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RENOVATED PARTIAL PLOTS AND HAT MATRIX FOR CENSORED REGRESSION MODEL

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ABSTRACT

Partial residual plots are a useful diagnostic tool in multiple regression to clarify the effect of the ith explanatory variable on the predictive ability of the response variable when the other explanatory variables are still present in the model. The usual diagnostic plots of residuals versus explanatory variable provide standard checks on model assumptions. Partial residual plots not only do this, but have the added enhancement of including whether violations to assumptions may be adequately modeled by including curvature in the model or should be modeled by transforming responses (Aziz and Wang, 2009) This paper outlines the construction of renovated partial residuals, plots and their properties for regression with censored data.

KEYWORDS

Renovated partial residual; censored regression; censored data; Buckley-James estimators; product-limit estimator; properties; diagnostic analysis; added variable plot and hat matrix.

Mathematics Subject Classification: 62H07; 62H99.

1 INTRODUCTION

Consider the linear regression model

$$Y = X\beta + \varepsilon \tag{1.1}$$

where Y represents an n-vector of responses, X is an $n \times (p+1)$ explanatory variables, the $(p+1) \times 1$ vector, β, represents the unknown parameter vector, $\beta^T = (\beta_0, \beta_1, ..., \beta_p)$, of the model, and the term ε is a random variable with independent and identically distribution F components with mean zero and variance σ^2, and unknown survival function S=1-F.

Suppose that we separate out X_i, the $(i+1)$th column of X, for special consideration $(0 \leq i \leq p$). If we use the notation that x_i represents the carrier which gives the n observations in X_i and y the response variable, then a common way of writing model (1.1) is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_i x_i + ... + \beta_p x_p + \varepsilon. \tag{1.2}$$

Partial residual plots will give us insight into whether x_i should be included in the model, or whether another unknown function, say $g(x_i)$, should instead include in the model to improve the fit. Possible choices of g may be $g(x) = c_1 x$ or $g(x) = c_1 x + c_2 x^2$, where c_1 and c_2 are constants. Following Cook (1993) and McKean and Sheather (1997), we adopt an approach that the correct model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + g(x_i) + ... + \beta_p x_p + \varepsilon^*. \tag{1.3}$$

This means that model (1.2) is a misspecification of the true model (1.3) unless g is a linear function. The aim of this paper is to present partial residual plots as a tool to help us to detect this. Certainly knowledge of g may assist in choosing a co-variate transformation towards the linearity of (1.2).

Suppose initially that we fit the misspecified model (1.2). Let $\hat{\beta}$ be the least squares estimator of β in (1.2). Then the residual R is given in terms of the vector $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_p)$ by

$$R = Y - X\hat{\beta}.$$

Furthermore, the ith partial residual vector is defined, for $1 \leq i \leq p$, by

$$R_{X_i} = R + X_i\hat{\beta}_i = Y - \sum_{j \neq i} X_j \hat{\beta}_j.$$

The plot of the n entries in R_{X_i} against the n entries in X_i, is called the ith partial residual plot, or the partial residual plot for X_i. This plot was first introduced by Ezekiel (1924) and later investigated by Larsen and McCleary (1972), Wood (1973) and Cook (1993). Atkinson (1985), for example, notes that the plot of R_{X_i} on X_i has slope $\hat{\beta}_i$. Plots pointing to curvature of the function g in (3) have been examined by Mansfield and Conerly (1987).

The aim of this paper is to build partial residual plot methods that are appropriate for linear regression where the regression are right-censored and the standard regression parameters are estimated by the Buckley-James procedure.

2 BUCKLEY-JAMES REGRESSION AND ESTIMATORS

Over the last twenty years there have been many regression methods proposed in the literature to cater for the commonly occurring situation where the response variable of interest has been right-censored. The proportional hazards method of Cox (1972) has certainly experienced wide usage. However, methodology based on the simple linear model enjoys considerable support: because the modeling assumptions underlying proportional hazards may not be tenable; because many models may be transformed to the linear; and simple linear regression is conceptually well understood.

