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THE OPTIMIZATION OF PARAMETERS FOR TARGET TRACKING FILTER BY GENETIC ALGORITHM AND TAGUCHI METHOD

Chun-Mu Wu[1], **Ching-Kao Chang**[2] and **Tung-Te Chu**[2,1]

[1] Department of Mechanical Engineering and Automation Engineering, Kao Yuan University, Taiwan, ROC. Email: ttca@seed.net.tw
No. 1821, Jhongshan Rd., Lujhu Township, Kaohsiung County, 821, Taiwan, ROC

[2] Institute of Engineering Science and Technology, National Kaohsiung First University of Science and Technology, No. 2, Juoyue Rd., Nantz District, Kaohsiung, 811, Taiwan, ROC

ABSTRACT

The paper searches for the parameter optimization of a new fourth-order target tracker α-β-γ-δ filter for numerical simulation as the current third-order α-β-γ filter system tracks only the position and the velocity of a target, rather than the acceleration. Although the new fourth-order α-β-γ-δ filter shows a significantly improved tracking accuracy over the conventional α-β-γ filter, more computation time is required in the optimization process. The developed GA-based α-β-γ-δ filter not only could find the optimal set of filter parameters to minimize position tracking errors, but could also reduce the computation time (Wu et al., 2009). Moreover, Taguchi method is applied to finding the optimal set of GA parameters for better tracking accuracy. Meanwhile, for more practical applications, the simulated trajectory, such as sin wave, illustrates the filter efficiency in this study.

KEYWORDS

Target tracker; Genetic Algorithm; the α-β-γ-δ filter; the α-β-γ filter; the GA-based α-β-γ-δ filter.

1. INTRODUCTION

Lin and Chang (2004) stated Target tracking has been studied and applied with many mathematical models in much research. It is regarded as an important tool in military and civilian fields to track fast-moving objects, such as airplanes, missiles, submarines, and air-traffic handlings. The dynamic model and the discrete-time data are used to describe and predict the kinematics of a dynamic object. Since the mid 1950's, the α-β filter system, a sampled data target tracking algorithms described by Sklansky (Sklansky, 2004), achieved a mathematic optimization process in radar systems. The relatively simple α-β and α-β-γ filter trackers (Jing et al., 2004; Ogata, 1987) were explored to deal with this problem that numerous optimal target-tracking algorithms were developed (Benedict and Bordner, 1962; Kalata, 1992) Recently, Tenne and Singh presented the optimal design of the third-order α-β-γ filter (Tenne and Singh, 2002); Neal and Benedict (Neal and Benedict, 1967) discussed an optimization relation of α-β-γ filter. In order to predict the acceleration and to improve the tracking accuracy, an additional state variable called jerk (a time derivative of acceleration) is used in this study. Nevertheless, since the new α-β-γ-δ filter would take more computation time in the optimization process, a practical technique of adding jerk in the new α-β-γ-δ filter for optimal simulation via GA is recommended to reduce the computation time. In recent years, several parameter estimation techniques were applied to discussing α-β-γ filters (Lin and Chang, 2004; Lu and Tzeng, 2000). As a numerical optimization method, the artificial intelligence-based GA is successively explored to offer an efficient optimization for finding the optimal parameters (i.e., α, β, γ and δ) as the final output values. The developed GA-based α-β-γ-δ filter not only shows the optimization of filter parameters to minimize position tracking errors, but could also reduce the computation time (Wu et al., 2009). Meanwhile, as the applicability of GA has been widely developed (Jung, Lee and Lee, 1998; Wang, 1997; Ma et al., 1996), Taguchi method is applied to finding the optimal set of GA parameters for better tracking accuracy in this paper.

2. MATHEMATICAL EQUATIONS

Based on Tenne and Singh (2002), Kalata (1981), and Wu et al. (2009), the following matrix equations are utilized for the object's position and velocity of the modeling target tracking.

