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DOUBLE QUARTILE RANKED SET SAMPLES

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ABSTRACT

Double quartile ranked set sampling procedure (DQRSS) and its properties for estimating the population mean are introduced. The performance of DQRSS with respect to simple random sapling (SRS), ranked set sampling (RSS) and quartile ranked set samples (QRSS) for estimating the population mean, is considered. The DQRSS estimator is unbiased of the population mean for symmetric distributions about its mean. In addition, the DQRSS method is more efficient than the SRS, RSS, and QRSS for all symmetric and asymmetric distributions considered in this study. For asymmetric distributions considered in this study, DQRSS estimator has a smaller bias.

KEYWORDS

Ranked set sampling, quartile ranked set sampling, double quartile ranked set sampling

1. INTRODUCTION

McIntyre (1952) introduced ranked set sampling method for estimating the mean of pasture yields. In situations where the experimental or sampling units in a study can be more easily ranked than quantified, McIntyre proposed that the mean of m sample units based on a RSS as an estimator of the population mean. This estimator is unbiased estimator with a smaller variance compared to the usual sample mean based on a SRS of the same size. Takahasi and Wakimoto (1968) provided the mathematical properties of RSS. Dell and Clutter (1972) showed that RSS estimator is an unbiased for the population mean regardless of error in ranking. Samawi et al. (1996) suggested using extreme ranked set sampling (ERSS) for estimating a population mean, and showed that for symmetric distributions, the ERSS estimator is unbiased and has a smaller variance than the SRS estimator. Muttlak (1997) suggested using median ranked set sampling (MRSS) to increase the efficiency of the estimator and to reduce errors in ranking. Al-Saleh and Al-Kadiri (2000) introduced double ranked set sampling for estimating the population mean, they showed that the ranking in the second stage is easier than the ranking in the first stage. Al-Saleh and Al-Omari (2002) introduced multistage ranked set sampling, that increase the relative efficiency for estimating the population mean for fixed sample size. Muttlak (2003) proposed QRSS for estimating the population mean and to reducing the errors in ranking comparing to RSS.

2. SAMPLING METHODS

2.1 Quartile ranked set samples

In QRSS method, select m units from the population and rank the units within each sample with respect to a variable of interest. If the sample size is even, select for measurement from the first $m/2$ samples the $(\mathrm{q}_1(m+1))$th smallest rank and from the second $m/2$ samples the $(\mathrm{q}_\mathrm{u}(m+1))$th smallest rank. If the sample size is odd, select from the first $(m-1)/2$ samples the $(\mathrm{q}_1(m+1))$th smallest rank and from the other $(m-1)/2$ samples the $(\mathrm{q}_\mathrm{u}(m+1))$th smallest rank, and from one sample the median for that sample for actual measurement.

2.2 Double ranked set sampling

DRSS can be described as follows:

1) Identify m^3 elements from the target population and divide these elements randomly into m sets each of size m^2 elements.
2) Use the usual RSS procedure on each set to obtain m ranked set samples of size m each.
3) Apply the RSS procedure again on step (2) to obtain a DRSS of size m.

In this article, we consider double quartile ranked set samples (DQRSS) as a modification of RSS for estimating the population mean. The performance of DQRSS with respect to SRS, RSS and QRSS for estimating the population mean, is considered. The results indicates that the use of DQRSS for estimating the population mean is more efficient than SRS, RSS and QRSS for all distributions considered in this study. For asymmetric distributions, the DQRSS estimator has smaller bias with variance smaller than that of the SRS estimator.

2.3 Double quartile ranked set samples

The DQRSS procedure can be described as follows:

Step 1: Select m^3 units from the population and divide them into m^2 samples each of size m .

Step 2: If the sample size is even, select from the first $m^2/2$ samples the $(\mathrm{q}_1(m+1))$th smallest rank, and from the second $m^2/2$ samples the $(\mathrm{q}_3(m+1))$th smallest rank. If the sample size is odd, select from the first $m(m-1)/2$ samples the $(\mathrm{q}_1(m+1))$th smallest rank, the median from the next m samples and the $(\mathrm{q}_3(m+1))$th smallest rank from the second $m(m-1)/2$ samples. This step yield m sets each of size m.

Step 3: Apply the QRSS procedure on the m sets obtained in step 2, to get a DQRSS sample of size *m*.

