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AN OPTIMAL DESIGN OF A SKIP LOT SAMPLING PLAN OF TYPE V BY MINIMIZING AVERAGE SAMPLE NUMBER

Muhammad Aslam[1], **Saminathan Balamurali**[2] **Chi-Hyuck Jun**[3], **Munir Ahmad**[4] and **Mujahid Rasool**[1]

[1] Department of Statistics, Forman Christian College University, Lahore, Pakistan. Email: aslam_ravian@hotmail.com

[2] Departments of Mathematics, Kalasalingam University, Krishnankoil 626190, TN, India. Email: sbmurali@rediffmail.com

[3] Department of Industrial and Management Engineering, POSTECH, Pohang 790-784, Republic of Korea. Email: chjun@postech.ac.kr

[4] National College of Business Administration & Economics, Lahore, Pakistan. Email: drmunir@ncbae.edu.pk

ABSTRACT

In this paper, we propose a designing methodology to find the optimal parameters of skip-lot sampling plan of type V (SkSP-V) in terms of reducing the average sample number. The two-points on the operating characteristic curve approach is used to find the design parameters of the proposed plan and the reference plan as well. The tables are presented and the results are explained using an example. The advantages of the proposed plan over the reference plan is also discussed and proved that the SkSP-V is better than the reference sampling plan in terms of probability of acceptance, average sample number and average total inspection.

KEY WORDS:

Binomial sampling; Consumer's risk; Producer's risk; Single sampling; Skip-lot sampling.

1. INTRODUCTION

Dodge (1955a) innovated the concept of continuous sampling and provided mathematical rationale and the rules of operation for the first continuous sampling plan (CSP) familiarly known as CSP-1. Continuous sampling plans can be applied for a product consisting of individual units and manufactured by an essentially continuous process. Later several modifications on continuous sampling plans were proposed and the resultant plans were designated as CSP-2, CSP-3, CSP-F, CSP-T, CSP-V etc. All these plans are available in the US military standard MIL-STD 1235C (1988). For more details about these continuous sampling plans one can refer Stephens (2001).

Dodge (1955b) later presented an extension of continuous sampling plans for individual units to a "skip-lot sampling plan (SkSP) that is applicable to bulk materials or products produced in successive batches or lots", and the plan is designated as "SkSP-1". One of the basic motivations for this extension is stated as "applied to chemical and physical analyses, SkSP-1 sampling plan provides a basis for reducing testing costs". So, skip-lot sampling is used for sampling chemical and physical processes in order to bring about substantial savings on inspection of products, which normally conform to specification. This particular sampling plan is useful when the lots are small or where inspection is slow and costly**.** The operation of the SkSP-1 plan can be seen in Dodge (1955b). Burnett (1967) had presented a Markov chain model for deriving the operating characteristic (OC) function of SkSP-1 plan. Based on the objectives of skip-lot sampling, Perry (1973) formalized the application of skip-lot sampling to the situation in which each lot to be inspected is sampled according to a lot inspection plan, called the reference plan. This plan is designated as SkSP-2 plan. For detailed information about skip-lot sampling plans, one can refer Schilling (1982), ISO 2859-3 (2005) and Balamurali et al (2008). Recently, Aslam et al. (2010) proposed a designing methodology to determine the optimal parameter of a SkSP-2 plan. In this paper, we consider the skip-lot sampling plan of type SkSP-V and propose the designing methodology to determine the optimal parameters using single sampling plan as the reference plan as no such designing methodology is available in the literature . The designing methodology proposed in this paper will satisfy both producer and consumer's risks simultaneously.

