$

In the special case that $A = B, \bar{X}_1 = \bar{X}_2, \left(V\left(\hat{P}_1\right) = V\left(\hat{P}_2\right) \right)$, the variance is simplified to the form

$$V\left(\Delta\right) = \frac{2}{\bar{X}_1^2 n}\left(A - pC\right) . \qquad (5.2)$$

If we consider the more general linear estimate of the change of the form (Hansen, Hurwitz y Madow (1953))

$$\Delta_2 = a\hat{P}_{1u} + b\hat{P}_{1m} + c\hat{P}_{2m} + d\hat{P}_{2u} ,$$

subject to the condition that this provides an unbiased estimate

$$E\left(\Delta_2\right) = aE\left(\hat{P}_{1u}\right) + bE\left(\hat{P}_{1m}\right) + cE\left(\hat{P}_{2m}\right) + dE\left(\hat{P}_{2u}\right)$$
$$= aP_1 + bP_1 + cP_2 + dP_2 = \left(a+b\right)P_1 + \left(c+d\right)P_2 = P_2 - P_1 .$$

We find that we must take

$$a + b = -1 \text{ and } c + d = 1 ,$$

then

$$\Delta_2 = a\hat{P}_{1u} - \left(a+1\right)\hat{P}_{1m} + c\hat{P}_{2m} + \left(1-c\right)\hat{P}_{2u} .$$

Following the same procedure as for the estimate of the population product in the second period

$$V\left(\Delta_2\right)=a^2V\left(\hat{P}_{1u}\right)+\left(a+1\right)^2V\left(\hat{P}_{1m}\right)+c^2V\left(\hat{P}_{2m}\right)$$
$$+\left(1-c\right)^2V\left(\hat{P}_{2u}\right)-2\left(a+1\right)c\,Cov\left(\hat{P}_{1m},\hat{P}_{2m}\right),$$

$$a_{opt}=\frac{-q\bar{X}_2\left(AB-qC^2\right)+BCpq\bar{X}_1}{\left(AB-q^2C^2\right)\bar{X}_2},$$

$$c_{opt}=\frac{pAB\bar{X}_1+C\bar{X}_2\,pqA}{\left(AB-q^2C^2\right)\bar{X}_1},$$

We find that the estimate that minimizes the variance is

$$\Delta_2=\frac{q\left(AB-qC^2\right)}{AB-q^2C^2}\left(\hat{P}_{2u}-\hat{P}_{1u}\right)+\frac{pAB}{AB-q^2C^2}\left(\hat{P}_{2m}-\hat{P}_{1m}\right)$$
$$+\frac{pqC}{AB-q^2C^2}\left[B\frac{\bar{x}_1}{\bar{x}_2}\left(\hat{P}_{1u}-\hat{P}_{1m}\right)-A\frac{\bar{x}_2}{\bar{x}_1}\left(\hat{P}_{2u}-\hat{P}_{2m}\right)\right].$$

In the special case that $A=B,\bar{X}_1=\bar{X}_2$, $\left(V\left(\hat{P}_1\right)=V\left(\hat{P}_2\right)\right)$, the estimate is greatly simplified to the form

$$\Delta_2=\frac{pA}{A-Cq}\left(\hat{P}_{2m}-\hat{P}_{1m}\right)+\frac{q\left(A-C\right)}{A-Cq}\left(\hat{P}_{2u}-\hat{P}_{1u}\right). \qquad (5.3)$$

This estimate of the change is obtained by taking the weighted mean of the two estimates $\hat{P}_{2m}-\hat{P}_{1m}$ and $\hat{P}_{2u}-\hat{P}_{1u}$. The weights to be assigned to these estimates are

$$\frac{pA}{A-Cq}\text{ and }\frac{q\left(A-C\right)}{A-Cq},$$

with

$$\frac{pA}{A-Cq}+\frac{q\left(A-C\right)}{A-Cq}=1.$$

The variance of Δ_2 is given by

$$V\left(\Delta_2\right)=\frac{2A}{\bar{X}_1^2n}\,\frac{A-C}{A-Cq}.$$

We can note that for $A/C>0$ the last expression is minimum for $q=0$, i.e., the variance of Δ_2 will be minimized if the units on both occasions are identical. In this case

$$V\left(\Delta_2\right)=\frac{2A}{\bar{X}_1^2n}\,\frac{A-C}{A}.$$

6. COMPARISON OF THE EFFICIENCY OF THE PROPOSED ESTIMATORS OF THE CHANGE

The efficiency of the estimate, (5.2), over the estimate

$$V\left(\Delta_2\right)=\frac{2A}{\bar{X}_1^2 n}\,\frac{A-C}{A-Cq}\,,$$

is

$$\frac{V\left(\Delta_2\right)}{V\left(\Delta\right)}=\frac{A\left(A-C\right)}{A^2-AC+C^2pq}\,.$$

If $p=0,\ q=1$, (no matching) and the two estimators are equally efficient. Identical conclusion is obtained if $q=0,\ p=1$, (perfect matching).

However, if $p\neq 0$ and $q\neq 0$ is obtained more efficient for Δ_2, when the correlation between the first and second periods for an observation on the same sampling units increases.

The expression

$$\frac{V\left(\Delta_2\right)}{V\left(\Delta\right)}=\frac{A\left(A-C\right)}{A^2-AC+C^2pq}\,,$$

is rewritten in terms of the correlation coefficients and assuming that

$$C_{y_1}=C_{y_2}=C_{x_1}=C_{x_2}=C_0\,,$$

$$\rho_1=\rho_2=\rho_3=\rho_4=\rho\,,$$

$$\rho_5=\rho_6=\rho_0\,.$$

Its become

$$\frac{V\left(\Delta_2\right)}{V\left(\Delta\right)}=\frac{\left(1-\rho\right)\left(1-\rho_0\right)}{\left(1-\rho\right)\left(1-\rho_0\right)+\left(\rho_0-\rho\right)^2 pq}\,.$$

If $\rho=\rho_0=0$, for different values of p and q,

$$\frac{V\left(\Delta_2\right)}{V\left(\Delta\right)}=1$$

and identical precision is obtained with the two estimators.