Step 4: The whole cycle may be repeated n times to obtain a sample of size mn from DQRSS.

Note that we will take the nearest integer of $(q_1(m+1))\text{th}$ and $(q_3(m+1))\text{th}$, where $q_1 = 0.25$ and $q_3 = 0.75$.

3. ESTIMATING OF THE POPULATION MEAN

Let $X_{11}, X_{12}, ..., X_{1m}$; $X_{21}, X_{22}, ..., X_{2m}$;…; $X_{m1}, X_{m2}, ..., X_{mm}$; be m independent random samples of size m and assume that each variable X_{ij} has the same distribution function $F(x)$ with mean μ and variance σ^2. Let $X_{i(1)}, X_{i(2)}, ..., X_{i(m)}$ $(i = 1, 2, ..., m)$ be the ordered statistics of the ith sample $X_{i1}, X_{i2}, ..., X_{im}$ $(i = 1, 2, ..., m)$. Let $Y_1, Y_2, ..., Y_m$ be RSS, then $Y_i \overset{d}{=} X_{(i)}$. The estimator of the population mean μ using RSS is defined by $\bar{Y}_{RSS} = \frac{1}{m}\sum_{i=1}^{m} Y_i$, with variance given by

$$\text{Var}(\bar{Y}_{RSS}) = \frac{\sigma^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}(\mu_{(i)} - \mu)^2 .$$

The estimator of the population mean μ using SRS is defined by $\bar{X}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} X_i$, with variance σ^2 / m.

At the kth cycle $(k = 1, 2, ..., n)$, for even sample size, let $Y^*_{i(q_1(m+1))k}$ be the first quartile of the ith sample $(i = 1, 2, ..., l; l = m/2)$, and let $Y^*_{i(q_3(m+1))k}$ be the third quartile of the ith sample $(i = l+1, ..., m)$,. The quantified sample $Y^*_{1(q_1(m+1))k}$, $Y^*_{2(q_1(m+1))k}$, …, $Y^*_{\frac{m}{2}(q_1(m+1))k}$, $Y^*_{\frac{m}{2}+1(q_3(m+1))k}$, …, $Y^*_{m(q_3(m+1))k}$, will denote the DQRSSE.

If the sample size is odd, let $Y^*_{i(q_1(m+1))}$ be the first quartile of the ith sample $(i = 1, 2, ..., h)$, where $h = (m-1)/2$, $Y^*_{i((m+1)/2)k}$ is the median of the ith sample ($i = (m+1)/2$), and $Y^*_{i(q_3(m+1))k}$ the third quartile of the ith sample $(i = h+2, ..., m)$. The quantified sample $Y^*_{1(q_1(m+1))k}$, $Y^*_{2(q_1(m+1))k}$, …, $Y^*_{\frac{m-1}{2}(q_1(m+1))k}$, $Y^*_{\frac{m-1}{2}+1(\frac{m+1}{2})k}$, $Y^*_{\frac{m-1}{2}+2(q_3(m+1))k}$, …, $Y^*_{m(q_3(m+1))k}$ will denote the DQRSSO.

The estimator of the population mean using DQRSS can be defined as

$$\bar{Y}^*_{DQRSS}=\begin{cases}\bar{Y}^*_{DQRSSE}=\dfrac{1}{mn}\displaystyle\sum_{k=1}^{n}\left(\sum_{i=1}^{l}Y^*_{i(q_1(m+1))k}+\sum_{i=l+1}^{m}Y^*_{i(q_3(m+1))k}\right), l=m/2\\ \bar{Y}^*_{DQRSSO}=\dfrac{1}{mn}\displaystyle\sum_{k=1}^{n}\left(\sum_{i=1}^{h}Y^*_{i(q_1(m+1))k}+Y^*_{(h+1)\left(\frac{m+1}{2}\right)k}+\sum_{i=h+2}^{m}Y^*_{i(q_3(m+1))k}\right), h=(m\text{-}1)/2\end{cases}$$