2.1 The third-order α-β-γ filter target tracker:

$$P_f(k+1) = \Gamma P_s(k) \tag{1}$$

$$P_f = \begin{bmatrix} p_f & \dot{p}_f & \ddot{p}_f \end{bmatrix}^T,\ P_s = \begin{bmatrix} p_s & \dot{p}_s & \ddot{p}_s \end{bmatrix}^T \tag{2}$$

$$\Gamma = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \end{bmatrix} \tag{3}$$

where T stands for time step or time increment, p for position, $\dot{p}$ for velocity, and $\ddot{p}$ for acceleration; subscript f and s denote the predicted and smoothed state values, respectively.

$$P_s(k) = S(k) + K(y_e(k) - HP_f(k)) \tag{4}$$

in which

$$P_s = \begin{bmatrix} p_s & \dot{p}_s & \ddot{p}_s \end{bmatrix}^T\ P_f = \begin{bmatrix} p_f & \dot{p}_f & \ddot{p}_f \end{bmatrix}^T \tag{5}$$

$$S(k) = \begin{bmatrix} p_f(K) & \dot{p}_f(K) & \ddot{p}_f(K) \end{bmatrix}^T \tag{6}$$

$$K = \begin{bmatrix} \alpha & \frac{\beta}{T} & \frac{\lambda}{2}T^2 \end{bmatrix}^T \tag{7}$$

$$y_e(k) = HP(k) + V(k)\quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

where subscript e denotes the exact value.

With Z-Transform and equations (1) to (8), the ratio (p_f/p_e) is solved, and the transfer function in the z-domain is given by

$$G(z)=\frac{p_f}{p_e}=\frac{\alpha+\left(-2\alpha-\beta+\frac{1}{4}\gamma\right)z+\left(\alpha+\beta+\frac{1}{4}\gamma\right)Z^2}{Z^3+\left(\alpha+\beta+\frac{1}{4}\gamma-3\right)Z^2+\left(-2\alpha-\beta+\frac{1}{4}\gamma+3\right)z+\alpha-1} \tag{9}$$

In the α-β-γ filter, the constraints of α, β, and γ parameters are yielded by Jury's stability test (Ogata, 1987) as shown below.

$$0<\alpha<2 \tag{10}$$

$$0<\beta<4-2\alpha \tag{11}$$

$$0<\gamma<\frac{4\alpha\beta}{2-\alpha} \tag{12}$$

2.2 The fourth-order α-β-γ-δ filter target tracker:

As shown in item 2.1, for improving the tracking accuracy, the equations of the fourth-order α-β-γ-δ filter target tracker are given by

$$P_f(k+1)=\Gamma P_s(k) \tag{13}$$

in which

$$P_f=\begin{bmatrix}p_f & \dot{p}_f & \ddot{p}_f & \dddot{p}_f\end{bmatrix}^T, P_s=\begin{bmatrix}p_s & \dot{p}_s & \ddot{p}_s & \dddot{p}_s\end{bmatrix}^T \tag{14}$$

$$\Gamma=\begin{bmatrix}1 & T & \frac{1}{2}T^2 & \frac{1}{6}T^3\\ 0 & 1 & T & \frac{1}{2}T^2\\ 0 & 0 & 1 & T\end{bmatrix} \tag{15}$$

$$P_s(k)=S(k)+K\left(y_e(k)-HP_f(k)\right) \tag{16}$$

where

$$P_s=\begin{bmatrix}p_s & \dot{p}_s & \ddot{p}_s & \dddot{p}_s\end{bmatrix}^T \tag{17}$$

$$S(k)=\begin{bmatrix}p_f(k) & \dot{p}_f(k) & \ddot{p}_s(k) & \dddot{p}_s(k-1)\end{bmatrix}^T \tag{18}$$

$$K=\begin{bmatrix}\alpha & \frac{\beta}{T} & \frac{\lambda}{2}T^2 & \frac{\delta}{6}T^3\end{bmatrix}^T, \tag{19}$$

$$y_e(k)=HP(k)+V(k)\text{ , }H=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} \tag{20}$$

$$P_f = \begin{bmatrix} p_f & \dot{p}_f & \ddot{p}_f & \dddot{p}_f \end{bmatrix}^T \tag{21}$$

In the same way, the transfer function in the z-domain is given by

$$G(z)=\frac{p_f}{p_e}=\frac{\left(\alpha+\beta+\frac{\gamma}{4}+\frac{\delta}{36}\right)Z^3+\left(-3\alpha-2\beta+\frac{\delta}{9}\right)Z^2+\left(3\alpha+\beta-\frac{1}{4}\gamma+\frac{\delta}{36}\right)Z-\alpha}{Z^4+\left(\alpha+\beta+\frac{\gamma}{4}+\frac{\delta}{36}-4\right)Z^3+\left(-3\alpha-2\beta+\frac{\delta}{9}+6\right)Z^2+\left(3\alpha+\beta-\frac{\gamma}{4}+\frac{\delta}{36}-4\right)Z+(1-\alpha)} \tag{22}$$