2. SkSP-V SKIP LOT SAMPLING PLAN

The continuous sampling plan of type CSP-V is one of the single level continuous sampling plans in which reduced inspection can be achieved by using a smaller clearance interval when reducing the sampling frequency has no advantage upon demonstration of good product quality. Since the skip-lot concept is sound and useful and it is economically advantageous to the skip-lot approach in the design of sampling plans, Balamurali and Jun (2010) developed a new system of skip-lot sampling plan designated as SkSP-V based on the principles of CSP-V plan. The SkSP-V sampling plan is having a provision for reducing a normal inspection. They have also studied the properties of the SkSP-V plan with single sampling plan as the reference plan. According to Balamurali and Jun (2010) the operating procedure of the SkSP-V plan is stated as follows:

(1) At the outset, start with normal inspection using the reference plan. During the normal inspection, lots are inspected one by one in the order of production or in the order of being submitted to inspection.
(2) When i consecutive lots are accepted on normal inspection, discontinue the normal inspection and switch to skipping inspection.
(3) During skipping inspection, inspect only a fraction f of the lots selected at random. Skipping inspection is continued until sampled lot is rejected.
(4) When a lot is rejected on skipping inspection before k consecutively sampled lots are accepted, revert to normal inspection as per (1) above.
(5) When a lot is rejected after k consecutive lots have been accepted revert to normal inspection with reduced clearance number x as per (6) given below.
(6) During normal inspection with clearance number x, lots are inspected one by one in the order of being submitted to inspection and continue the inspection until a lot is rejected or x lots are accepted whichever occurs earlier.
(7) When a lot is rejected, immediately revert to normal inspection with clearance number i as per (1) given above.

(8) When x lots are accepted, discontinue normal inspection and switch to skipping inspection as per (3) above.
(9) Replace or correct all the non-conforming units found with conforming units in the rejected lots.

Associated with this plan are a reference plan and four parameters f, i, k and x. In general, $0 < f < 1$ and i, k and $x(\leq i)$ are positive integers and the plan is designated as SkSP-V (i, f, k, x). The proposed plan is generalization of SkSP-2 plan. When $k = x = i$, the present plan reduces to SkSP-2 sampling plan. It is also important to note that when $f = 1$ the SkSP-V plan reduces to a reference sampling plan.

3. DESIGNING OF SkSP-V PLAN

The probability of accepting a lot based on SkSP-V plan and other performance measures of the SkSP-V sampling plan were derived by Balamurali and Jun (2010) using a Markov chain model. To simplify the number of design parameters, it can be assumed that $k = x$. According to Balamurali and Jun (2010), the probability of acceptance of the SkSP-V plan when $k = x$ is given by

$$P_a(p) = \frac{fP + (1-f)P^i + fP^{k+1}\left(P^i - P^k\right)}{f\left(1 + P^{i+k} - P^{2k}\right) + (1-f)P^i} \tag{3.1}$$

where P is the acceptance probability based on single sampling plan and $Q = 1 - P$. The acceptance probability of a lot under binomial model for the single sampling plan is given by

$$P = \sum_{j=0}^{c} \binom{n}{j} p^j (1-p)^{n-j} \tag{3.2}$$

The two-points on the OC curve approach is considered as a reasonable approach because the lot acceptance probability obtained by one risk may not satisfy the other risk. Further, producer wants that the probability of acceptance should be larger than $1-\alpha$ if the process fraction nonconforming is at the acceptable quality level (AQL) and the consumer demands that the lot acceptance probability should be less than β if the process fraction non-conforming is at the limiting quality level (LQL), see for example Balamurali et al. (2005). According to ANSI/ASQC standard A2 (1987) defines AQL as “the maximum percentage or proportion of variant units in a lot or batch that, for the purpose of acceptance sampling, can be considered as a process average”. Similarly LQL is defined as “the percentage or proportion of variant units in a batch or lot for which, for the purposes of acceptance sampling, the consumer wishes the probability of acceptance to be restricted to a specified low value”. Under the conditions of AQL (p_1) and LQL (p_2), equation (3.2) can be re-written as

$$P_1 = \sum_{j=0}^{c} \binom{n}{i} p_1^{\,j} \left(1-p_1\right)^{n-j} \tag{3.3}$$

$$P_2 = \sum_{j=0}^{c} \binom{n}{i} p_2^{\,j} \left(1-p_2\right)^{n-j} \tag{3.4}$$

Under the specified values AQL and LQL, we want to determine the design parameters of the SkSP-V sampling plan (i, f, k, x, n, c) such that the producer's and the consumer's risks should be satisfied simultaneously.