The variance of $\bar{Y}^*_{DQRSS}$ for even and odd sample size can be given respectively by

$$\sigma^{*2}_{DQRSSE}=\frac{1}{nm^2}\sum_{k=1}^{n}\left(\sum_{i=1}^{l}\sigma^{*2}_{i(q_1(m+1))k}+\sum_{i=l+1}^{m}\sigma^{*2}_{i(q_3(m+1))k}\right),\ l=m/2\,.$$

$$\sigma^{*2}_{DQRSSO}=\frac{1}{nm^2}\sum_{k=1}^{n}\left(\sum_{i=1}^{h}\sigma^{*2}_{i(q_1(m+1))k}+\sigma^{*2}_{(h+1)\left(\frac{m+1}{2}\right)k}+\sum_{i=h+2}^{m}\sigma^{*2}_{i(q_3(m+1))k}\right),\ h=(m\text{-}1)/2$$

Assume that Y_i^* has the mean μ_i^* and the variance $\sigma^{*2}_{(i)}$, Al-Saleh and Al-Kadiri (2000) showed that

$$\mu=\sum_{i=1}^{m}\mu^*_{(i)}\ ,\ \sigma^2=\frac{1}{m}\left[\sum_{i=1}^{m}\sigma^{*2}_{(i)}+\sum_{i=1}^{m}(\mu^*_{(i)}-\mu)^2\right]$$

where μ and σ^2 are the mean and the variance of the population respectively.

Lemma 1:

Let X be a random variable of pdf $f(x)$ and cdf $F(x)$. Its mean and variance are μ and σ^2 respectively. A random sample of size m was selected and ranked, let $X_{r:m}$ be the rth smallest value of the sample, where $r=1,...,m$. The pdf and cdf for $X_{r:m}$ are

$$f_{r:m}(x)=\frac{1}{B(r,m-r+1)}F^{r-1}(x)(1-F(x))^{m-r}f(x)\,,$$
$$F_{r:m}(x)=FB(F(x);r,m-r+1)\,,$$

respectively, where $FB(F(x);r,m-r+1)$ is a beta distribution function with parameters $(r,m-r+1)$. Let denote the mean and the variance of $X_{r:m}$ as $\mu_{r:m}$ and $\sigma^2_{r:m}$ respectively. Then

a. $\mu_{r:m}=F^{-1}[\alpha(r)]$
b. $\mu_{m-r+1:m}=F^{-1}[1-\alpha(r)]$

c. $\sigma^2_{r:m} + (\mu_{r:m} - \mu)^2 < \sigma^2$

where $\alpha(r) = QB(p_r; r, m-r+1))$ which is a quartile function for beta distribution and $p_r = r/(m+1)$.

If $f(x)$ is symmetry then

d. $\mu_{r:m} + \mu_{m-r+1:m} = 2\mu$

e. $\sigma^2_{r:m} = \sigma^2_{m-r+1:m}$

Proof:

The variance of $X_{r:m}$ is given by

$$\sigma^2_{r:m} = \int (x-\mu_{r:m})^2 f_{r:m}(x)dx = \int (x-\mu)^2 f_{r:m}(x)dx - (\mu_{r:m} - \mu)^2$$

Substituting $f_{r:m}(x)$ and rearranging the above equation produces

$$\sigma^2_{r:m} + (\mu_{r:m} - \mu)^2 = \int (x-\mu)^2 \left(\frac{1}{B(r, m-r+1)} F^{r-1}(x)(1-F(x))^{m-r} \right) f(x)dx$$

As $\dfrac{F^{r-1}(x)[1-F(x)]^{m-r}}{B(r, m-r+1)} < 1$, so

$$\sigma^2_{r:m} + (\mu_{r:m} - \mu)^2 < \sigma^2 = \int (x-\mu)^2 f(x)dx$$

Using Taylor series, as given in David & Nagarajah (2003), can be shown that

$$E(X_{r:m}) = \mu_{r:m} = \int x f_{r:m}(x)dx \approx F^{-1}_{r:m}(p_r)$$

Let $F_{r:m}(x) = FB(F(x); r, m-r+1) = p_r$

Utilizing this relationship produces

$$\mu_{r:m} = F^{-1}_{r:m}(p_r) = F^{-1}[\alpha(r)] \text{ where } \alpha(r) = QB(p_r; r, m-r+1))$$