7. AN EMPIRICAL STUDY

The data under consideration is taken from census 1951 and census 1961, West Bengal, District Census Hand Book, Midnapore. (Das (1982)).

The characters x_1 and x_2 are numbers of houses for 1951 and 1961 respectively and the characters y_1 and y_2 are numbers of literate persons for 1951 and 1961 respectively. For this population we obtain

$$\begin{array}{llll}
\bar{x}_1 = 38.3696 & \hat{C}_{x_1} = 1.3916 & \hat{\rho}_5 = 0.7990 & \hat{\rho}_3 = 0.5471 \\
\bar{x}_2 = 50.4321 & \hat{C}_{x_2} = 1.0585 & \hat{\rho}_6 = 0.5392 & \hat{\rho}_4 = 0.7028 \\
\bar{y}_1 = 31.4321 & \hat{C}_{y_1} = 2.2129 & \hat{\rho}_1 = 0.9187 & \\
\bar{y}_2 = 42.5761 & \hat{C}_{y_2} = 1.5048 & \hat{\rho}_2 = 0.7952 &
\end{array}$$

From these data we state that

$$\hat{V}_{\min}\left(\hat{P}_2'\right) = 0.87\frac{B}{\bar{x}_2^2 n} < \frac{B}{\bar{x}_2^2 n} = \hat{V}\left(\hat{P}\right).$$

(G_{opt} = 1.1494%) which means a gain in precision of 14.94% of the proposed estimator over the usual estimator.

We have also calculated the optimum matching fraction

$$\hat{p}_{opt} = 42.59\% .$$

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Ajit Kumar Das for the use of his numerical data.

REFERENCES

1. Arnab, R. and Okafor, F.C. (1992). A note on double sampling over two occasions. *Pak. J. Statist.* 8(3) B, 9-18.
2. Artés, E.M. and García, A.V. (2001). Estimation of current population ratio in successive sampling. *J. Ind. Soc. Agri. Statis.* 54(3), 342-354.
3. Artés, E.M. and García, A.V. (2002). Diseños muestrales en el tiempo. *Servicio de Publicaciones de la Universidad de Almería*.
4. Artés, E.M. and García, A.V. (2005). Multivariate indirect methods of estimation in successive sampling. *J. Ind. Soc. Agri. Statis.* 59(2), 97-103.
5. Cochran, W.G. (1977). *Sampling Techniques*, Third Edition, John Wiley & Sons, New York.
6. Das, K. (1982). Estimation of population ratio on two occasions. *J. Ind. Soc. Agri. Statis.* 34(2), 1-9.
7. Gandge, S.N., Varghese, T. and Prabhu-Ajgaonkar, S.G. (1993). A note on Modified Product Estimator. *Pak. J. Statist.* 9(3) B, 31-36.

8. García, A.V. (2001). *Mejora de estimadores en muestreo en ocasiones sucesivas*. Servicio de Publicaciones de la Universidad de Almería.
9. García, A.V. and Artés, E.M. (2002). Improvement on estimating of current population ratio in successive sampling. *Brazilian J. Probab. and Statist.* 16(2), 107-122.
10. García, A.V. and Artés, E.M. and Oña, I. (2004). Estimación de la Varianza poblacional en el Muestreo en Ocasiones Sucesivas. *Metodolog´ıa de Encuestas*, Volumen 6, No. 1, 9-22.
11. Hansen, M.H., Hurwitz, W.N. and Madow, W.G. (1953). *Sample Survey Methods and Theory*, Vol. 1 y 2, John Wiley and Sons, Inc., New York and London.
12. Okafor, F.C. and Arnab, R. (1987). Some strategies of two-stage sampling for estimating population ratios over two occasions. *Austrl. J. Statist.* 29(2), 128-142.
13. Okafor, F.C. (1992). The theory and application of sampling over two occasions for the estimation of current population ratio. *Statistica*. 1, 137-147.
14. Rao, P.S.R.S. and Mudholkar, G.S. (1967). Generalized multivariate estimators for the Mean of Finite Population Parameters. *J. Amer. Statist. Assoc.* 62, 1008-1012.
15. Ray, S.K. and Singh, R.K. (1985). Some Estimators for the Ratio and Product of Population Parameters. *J. Ind. Soc. Agri. Stat.* 37(1), 1-10.
16. Shah, S.M. and Shah, D.N. (1973). Ratio-cum-product estimators for estimating ratio (product) of two population parameters. *The Ind. J. Statist.* 40, Series C, Pt. 2, 156-166.
17. Singh, R.K. (1982). Generalized estimators for the estimation of ratio and product of population parameters. *J. Statist. Research*, 16, (1 & 2), 15-23.
18. Singh, P. and Yadav, R.J. (1992). Generalised Estimation Under Successive Sampling. *J. Ind. Soc. Agri. Statis.* 44, 27-36.
19. Singh, V.K., Singh, H.P. and Shukla, D. (1994), A general class of chain estimators for ratio and product of two means of finite population. *Commun. Statist.-Theory Meth.* 23(5), 1341-1355.
20. Sukhatme, P.V., Sukhatme, B.V., Sukhatme, S. and Asok, C. (1984), *Sampling Theory of Surveys with Applications 3a Edic.* Iowa State University Press , USA; Indian Society of Agricultural Statistics, India.