The α, β, γ, and δ parameters appear as follows

$$0<\alpha<2 \tag{23}$$

$$0<\beta<\frac{13}{6}(4-2\alpha) \tag{24}$$

$$0<\gamma<\frac{4\alpha\beta}{2-\alpha} \tag{25}$$

$$0<\delta<24(2-\alpha) \tag{26}$$

3. GENETIC ALGORITHM

As for Evolutionary Programming and Evolutionary Strategies, GA is one of the Evolution Algorithms, which can be traced back to the early 1950s and formally introduced in 1970s by John Holland at university of Michigan, United States. He utilized the principles of biological evolution in the nature-natural selection and the survival of the fittest for finding the best solutions in the optimization process (Wu et al., 2010). The ideals of GA are derived from the principles of “natural selection “and “survival of the fittest” in the world of nature, imitating competitions among creatures which show the survivors as being able to propagate their next generations. Then, it is applied to the optimal design which randomly generates abundant compositions that more optimal results are kept in the operations (Wu et al., 2010). Thus, after an iterative process of selections, the most optimal result can eventually be generated in the operations. Jing et al., (2004) presented Genetic algorithms imitate the way creatures going through the procedures of inheritances. In GA, there are three main operators, namely reproduction, crossover, and mutation.

4. TAGUCHI METHOD

Taguchi method is a statistical process perturbing a parameter for studying the effects on the whole output and has been widely used in engineering analyses to optimize the performance characteristics through the setting of design parameters (Erzurumlu and Ozcelik, 2006). When performing Taguchi method analyses, the S/N ratio for each experimental run is computed and recorded that the S/N ratio is an indication of significance. The design objective function is the-small-the-better in this study that the

S/N ratio with the-small-the-better characteristic is selected and expressed as follows (Wu et al., 2010).

$$\eta_{ij} = -10\log\left(\frac{1}{n}\sum_{j=1}^{n} y_{ij}^2\right) \tag{27}$$

where y_{ij} is the i th experiment at the j th test, n is the total number of the tests, and s is the standard deviation.

This paper intends to find the optimal parameter combination of GA parameters, namely Population, Generation, Crossover, and Mutation, via Taguchi method, and to use them as the optimization of tracking accuracy. Wu et al. (2010) presented each parameter presents three levels, table 1. The L_9 orthogonal array is chosen for the approach. Finally, the tracking accuracy is compared for each type of error with and without Taguchi method.

5. NUMERICAL SIMULATION EXAMPLE

Lee et al. (2009) presented a searching method for the parameter optimization of a third-order α-β-γ filter. As extended, it is desirable to compare the position tracking errors of GA-α-β-γ filter with GA-α-β-γ-δ filter via a numerical simulation process as well as to know the difference between GA-based α-β-γ-δ filter tracker with and without Taguchi method in this paper. Consider, for example, a case of the signal in sin wave Y = sin (t) + sin (3t) + sin (5t).

Wu et al. (2010) said that simulation of the object's position is performed with each of the two trackers, and the associated errors are determined. The limits of α, β, γ and δ are determined by the constraints given by equations (10)-(12) for the three-order α-β-γ filter and by equations (23)-(26) for the fourth-order α-β-γ-δ filter. In the numerical simulation process, the increment for each simulated parameter value is set 0.1.

In the numerical simulation, with t_{start} =0 and t_{max} =10, the error in each filter is calculated in different time steps (i.e. T = 0.2, 0.1, 0.05, 0.025). Figures 1, 2, 3, and 4 show a position tracking plot at T = 0.2 and 0.05 with GA-based α-β-γ and GA-based α-β-γ-δ, showing the acceleration changes with time and the change of the state variable, jerk. It is noted that the simulation error in acceleration is related to the second order derivative with respect to time (Wu et al., 2010).

