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MATHEMATICAL MODELING OF AGE SPECIFIC MARITAL FERTILITY RATES IN URBAN AREA OF BANGLADESH

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ABSTRACT

The purpose of this study is to build mathematical modeling of age specific marital fertility rates (ASMFRs) in urban area of Bangladesh. For this, the secondary data of ASMFRs have been taken from Bangladesh Demographic Health Survey. It is observed that they follow polynomial models. Model validation technique, cross-validity prediction power (CVPP), ρ_{cv}^2, has been applied to these mathematical models to check the validity of the models.

KEYWORDS AND PHRASES

Age specific marital fertility rates (ASMFRs) Mathematical Modeling Polynomial Variance explained (***R*²**) Cross validity prediction power (CVPP).

1. INTRODUCTION

It is to be noted that mathematical modeling in demography in our country have rarely been used. In the modern era, mathematical model is very sophisticated mechanism to express data in mathematics. Mathematical model is very important for the estimation of population projections and estimations. In fact, mathematical model is essentially an attempt to find out structural relationships and their dynamic behaviour among the various elements in demographic processes. Mathematical models are two types: non-deterministic or stochastic and deterministic. Some deterministic models have been fitted in the present study.

Ali (1994) reviewed the relationship of total separation rates and separation rates due to death with their age variable and found a semi-log function of the type $\log_e y = \alpha + \beta x$. Age distribution, age specific death rates (ASDRs) and the number of persons surviving at an exact age x (l_x) for male population of Bangladesh in 1991 follow modified negative exponential model, 4th degree polynomial model and 3rd degree polynomial model, respectively (Islam, Islam, Ali and Mostofa, 2003). In (Islam and Ali, 2004) it was found that age specific fertility rates (ASFRs) follows slightly modified biquadratic polynomial model where as forward and backward cumulative ASFRs follow quadratic and cubic polynomial model, respectively in the rural area of Bangladesh.

Traditionally, one can draw some graphs of the demographic parameters. But, very few of us know what types of functional form are more appropriate for the parameters in the context of Bangladesh. Thus, the objectives of this study have been briefly mentioned below:

i) to build up mathematical models to ASMFRs and forward cumulative ASMFRs in the urban area of Bangladesh, and
ii) to apply CVPP to these models.

2. METHODOLOGY

A secondary data on ASMFRs in 1996 and 1999-2000 in urban area of Bangladesh have been taken from Bangladesh Demographic Health Survey (Mitra and Associates, 1997 & 2001) which is shown in Table 1. In this section, polynomial are briefly discussed in the following:

An expression of the form

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n \quad \left(a_n \neq 0\right) \text{ (Waerden, 1948)}$$

where a_0 is the constant term; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) but a_1, a_2,..., a_n are also constants and n is the positive integer, is called a polynomial of degree n and the symbol x is called an indeterminate. If n=0 then it is called constant function. If n=1 then it is called polynomial of degree 1 i.e. simple linear function. If n=2 then it is called polynomial of degree 2 i.e. quadratic polynomial, etc. (Spiegel, 1992). Using the scattered plot of ASMFRs by age group in years in urban area of Bangladesh (Figures 1 and 2), it is observed that ASMFRs can be fitted by polynomial model Therefore, an nth degree polynomial model is considered and the form of the model is $y = a_0 + \sum_{i=1}^{n} a_i \quad x^i + u$ (Gupta and Kapoor, 1997) where, x is age group y is ASMFRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the stochastic error term of the model. Here we have to select a suitable n for which the error sum of square is minimum.

Using the dotted plot of forward cumulative ASMFRs by age group in years in urban area of Bangladesh (Figures 3 and 4), it seems that forward cumulative ASMFRs follows an nth degree polynomial model. Therefore, the structure+ of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \quad x^i + u$$

where, x is age group in years; y is forward cumulative ASMFRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here we have to choose a suitable n for which the error sum of square is minimum. The information on model fitting has been demonstrated in Table 2. The software STATISTICA was used to fit these mathematical models. To check how much these models are stable, the cross validity prediction power (CVPP), ρ^2_{cv}, is applied. Here, $\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$; where, n is the number of cases, k is the number of regressors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 has been summarized in Table 3.

RESULTS AND DISCUSSION

The polynomial model is assumed for ASMFRs in urban area of Bangladesh. The fitted equations are as follows:

y = 1.047466-0.0475514x+0.0005493x^2 in 1996 (2.1)

The coefficient of determination R^2=0.99449 and ρ^2_{cv} = 0.978572. Which is the polynomial of degree two i.e. quadratic polynomial.

y = 0.4346714-0.0104057x in 1999-2000 (2.2)

giving R^2=0.99397 and ρ^2_{cv} =0.988275. Which is the polynomial of degree 1 i.e. simple linear regression model.