$$\frac{fP_1 + (1-f)P_1^i + fP_1^{k+1}\left(P_1^i - P_1^k\right)}{f\left(1+P_1^{i+k} - P_1^{2k}\right) + (1-f)P_1^i} \geq 1-\alpha \tag{3.5}$$

$$\frac{fP_2 + (1-f)P_2^i + fP_2^{k+1}\left(P_2^i - P_2^k\right)}{f\left(1+P_2^{i+k} - P_2^{2k}\right) + (1-f)P_2^i} \leq \beta \tag{3.6}$$

The values of P_1 and P_2 are determined from (3.3) and (3.4). There may exist multiple solutions, we alternatively determine these parameters to minimize the average sample number at the quality level p_2, which is analogous to minimizing the average sample number (ASN) in a usual single and double sampling plans. Obviously, a sampling plan having smaller ASN would be more desirable. According to Balamurali and Jun (2010) the ASN of the SkSP-V plan at LQL is given as

$$ASN\left(p_2\right) = \frac{nf + nf\left(P_2^{i+k} - P_2^{2k}\right)}{f\left(1+P_2^{i+k} - P_2^{2k}\right) + (1-f)P_2^i} \tag{3.7}$$

The design parameters of the SkSP-V sampling plan are determined for various combinations of AQL and LQL. Therefore, we consider the following optimization problem to determine parameters of the SkSP-V plan.

$$\text{Minimize } ASN\left(p_2\right) = \frac{nf + nf\left(P_2^{i+k} - P_2^{2k}\right)}{f\left(1+P_2^{i+k} - P_2^{2k}\right) + (1-f)P_2^i} \tag{3.8}$$

Subject to

$$\frac{fP_1 + (1-f)P_1^i + fP_1^{k+1}\left(P_1^i - P_1^k\right)}{f\left(1+P_1^{i+k} - P_1^{2k}\right) + (1-f)P_1^i} \geq 1-\alpha$$

$$\frac{fP_2 + (1-f)P_2^i + fP_2^{k+1}\left(P_2^i - P_2^k\right)}{f\left(1+P_2^{i+k} - P_2^{2k}\right) + (1-f)P_2^i} \leq \beta$$

$$n > 1, c \geq 0, i, k, x > 1, 0 < f < 1 \tag{3.9}$$

The design parameters values along with the OC values at the producer's risks of 5% and the consumer's risks of 10% are placed in Table 1. The design parameters of the single sampling plan can be obtained by a similar approach, that is shown in Table 2. From Table 1, we can see the various trends in design parameters. For the same value of p_1, as we increase the value of p_2, we noted the decreasing trends in n, c and increasing trend in f. We noted the same behavior in the design parameters of the single sampling plan.

3.1 Example

Suppose one wants to determine parameters of an SkSP-V plan from Table 1 according to the conditions given that $p_1 = 0.005$, $p_2 = 0.025$, $\alpha = 0.05$ and $\beta = 0.10$. From this table, one can find the optimal parameters as $n = 91$, $c = 0$, $i = 6$, $k = 5$, $x = 5$ and $f = 0.01$ corresponding to the above mentioned AQL and LQL values. Based on these parameters, the SkSP-V plan is operated as follows.

Step 1. Start with normal inspection (inspecting every lot) using the single sampling plan (91, 0).

Step 2. When 6 consecutive lots are accepted on normal inspection, discontinue the normal inspection and switch to skipping inspection.

Step 3. During skipping inspection, inspect 1 lot out of every 100 lots selected at random. Skipping inspection is continued until sampled lot is rejected.

Step 4. When a lot is rejected on skipping inspection before 5 consecutively sampled lots are accepted, revert to normal inspection as per (1) above.

Step 5. When a lot is rejected after 5 consecutive lots have been accepted revert to normal inspection with reduced clearance number 5 as per (6) given below.

Step 6. During normal inspection with clearance number 5, all the four lots are inspected one by one in the order of being submitted to inspection and continue the inspection until a lot is rejected or 5 lots are accepted whichever occurs earlier.

Step 7. When a lot is rejected, immediately revert to normal inspection with clearance number 5 as per (1) given above.

Step 8. When 4 consecutive lots are accepted, discontinue normal inspection and switch to skipping inspection as per (3) above.

Step 9. Replace or correct all the non-conforming units found with conforming units in the rejected lots.