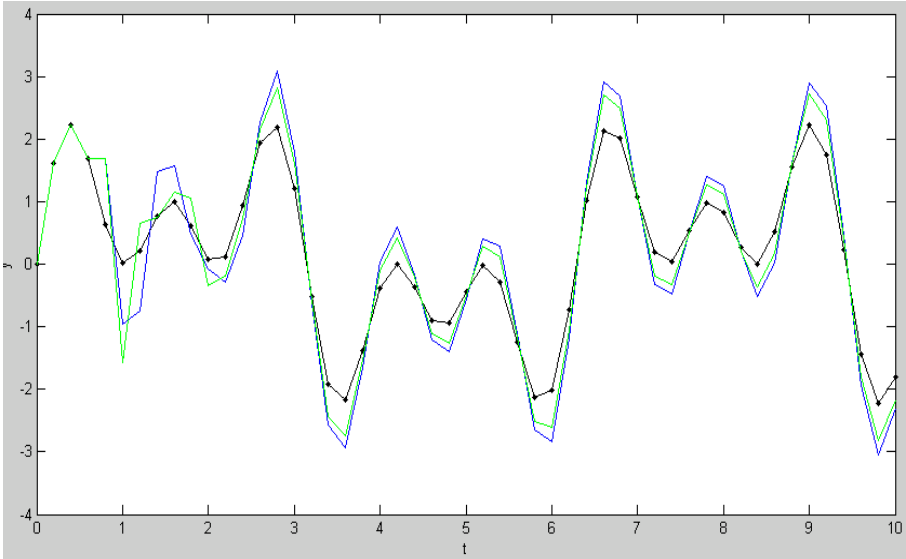

Fig. 1: A Tracking Plot at T=0.2 (L2 and Lm) with the GA-based α-β-γ filter (-: de, -: Lm, -: L2)

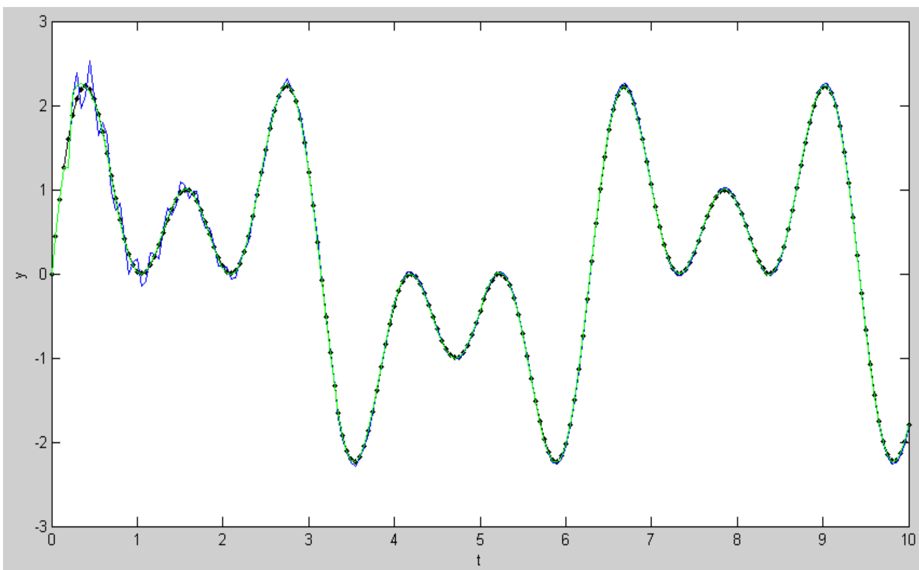

Fig. 2: A Tracking Plot at T=0.05 (L2 and Lm) with the GA-based α-β-γ filter (-:de, -: Lm, -: L2)