In this regard, the linear regression method of Buckley and James (1979) for regression with censored response variable data has performed well in many simulation studies and comparisons (Weissfeld and Schneider, 1986; Heller and Simonoff, 1990; Hillis, 1993; Wu and Zubovic, 1995 and Currie, 1996). In particular, Heller and Simonoff (1992) discuss how to make an appropriate choice between the distribution-free regression methodology of Buckley-James and proportional hazards when estimating life-times in the presence of a continuous explanatory variable.

In recent developments, the Buckley-James methodology has been enhanced by examining its association with classical least squares in areas such as scatter-plots, residual analysis and diagnostics (Smith, 2004). Much of this work has developed through the process of 'renovation' of the censored response data in the scatter-plot. In this process, based on Buckley-James linear regression estimators, censored responses are adjusted to estimated expected positions had they been unaffected by censoring, while uncensored responses are not moved. These estimated positions will be consistent under the conditions that the Buckley-James parameter estimates are consistent; essentially, that during the linear model fitting process, the expected numbers of censored residuals and uncensored residuals be large over the support of the residual distribution. Buckley-James estimation and the renovation process are outlined as follows.

Suppose that for each $i = 1,2,...,n$, the response Y_i is not completely observed but is rather right-censored by a fixed censor time t_i which does not depend on the response. This means that the observed responses and co-variates are in the form

$$(Z_i, \delta_i, X_i), \quad i = 1,2,...,n,$$

where $Z_i = min\{Y_i, t_i\}$, $\delta_i = I_{(y_i \le t_i)}$ are the indicators of censoring, and X_i^T is the *ith* row of X. The censor variable t_i need not be random in general. We fit the model (1.1) to the data set (Z_i, δ_i, X_i).

In the Buckley-James approach (Buckley and James, 1979), censored responses in the scatter-plot of data (Z_i, δ_i, X_i) are lifted higher in the scatter-plot by replacing them with their estimated conditional expected responses, $Y^*(b)$, using a weighted linear combination

$$Y^*(b) = Xb + W(b)E(b) \tag{2.1}$$

of observed residuals $E(b) = (e_1(b), e_2(b), ..., e_n(b))^T = Z - Xb$ from a fitted line of slope $b = (b_0, b_1, ..., b_p)^T$ through the observed responses $Z = (Z_1, Z_2, ..., Z_n)^T$. The entries in the weights matrix $W(b) = diag(\delta) + [w_{ik}(b)]$ in (2.1) are defined by

$$w_{ik}(b) = \begin{cases} \frac{d\hat{F}_b(e_k(b))\delta_k(1-\delta_i)}{1-\hat{F}_b(e_i(b))}, & \text{if} \quad e_k(b) > e_i(b); \\ 0 & \text{otherwise.} \end{cases}$$

involving $d\hat{F}_b(e_k(b))$, the probability mass assigned by $\hat{F}$ to $e_k(b))$. The weight $W(b)$ is upper-triangular matrix and satisfies: (i) W is idempotence matrix. (ii) Row sum $W\mathbf{1} = \mathbf{1}$. Using $\hat{F}_b$, the product-limit estimator based on $E(b)$. The Buckley-James estimator are found by determine an iterative solution $b = \hat{\beta}_{bj}$ to the equation

$$\Phi(b) = (X^T X)^{-1} X^T Y^*(b) - b = 0. \tag{2.2}$$

A solution $b = \hat{\beta}_{bj}$ to (2.2) is reached iteratively when the norm of the left side is minimum (James and Smith, 1984; Lin and Wei, 1992). This solution is of the form

$$\hat{\beta}_{bj} = [X^T W(\hat{\beta}_{bj})X]^{-1} X^T W(\hat{\beta}_{bj})Z$$

when (2.2) has an exact solution. If solution to (2.2) take the form of multiple zero-crossings, or points where Φ changes sign, it is customary to take $\hat{\beta}_{bj}$ as their average (see, for example, Wu and Zubovic, 1995).