Table 1:
Parameters of SkSP-V plan for specified AQL and LQL

p_1	p_2	Optimal Parameters							
		i	k	f	n	c	ASN	$(1-\alpha)\%$	$\beta\%$
0.001	0.002	7	6	0.0002	1946	1	1946	95.42	10.00
	0.005	6	5	0.010	460	0	460.00	95.02	9.98
	0.010	4	3	0.119	230	0	229.98	95.00	9.97
	0.015	4	3	0.238	153	0	152.00	95.00	9.93
	0.020	5	4	0.338	114	0	114.00	95.00	10.00
	0.030	3	2	0.644	76	0	75.97	95.00	9.93
0.005	0.010	6	5	0.0005	390	1	390.00	95.09	9.96
	0.025	6	5	0.01	91	0	91.00	95.08	10.00
	0.050	6	5	0.080	45	0	45.00	95.00	9.95
	0.100	3	2	0.414	22	0	21.99	95.00	9.97
	0.150	2	1	0.671	15	0	14.96	95.00	9.08
0.01	0.020	5	4	0.001	198	1	198.00	95.32	9.84
	0.050	5	4	0.016	45	0	45.00	95.13	10.00
	0.100	2	1	0.168	25	0	21.89	95.02	9.50
	0.200	2	1	0.445	11	0	10.95	95.00	9.43
	0.300	2	1	0.720	7	0	6.98	95.00	8.48
0.05	0.100	5	4	0.001	39	1	39.00	95.48	9.23
	0.250	3	2	0.039	9	0	9.00	95.10	8.46
	0.500	2	1	0.218	4	0	3.99	95.01	7.55
0.100	0.200	4	3	0.0003	13	0	13.00	95.00	8.29
	0.500	3	2	0.048	4	0	4.00	95.08	6.70

Table 2:
Parameters of Single Sampling Plan for Specified AQL and LQL

p_1	p_2	Optimal Parameters	
		n	c
0.001	0.002	12375	18
	0.005	1135	3
	0.010	531	2
	0.015	258	1
	0.020	194	1
	0.030	129	1
0.005	0.010	2478	18
	0.025	266	3
	0.050	105	2
	0.100	38	1
	0.150	25	1
0.01	0.020	1235	18
	0.050	132	3
	0.100	52	2
	0.200	18	1
	0.300	12	1
0.05	0.100	233	17
	0.250	25	3
	0.500	7	1
0.100	0.200	109	16
	0.500	12	3

4. ADVANTAGES OF THE SKSP-V PLAN

In this section, we discuss the advantages of the SkSP-V sampling over the single sampling plan. For this purpose, we have calculated ASN values of SkSP-V plan and compared with the sample size required for a single sampling plan for different values of p_1 and p_2. Table 3 summarizes the results.

From this table, we can see that for the same values of AQL and LQL, the SkSP-V sampling plan provides much smaller sample size as compared to single sampling plan (or reference sampling plan). For an example, when p_1=0.001 and p_2=0.002, the required sample size $n = 1946$ from Table 1 for SkSP-V plan and it is 12375 when we test the items using the single sampling plan. So, the SkSP-V sampling is more economic than the single sampling in saving the time, cost and the efforts for an experiment.

Table 3:
Comparison of Sample Size

p_1	p_2	SkSP-V Plan	Single Sampling Plan
0.001	0.002	1946	12375
	0.005	460	1135
	0.010	230	531
	0.015	153	258
	0.020	114	194
	0.030	76	129
0.005	0.010	390	2478
	0.025	91	266
	0.050	45	105
	0.100	22	38
	0.150	15	25
0.01	0.020	198	1235
	0.050	45	132
	0.100	25	52
	0.200	11	18
	0.300	7	12
0.05	0.100	39	233
	0.250	9	25
	0.500	4	7
0.100	0.200	13	109
	0.500	4	12