Let $F_{m-r+1:m}(x) = FB(F(x); m-r+1, r) = q_r, \quad q_r + p_r = 1$

$$\mu_{m-r+1:m} = F^{-1}_{m-r+1:m}(q_r) = F^{-1}[QB(q_r; m-r+1, r)]$$

Since $QB(1-p_r; m-r+1, r) = 1 - QB(p_r; r, m-r+1)$, then

$$\mu_{m-r+1:m} = F^{-1}[1-\alpha(r)]$$

If $f(x)$ is symmetry for any $0 \le \alpha(r) \le 1$

$$F^{-1}[1-\alpha(r)]-\mu=\mu-F^{-1}[\alpha(r)]$$

So $$F^{-1}[1-\alpha(r)]+F^{-1}[\alpha(r)]=\mu_{r:m}+\mu_{m-r+1:m}=2\mu$$

The variance of $X_{r:m}$ is given by

$$\sigma^2_{r:m}=\int\frac{\left(F^{-1}(u)-F^{-1}[\alpha(r)]\right)^2}{B(r,m-r+1)}u^{r-1}(1-u)^{m-r}\,du$$

For symmetrical $f(x)$,

$$\sigma^2_{r:m}=\int\frac{\left(F^{-1}(1-u)-F^{-1}[1-\alpha(r)]\right)^2}{B(r,m-r+1)}u^{r-1}(1-u)^{m-r}\,du$$

$$=\int\frac{\left(F^{-1}(u)-F^{-1}[1-\alpha(r)]\right)^2}{B(m-r+1,r)}u^{m-r}(1-u)^{r-1}du=\sigma^2_{m-r+1:m}$$

Lemma 2:

Let $Y_{r:m}$ be the rth smallest value of a random sample of size m. The sample was selected from a population of pdf

$$f_{r:m}(x)=\frac{1}{B(r,m-r+1)}F^{r-1}(x)(1-F(x))^{m-r}f(x)$$

where the mean and variance correspond to pdf $f(x)$ are μ and σ^2 respectively. In addition, let $Y_{m-r+1:m}$ be the $(m-r+1)$th smallest value, $E(Y_{r:m})=\mu^*_{r:m}$, $E(Y_{m-r+1:m})=\mu^*_{m-r+1:m}$, $\mathrm{Var}(Y_{r:m})=\sigma^{*2}_{r:m}$, $\mathrm{Var}(Y_{m-r+1:m})=\sigma^{*2}_{m-r+1:m}$. Then,

a. $\mu^*_{r:m}=F^{-1}[\alpha\circ\alpha(r)]$
b. $\mu^*_{m-r+1:m}=F^{-1}[1-\alpha\circ\alpha(r)]$
c. $\sigma^{*2}_{r:m}+(\mu^*_{r:m}-\mu_{r:m})^2+(\mu_{r:m}-\mu)^2<\sigma^2$.

If $f(x)$ is symmetry then

d. $\mu^*_{r:m}+\mu^*_{m-r+1:m}=2\mu$
e. $\sigma^{*2}_{r:m}=\sigma^{*2}_{m-r+1:m}$

Proof:

Using the results of lemma 1

$$\mu^*_{r:m}=F^{-1}_{r:m}[\alpha(r)]=F^{-1}[\alpha\circ\alpha(r)]$$
$$\mu^*_{m-r+1:m}=F^{-1}_{r:m}[1-\alpha(r)]=F^{-1}[1-\alpha\circ\alpha(r)]$$

$$\sigma_{r:m}^{*2} + (\mu_{r:m}^{*} - \mu_{r:m})^2 < \sigma_{r:m}^{2}$$

Since $\sigma_{r:m}^{2} + (\mu_{r:m} - \mu)^2 < \sigma^2$ so

$$\sigma_{r:m}^{*2} + (\mu_{r:m}^{*} - \mu_{r:m})^2 + (\mu_{r:m} - \mu)^2 < \sigma^2$$

For any symmetry distribution, and $\alpha \in [0,1]$

$$(\mu - F^{-1}(\alpha)) = (F^{-1}(1-\alpha) - \mu)$$

so, $\mu_{r:m}^{*} + \mu_{m-r+1:m}^{*} = 2\mu$

The variance of $Y_{r:m}$ is equal to

$$\begin{aligned}\sigma_{r:m}^{*2} &= \int \frac{\left(F_{r:m}^{-1}(u) - F_{r:m}^{-1}[\alpha(r)]\right)^2}{B(r, m-r+1)} u^{r-1}(1-u)^{m-r}\, du \\ &= \int \frac{\left(F^{-1}(u) - F^{-1}[\alpha \circ \alpha(r)]\right)^2}{B(r, m-r+1)} u^{r-1}(1-u)^{m-r}\, du \\ &= \int \frac{\left(F^{-1}(u) - F^{-1}[1-\alpha \circ \alpha(r)]\right)^2}{B(m-r+1, r)} u^{m-r}(1-u)^{r-1}\, du = \sigma_{m-r+1:m}^{*2}\end{aligned}$$