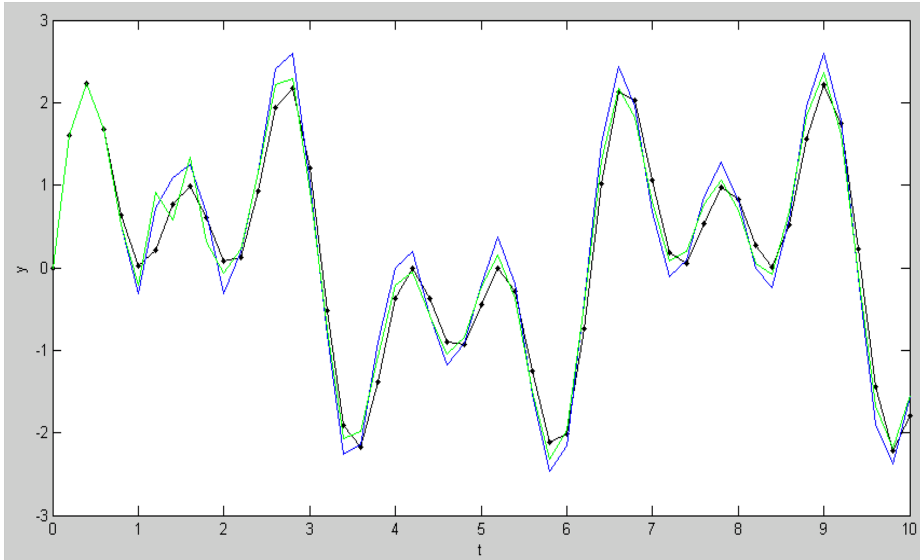

Fig. 3: A Tracking Plot at T=0.2 (L2 and Lm) with the GA-based α-β-γ-δ filter (-:de, -: Lm, -: L2)

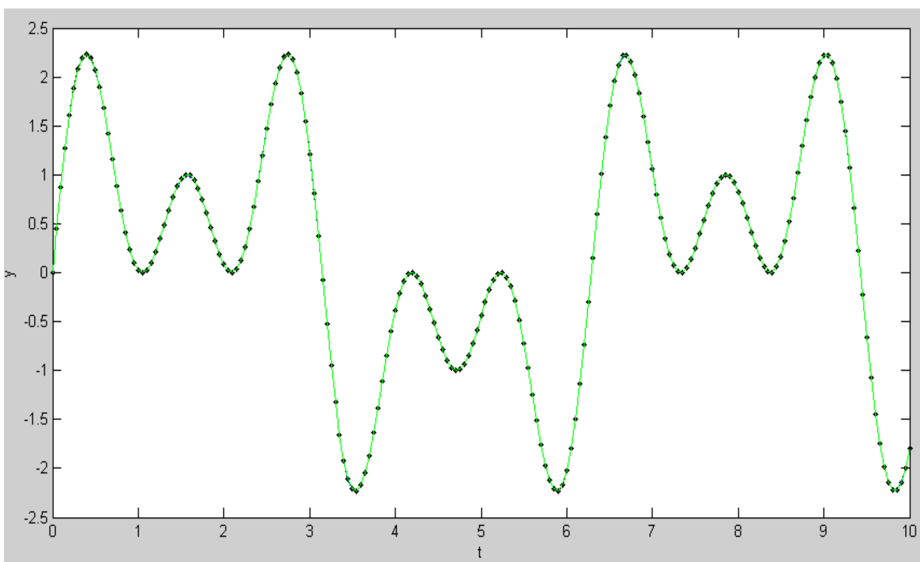

Fig. 4: A Tracking Plot at T=0.05 (L2 and Lm) with the GA-based α-β-γ-δ filter (-:de, -: Lm, -: L2)

Tables 2 and 3 summarize the tracking errors on L_2 and L_{max} from two filters, respectively. As shown, the GA-α-β-γ-δ filter outperforms the GA-α-β-γ filter at every time step. Clearly, a fourth-order target tracker of α-β-γ-δ filter presents a significantly better tracking accuracy than the α-β-γ filter does. Table 4 and 5 summarize the tracking errors on L_2 and L_{max} w/o Taguchi method in GA-based α-β-γ-δ filter tracker, respectively. The data indicates that the GA-based α-β-γ-δ filter tracker with Taguchi method outperforms the GA-based α-β-γ-δ filter tracker without Taguchi method for every given time step. Therefore, it shows that the two heuristic methods can be combined to improve the tracking accuracy.

Table 1:
The three levels of GA parameters

Parameters	Level 1	Level 2	Level 3
Population	50	150	200
Generation	25	35	50
Crossover	0.6	0.7	0.8
Mutation	0.1	0.2	0.3

Table 2:
Tracking Error Comparisons on L_2 among Two Filters

Time Step	GA-based-α-β-γ filter (1)	GA-based-α-β-γ-δ filter (2)	Improvement [(1)-(2)]/(1)%
T=0.2	α=0.5,β=2.6,γ=0.1 L_2=0.0600	α=0.5,β=3.0,γ=0.8,δ=0.8 L_2=0.0561	6.5%
T=0.1	α=1.2,β=1.2,γ=4.1 L_2=0.0078	α=0.3,β=3.0,γ=1.9,δ=10.9 L_2=0.0068	12.82%
T=0.05	α=1.1,β=1.1,γ=1.6 L_2=0.0025	α=0.2,β=3.0,γ=1.0,δ=12.4 L_2=0.0019	24.00%
T=0.025	α=1.1,β=1.0,γ=0.8 L_2=6.2348e-004	α=0.2,β=2.9,γ=1.0,δ=13.0 L_2=4.1660e-004	33.18%