In order to show the better efficiency of the SkSP-V plan in terms of probability of acceptance, average sample number and average total inspection (ATI) three figures are provided. Figure 1 gives the OC curves of the SkSP-V plan with parameters $i=6$, $k=3$, $x=3$, $f=0.01$ along with single sampling plan with parameters $N=1000$, $n=50$ and $c=1$ as the reference plan. Figure 2 gives the ASN curves while Figure 3 shows the ATI curves of the above mentioned plans. From Figure 1, it can be observed that the SkSP-V plan increases the probability of acceptance in the region of principal interest, i.e. for good quality levels and maintains the consumer's risk at poor quality levels compared with the single sampling plan. It implies that SkSP-V plan gives comparatively lesser producer's risk while safeguarding the consumer's interest than the single sampling plan. From Figures 2 and 3, it is easily observed that when the lot quality is good, reduction in ASN as well as ATI are achieved through the SkSP-V plan over the single sampling plans. When the lot quality deteriorates, the ASN and ATI of the SkSP-V plan converge with the single sampling plan.

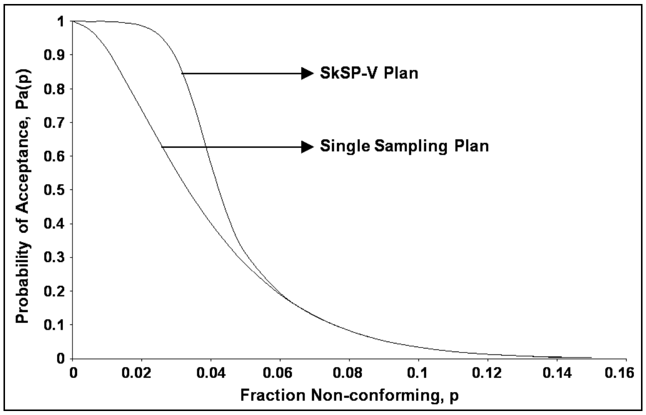

Fig.1: Operating Characteristic (OC) Curves of SkSP-V & Single Sampling Plans

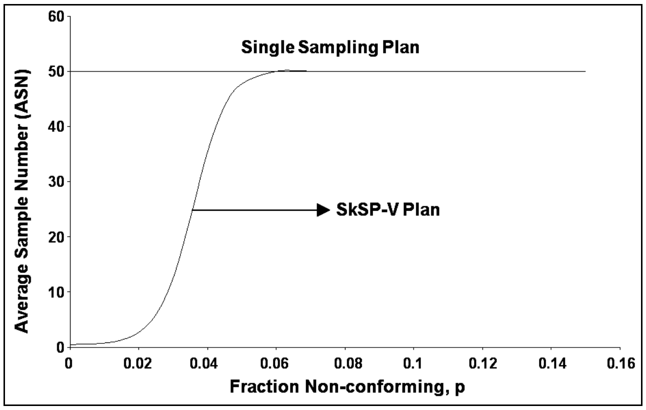

Fig.2: Average Sample Number (ASN) Curves of SkSP-V & Single Sampling Plans

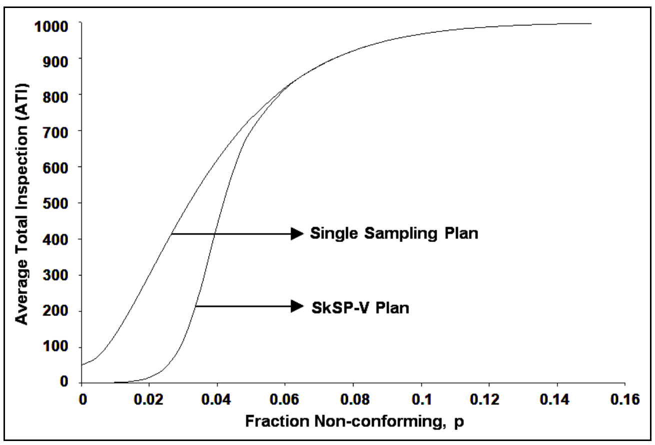

Fig.3: Average Total Inspection (ATI) Curves of SkSP-V & Single Sampling Plans

5. CONCLUDING REMARKS

In this paper, we have considered the problem of designing the SkSP-V sampling plan. The two points approach is used to find the design parameters of the SkSP-V plan, which is considering the producer's and the consumer's simultaneously. Tables for showing design parameters of both SkSP-V and single sampling plans have been presented and comparison has been made between two plans. The procedure was described to use the proposed methodology in practice. It has been proved that the proposed plan is better than the single sampling plan in terms of the sample size requires.

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