Lemma 3:

1. $\hat{\mu}_{DQRSS}^{*}$ is an unbiased estimator of the population mean, under the assumption that the population is symmetric about its mean**.**
2. $\mathrm{Var}\left(\overline{Y}_{DQRSS}^{*}\right)$ is less than each of $\mathrm{Var}(\overline{X}_{SRS})$, $\mathrm{Var}(\overline{X}_{RSS})$ and $\mathrm{Var}(\overline{Y}_{QRSS})$.
3. The mean square error of DQRSS estimator is less than the variance of the SRS estimator for asymmetric distributions i.e., $\mathrm{MSE}\left(\overline{Y}_{DPRSS}^{*}\right) < \mathrm{Var}(\overline{X}_{SRS})$.

Proof:

For m even

$$\hat{\mu}_{DQRSSE} = \frac{1}{m}\left(\sum_{i=1}^{l} Y_{i(r:m)} + \sum_{i=l+1}^{m} Y_{i(m-r+1:m)}\right)$$

$$\begin{aligned}E\left(\hat{\mu}_{DQRSSE}\right) &= \frac{1}{m}\left(\sum_{i=1}^{\frac{m}{2}} E\left(Y_{i(r:m)}\right) + \sum_{i=\frac{m}{2}+1}^{m} E\left(Y_{i(m-r+1:m)}\right)\right) \\ &= \frac{1}{m}\left(\frac{m}{2}\mu_{r:m}^{*} + \frac{m}{2}\mu_{m-r+1:m}^{*}\right) = \mu\end{aligned}$$

and

$$\operatorname{Var}\left(\hat{\mu}_{DQRSSE}\right)=\frac{1}{m^2}\left(\sum_{i=1}^{\frac{m}{2}}\operatorname{Var}\left(Y_{i(r:m)}\right)+\sum_{i=\frac{m}{2}+1}^{m}\operatorname{Var}\left(Y_{i(m-r+1:m)}\right)\right)$$
$$=\frac{1}{2m}\left(\sigma_{r:m}^{*2}+\sigma_{m-r+1:m}^{*2}\right)<\frac{\sigma^2}{m}$$

For m odd

$$\hat{\mu}_{DQRSSO}=\frac{1}{m}\left(\sum_{i=1}^{\frac{m-1}{2}}Y_{i(r:m)}+\sum_{i=\frac{m+3}{2}}^{m}Y_{i(m-r+1:m)}+Y_{\frac{m+1}{2}:m}\right)$$

$$E(\hat{\mu}_{DQRSSO})=\frac{1}{m}\left(\sum_{i=1}^{\frac{m-1}{2}}E\left(Y_{i(r:m)}\right)+\sum_{i=\frac{m+3}{2}}^{m}E\left(Y_{i(m-r+1:m)}\right)+E\left(Y_{\frac{m+1}{2}:m}\right)\right)$$
$$=\frac{1}{m}\left(\frac{m-1}{2}\left(\mu_{r:m}^{*}+\mu_{m-r+1:m}^{*}\right)+\mu\right)=\mu$$

and

$$\operatorname{Var}\left(\hat{\mu}_{DQRSSO}\right)=\frac{1}{m^2}\left(\sum_{i=1}^{\frac{m-1}{2}}\operatorname{Var}\left(Y_{i(r:m)}\right)+\sum_{i=\frac{m+3}{2}}^{n}\operatorname{Var}\left(Y_{i(m-r+1:m)}\right)+\operatorname{Var}\left(Y_{\frac{m+1}{2}:m}\right)\right)$$
$$=\frac{1}{m^2}\left(\frac{m-1}{2}\left(\sigma_{r:m}^{*2}+\sigma_{m-r+1:m}^{*2}\right)+\sigma_{\frac{m+1}{2}:m}^{*2}\right)<\frac{\sigma^2}{m}$$