And, another polynomial model is assumed for forward cumulative ASMFRs in urban area of Bangladesh and the fitted equations are

y = -0.806258+0.0888543x-0.0011336x^2 in 1996 (2.3)

providing R^2=0.99475 and ρ^2_{cv} =0.979583. This is the polynomial of degree two i.e. quadratic polynomial.

y = -0.8752017+0.0831314x-0.00110671x^2in 1999-2000 (2.4)

with R^2=0.99928 and ρ^2_{cv} =0.9972. Which is also two degree polynomial of i.e. quadratic or parabolic function.

It should be mentioned here that the usual models, i.e. Gompertz model, Makeham model and logistic model were also applied but seem to be worse fitted with respect to their shrinkages. Therefore, the results of those models were not shown here.

From this Table 2, it is shown that all the parameters of the fitted models in equations (2.1)-(2.4) are statistically significant with more than 99% of variance explained.

Table 3 shows that the fitted models in equations (2.1)-(2.4) are highly cross-validated and their shrinkage are 0.015918, 0.005695, 0.015167 and 0.00208, respectively. These imply that fitted models in equations (2.1) to (2.4) will be stable more than 97%. Moreover, the stability of R^2 of these models is more than 98%.

Table 1:
ASMFRs and its Cumulative Distribution in Urban Area of Bangladesh

Age Group $a - a+5$	ASMFRs		Cumulative ASMFRs	
	1996	1999-2000	1996	1999-2000
15-19	0.388	0.25	0.388	0.25
20-24	0.252	0.207	0.64	0.457
25-29	0.149	0.153	0.789	0.61
30-34	0.078	0.087	0.867	0.697
35-39	0.056	0.037	0.923	0.734
40-44	0.009	0.001	0.932	0.735

Source: (Mitra and Associates, 1997 and 2001)

Table 2:
Information on Model Fitting

Models	Percentage of Variance Explained	Parameters	Significant Probability (p)
Model 1	99.449		
		a_0	0.00084
		a_1	0.00305
		a_2	0.008571
Model 2	99.397		
		a_0	0.0000
		a_1	0.0000
Model 3	99.475		
		a_0	0.00532
		a_1	0.00144
		a_2	0.00309
Model 4	99.928		
		a_0	0.0002
		a_1	0.00007
		a_2	0.0002

Table 3:
Estimated Cross-Validity Prediction Power, ρ^2_{cv}, of the Predicted Equations of ASMFRs and its Forward Cumulative Distribution in Urban Area of Bangladesh

Models	N	k	R^2	ρ^2_{cv}	Shrinkage
Equation 1	6	2	0.99449	0.978572	0.015918
Equation 2	6	1	0.99397	0.988275	0.005695
Equation 3	6	2	0.99475	0.979583	0.015167
Equation 4	6	2	0.99928	0.9972	0.00208