Table 3:
Tracking Error Comparisons on L_{max} among Two Filters

Time Step	GA-based-α-β-γ filter (1)	GA-based-α-β-γ-δ filter (2)	Improvement [(1)-(2)]/(1)%
T=0.2	α=0.4,β=2.1,γ=0.1 L_{max}=1.0436	α=0.3,β=3.0,γ=1.1,δ=1.1 L_{max} =0.8898	14.73%
T=0.1	α=0.6,β=1.4,γ=0.1 L_{max} =0.3136	α=0.5,β=3.0,γ=2.7,δ=9.4 L_{max} =0.2425	22.67%
T=0.05	α=0.1,β=2.3,γ=0.1 L_{max} =0.2956	α=0.5,β=3.0,γ=2.1,δ=10.7 L_{max} =0.1912	35.31%
T=0.025	α=0.1,β=1.4,γ=0.1 L_{max} =0.1805	α=0.2,β=3.0,γ=1.0,δ=13.1 L_{max} =0.0960	46.81%

Table 4:
Tracking Error Comparisons of L_2 on GA-based α-β-γ-δ filter w/o Taguchi method

Time step	GA α-β-γ-δ filter without Taguchi method (1)	GA α-β-γ-δ filter with Taguchi method (2)	improvement ((1)-(2))/(1)%
T= 0.2	α=0.5,β=3.0,γ=0.8,δ=0.8 L_2=0.0561	α=0.85,β=2.57,γ=1.94,δ=0.50 L_2=0.0528	5.88%
T= 0.1	α=0.3,β=3.0,γ=1.9,δ=10.9 L_2=0.0068	α=0.59,β=2.58,γ=1.71,δ=0.42 L_2=0.0062	8.82%
T= 0.05	α=0.2,β=3.0,γ=1.0,δ=12.4 L_2=0.0019	α=1.06,β=1.97,γ=3.98,δ=3.67 L_2=0.00177	6.84%
T= 0.025	α=0.2,β=2.9,γ=1.0,δ=13.0 L_2=4.16606e-004	α=1.37,β=1.77,γ=4.87,δ=5.23 L_2=3.86770e-004	7.16%

Table 5:
Tracking Error Comparisons of L_{max} on GA-based α-β-γ-δ filter w/o Taguchi method

Time step	GA α-β-γ-δ filter without Taguchi method (1)	GA α-β-γ-δ filter with Taguchi method (2)	Improvement ((1)-(2))/(1)%
T= 0.2	α=0.3,β=3.0,γ=1.1,δ=1.1 L_{max} =0.8898	α=0.98,β=2.53,γ=0.82,δ=2.86 L_{max} =0.8218	7.64%
T= 0.1	α=0.5,β=3.0,γ=2.7,δ=9.4 L_{max} =0.2425	α=0.35,β=3.70,γ=2.98,δ=4.68 L_{max} =0.2266	6.56%
T= 0.05	α=0.5,β=3.0,γ=2.1,δ=10.7 L_{max} =0.1912	α=0.65,β=2.79,γ=1.67,δ=4.98 L_{max} =0.1807	5.49%
T= 0.025	α=0.2,β=3.0,γ=1.0,δ=13.1 L_{max} =0.0960	α=0.71,β=2.18,γ=0.86,δ=2.69 L_{max} =0.0895	6.77%

7. CONCLUSION

The new fourth-order GA-based α-β-γ-δ filter is studied and compared with the GA-based third-order α-β-γ filter for tracking accuracy. The results clearly show the improvement in tracking accuracy of the former exhibits a significant, indicating that the fourth-order α-β-γ-δ filter is a highly accurate tracker especially when the moving target exhibits a large variation in its speed (acceleration), because it could predict the target trajectory in a pure data-driven system. The other result shows that GA and Taguchi method can be combined to improve the tracking accuracy; and, the searched methods could be applied to various fields.

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