4. EFFICIENCY OF DQRSS

To compare the considered estimators for the population mean using DQRSS with respect to the SRS, RSS, and QRSS procedures. Three symmetric distributions, namely, uniform, normal and logistic and three asymmetric distributions, namely, exponential, gamma and weibull are considered. The relative efficiency of the unbiased estimators using ranked set samples procedures for estimating the population mean with respect to SRS is defined as $eff\left(\bar{X}_{SRS},\bar{Y}_{RSS}\right)=\dfrac{\operatorname{Var}(\bar{X}_{SRS})}{\operatorname{Var}\left(\bar{Y}_{RSS}\right)}$, and for biased estimators the relative efficiency is defined as $eff\left(\bar{X}_{SRS},\bar{Y}_{RSS}\right)=\dfrac{\operatorname{Var}(\bar{X}_{SRS})}{\operatorname{MSE}\left(\bar{Y}_{RSS}\right)}$.

Assume the cycle is repeated once, Tables 1 and 2 summarize the relative efficiency of the RSS, QRSS and DQRSS estimators with sample sizes $m=6,7,10,11$ and 12, for each simulation, 60,000 iterations were performed.

Table 1:
The relative efficiency for estimating the population mean using RSS, QRSS, and DQRSS with respect to SRS with sample size $m = 6$ and 7.

Distribution		$m = 6$			$m = 7$		
		RSS	***QRSS***	***DQRSS***	***RSS***	***QRSS***	***DQRSS***
Uniform (0,1)	*eff*	3.500	3.214	16.966	4.000	3.809	23.445
	Bias						
Uniform (0,2)	*eff*	3.500	3.232	17.267	4.000	3.770	23.021
	Bias						
Normal (0,1)	*eff*	3.191	3.639	11.906	3.658	4.065	14.669
	Bias						
Normal (1,2)	*eff*	3.210	3.645	11.950	3.631	4.051	14.590
	Bias						
Logistic (-1,1)	*eff*	2.868	3.729	11.707	3.259	4.144	13.845
	Bias						
Exponential (1)	*eff*	2.430	3.009	9.549	2.746	3.321	8.692
	Bias		0.092	0.016		0.075	0.059
Exponential (2)	*eff*	2.407	3.016	9.569	2.735	3.327	8.598
	Bias		0.046	0.008		0.038	0.029
Exponential (3)	*eff*	2.467	3.051	9.764	2.693	3.293	8.513
	Bias		0.031	0.005		0.025	0.020
Gamma (1,2)	*eff*	2.391	3.022	9.395	2.715	3.333	8.575
	Bias		0.183	0.033		0.150	0.119
Gamma (1,3)	*eff*	2.416	3.025	9.572	2.669	3.282	8.496
	Bias		0.279	0.047		0.230	0.178
Weibull (1,3)	*eff*	2.459	3.029	9.660	2.755	3.334	8.503
	Bias		0.274	0.047		0.227	0.178

Table 2:
The relative efficiency for estimating the population mean using RSS, QRSS and DQRSS with respect to SRS with sample size m = 10,11 and 12

Distribution		m = 10			m = 11			m = 12		
		RSS	***QRSS***	***DQRSS***	***RSS***	***QRSS***	***DQRSS***	***RSS***	***QRSS***	***DQRSS***
Uniform (0,1)	*eff*	5.500	5.085	38.097	6.000	5.637	47.852	6.500	6.730	66.637
	Bias									
Uniform (0,2)	*eff*	5.500	5.128	38.463	6.000	5.680	47.627	6.500	6.667	66.234
	Bias									
Normal (0,1)	*eff*	4.827	5.736	31.288	5.197	6.067	35.034	5.673	6.338	37.261
	Bias									
Normal (1,2)	*eff*	4.844	5.850	31.721	5.195	6.240	35.046	5.652	6.412	36.958
	Bias									
Logistic (-1,1)	*eff*	4.198	6.270	32.220	4.533	6.755	34.801	4.911	6.728	34.315
	Bias									
Exponential (1)	*eff*	3.440	3.281	15.024	3.671	3.542	28.555	3.922	4.693	8.303
	Bias		0.117	0.056		0.105	0.001		0.061	0.083
Exponential (2)	*eff*	3.426	3.288	14.916	3.659	3.521	28.406	3.962	4.735	8.409
	Bias		0.059	0.028		0.053	0.000		0.031	0.042
Exponential (3)	*eff*	3.394	3.252	14.844	3.653	3.535	28.775	3.964	4.773	8.452
	Bias		0.039	0.019		0.035	0.000		0.020	0.028
Gamma (1,2)	*eff*	3.440	3.276	14.878	3.723	3.594	28.877	3.919	4.697	8.372
	Bias		0.234	0.113		0.210	0.001		0.123	0.167
Gamma (1,3)	*eff*	3.460	3.274	14.963	3.638	3.539	28.510	3.990	4.711	8.350
	Bias		0.354	0.170		0.314	0.002		0.184	0.250
Weibull (1,3)	*eff*	3.471	3.245	14.808	3.699	3.576	28.675	3.960	4.751	8.480
	Bias		0.352	0.170		0.313	0.002		0.185	0.249