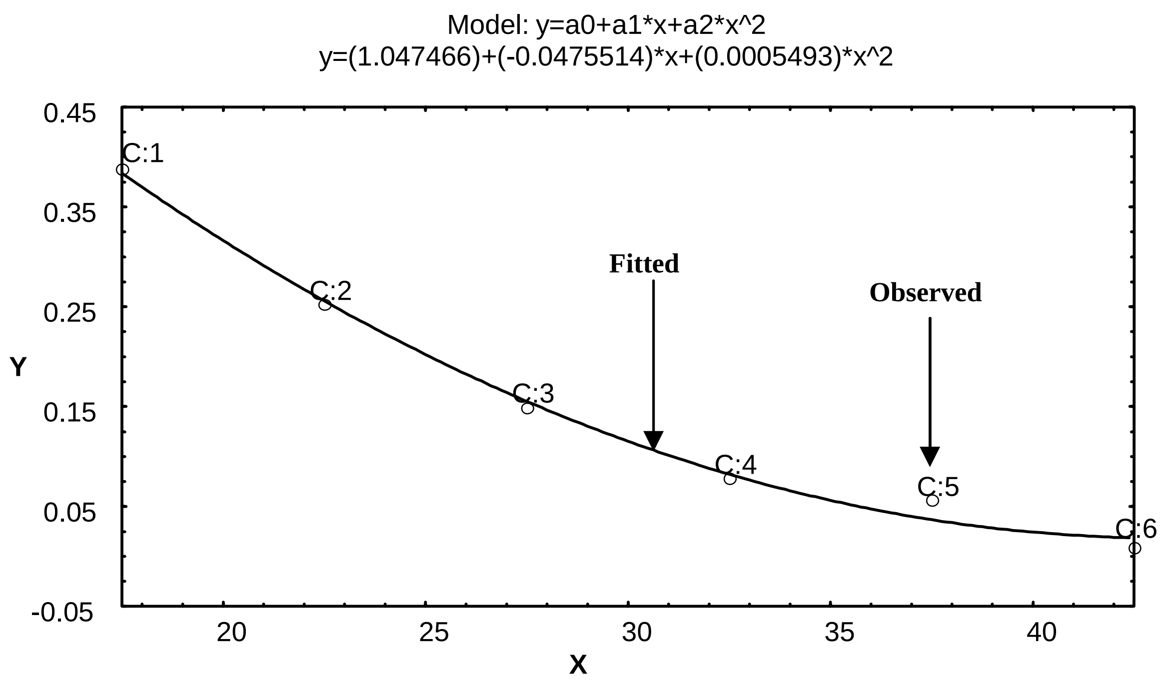

Fig. 1: Observed and Fitted ASMFRs in Urban Area of Bangladesh in 1996.
X: Age Group in Years and Y: ASMFRs.

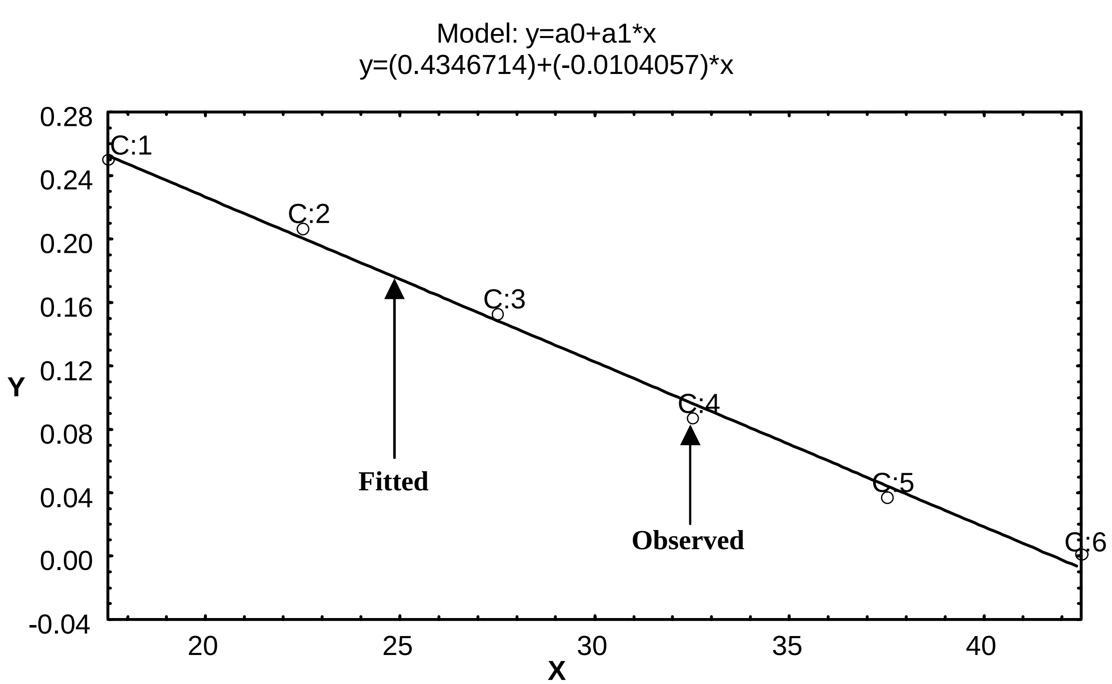

Fig. 2: Observed and Fitted ASMFRs in Urban Area of Bangladesh in 1999-2000.
X: Age Group in Years and Y: ASMFRs.

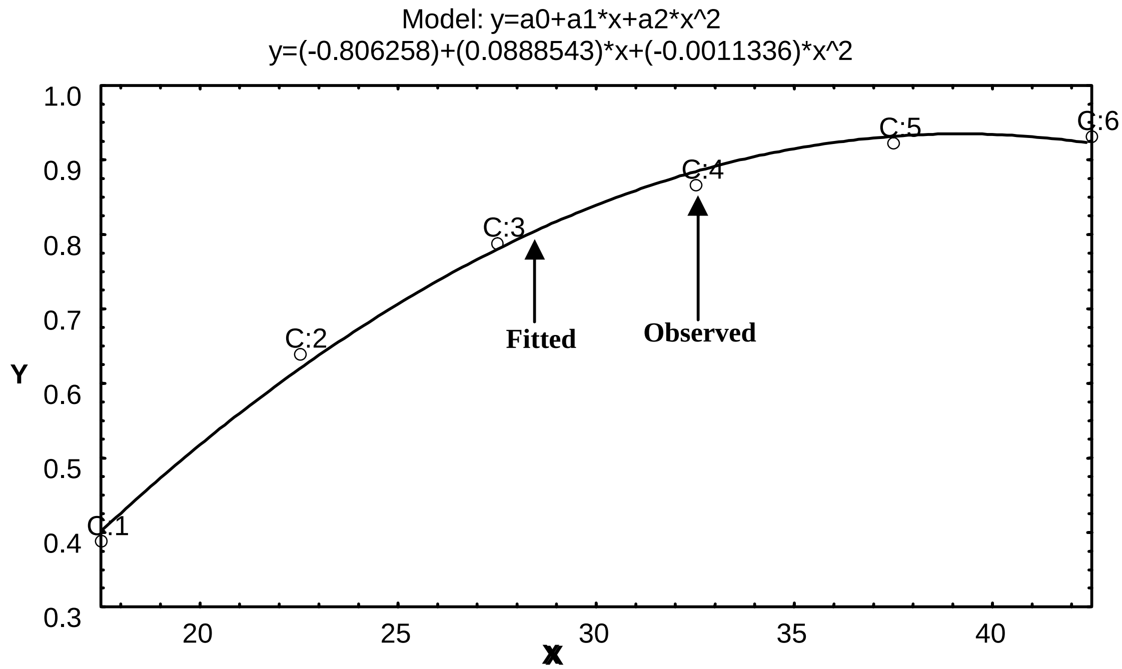

Fig. 3: Observed and Fitted Forward Cumulative ASMFRs in Urban Area of Bangladesh in 1996. X: Age Group in Years and Y: Forward Cumulative ASMFRs.

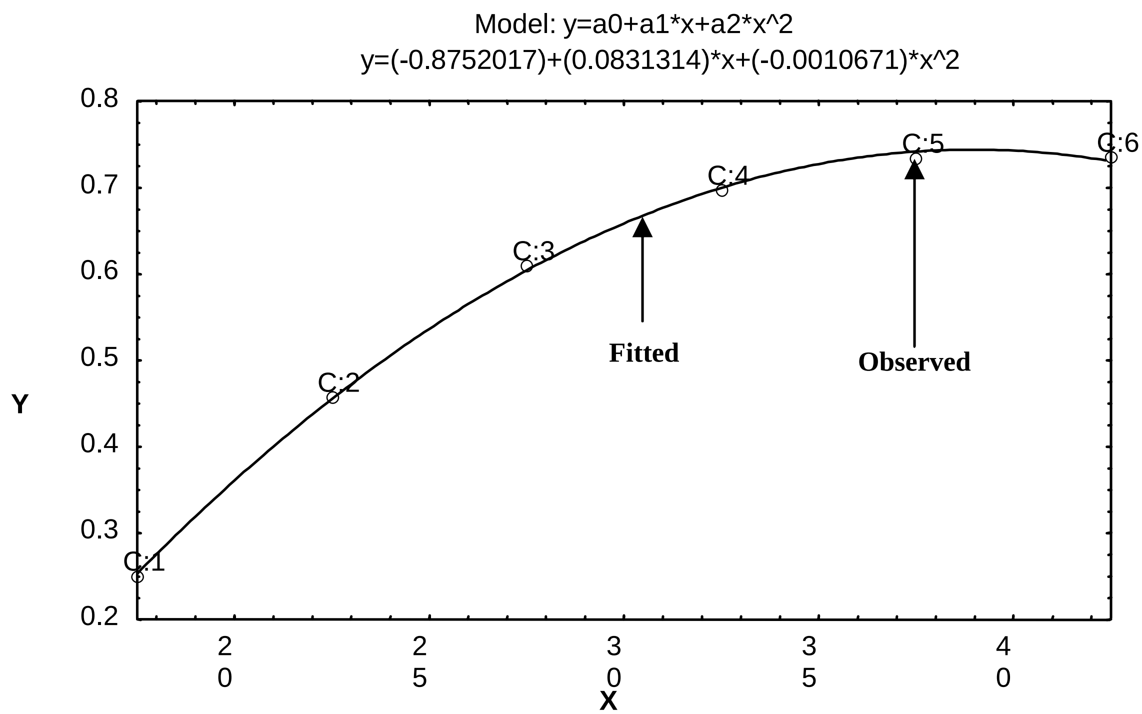

Fig. 4: Observed and Fitted Forward Cumulative ASMFRs in Urban Area of Bangladesh in 1999-2000. X: Age Group in Years and Y: Forward Cumulative ASMFRs.

CONCLUSION

In this study, it is observed that the ASMFRs in urban area of Bangladesh follows 2nd degree polynomial model and simple linear regression model, respectively. On the other hand, forward cumulative ASMFRs follows 2nd degree polynomial in both cases.

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