From simulation results, we conclude the following:

1. A gain in efficiency is attained using DQRSS for estimating the population mean for all cases that considered in this study. As an example for normal (0,1), with $m = 11$, the relative efficiency of the DQRSS 53.034 for estimating the population mean comparing this value with its counterpart 5.197, 6.067 using RSS and QRSS respectively.
2. If the underlying distribution is asymmetric, again in efficiency is attained using DQRSS, regardless of a smaller bias. As an example, for $m = 11$ the relative efficiency of the DQRSS 28.877 with bias 0.001 for estimating the population mean of a gamma distribution with parameters 1 and 2, while for $m = 11$ the relative efficiency using RSS is 3.723 and by using QRSS its 3.594 with bias 0.210.

5. DOUBLE QUARTILE RANKED SET SAMPLING WITH ERRORS IN RANKING

Dell and Clutter (1972) showed that the sample mean using RSS is unbiased estimator of the population mean regardless of whatever the ranking is perfect or not, and has a smaller variance than its counterpart SRS with the same sample size.

Muttlak (2003) showed that QRSS with errors in ranking is unbiased estimator of the population mean when the underlying distribution is assumed to be symmetric about its mean.

Let $Y^*_{i[q_1(m+1)]}$ and $Y^*_{i[q_3(m+1)]}$ be the first and third judgment quartile of the ith sample $(i=1,2,\ldots,m)$ respectively with errors in ranking. The estimator of the population mean with error in ranking using DQRSS can be defined as

$$\hat{Y}^*_{DQRSS_e}=\begin{cases}\hat{Y}^*_{DQRSSE_e}=\dfrac{1}{mn}\sum\limits_{k=1}^{n}\left(\sum\limits_{i=1}^{l}Y^*_{i[q_1(m+1)]k}+\sum\limits_{i=l+1}^{m}Y^*_{i[q_3(m+1)]k}\right), l=m/2\\[2ex] \hat{Y}^*_{DQRSSO_e}=\dfrac{1}{mn}\sum\limits_{k=1}^{n}\left(\sum\limits_{i=1}^{h}Y^*_{i[q_1(m+1)]k}+Y^*_{(h+1)\left[\left(\frac{m+1}{2}\right)\right]k}+\sum\limits_{i=h+2}^{m}Y^*_{i[q_3(m+1)]k}\right), h=(m-1)/2\end{cases}$$

The estimator of the population mean μ with errors in ranking has the following properties:

1. $\hat{Y}^*_{DQRSS_e}$ is unbiased estimator of the population mean if the population is symmetric about its mean.
2. $\mathrm{Var}\left(\hat{Y}^*_{DQRSS_e}\right)$ is less than $\mathrm{Var}\left(\bar{X}_{SRS}\right)$.
3. For asymmetric distribution about its mean, $\mathrm{MSE}\left(\hat{Y}^*_{DQRSS_e}\right)<\mathrm{Var}\left(\bar{X}_{SRS}\right)$

The above properties can be proved based on Takahasi and Wakimoto (1968), Dell and Clutter (1972), Muttlak (2003) and AL-Saleh and AL-Kadiri (2000).

In this article, it is observed that the DQRSS estimator is unbiased of the population mean if the underlying distribution is symmetric, and more efficient than the SRS, RSS and QRSS. The authors suggest using the DQRSS for estimating the population mean of symmetric distribution and asymmetric distribution when the biased is small; also, we can use DQRSS to reduce the errors in ranking than RSS.

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