Once the Buckley-James solution $b = \hat{\beta}_{bj}$ has been found, the response data may be renovated to construct a scatter-plot depicting new estimated conditional expected value positions for the censored points and unchanged locations for the uncensored points. This may be done by calculating $W = W(\hat{\beta}_{bj})$ for using in the renovation slope estimate

$$\hat{\beta} = (X^T WX)^{-1} X^T WZ$$

and then applying both W and $b = \hat{\beta}$ to (2.1). Since $WY^* = WZ$, it follows that the renovated responses $Y^* = (Y_1^*, Y_2^*, ..., Y_n^*)^T$ satisfy

$$\hat{\beta} = (X^T WX)^{-1} X^T WY^*$$

and

$$Y^* = X\hat{\beta} + W(Y^* - X\hat{\beta}). \tag{2.3}$$

By a renovated scatter-plot we mean a diagram of the pair (X, Y^*) with Y^* in formula (2.3), and renovated residuals $r^* = Y^* - X\hat{\beta}$ satisfy $r^* = Wr^*$ and produce the same renovated scatter-plot (X, Y^*). We assume this structure for W, $\hat{\beta}$, and Y^* in what follows. Note that the results (Smith, 2004) show that the Buckley-James estimate is equal to the renovated slope estimate of β if and only if the zero crossing in (2.2) is exact.

3 PARTIAL RESIDUALS AND THEIR PROPERTIES

Given $\hat{\beta} = (X^T WX)^{-1} X^T WY^*$, the fitted values may be written as

$$\hat{Y}^* = X\hat{\beta} = X(X^T WX)^{-1} X^T WY^* = H^* Y^*,$$

where $H^* = X(X^T WX)^{-1} X^T W$ is called the renovated hat matrix. As is the case for linear regression with no censoring, H^* is idempotent and satisfies $tr(H^*) = p + 1$. Partial residual plots are based on a partitioned structure for X and H^*. The development is centered around the weights matrix W which carries the censoring information through the ranking of the censored and uncensored residuals.

Let $X = (X_1, X_2)$ where X_1 is an $n \times r$ matrix of rank r and X_2 is an $n \times (p + 1 - r)$ matrix of rank $p + 1 - r$, and let H_1^* be defined by $H_1^* = X_1(X_1^T WX_1)^{-1} X_1^T W$. We know that

$$H^* = H_1^* + (I - H_1^*)X_2[X_2^T W(I - H_1^*)X_2]^{-1} X_2^T W(I - H_1^*). \tag{3.1}$$

So that we are able to isolate on variable at a time in the model (1.1), we consider X_2 in the structure $X = (X_1, X_2)$ to be a $n \times 1$ vector with associated parameter β_2. Then (1.1) becomes

$$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon \tag{3.2}$$

with X_1 of full rank with a possible first column of ones. We establish result for the Buckley-James estimation of the component β_2 in $\beta = (\beta_1, \beta_2)^T$ as

$$\hat{\beta}_2 = [X_2^T W(I - H_1^*)X_2]^{-1}[X_2^T W(I - H_1^*)]Y^*.$$

Consider a renovated form of the left-side of (3.2) and replace Y by the renovated response vectors Y^*, then the Y^* still follow a linear model with a possibly changed error distribution. Without loss of generality, this may be modeled as

$$Y^* = X_1\beta_1 + X_2\beta_2 + \varepsilon^*.$$

Then we may define the renovated partial residual vector for X_2 by

$$R^*_{X_2} = Y^* - X_1\hat{\beta}_1 = R^* + X_2\hat{\beta}_2 = (I - H^*)Y^* + X_2\hat{\beta}_2$$

where $R^* = Y^* - X\hat{\beta}$ are renovated residuals and $H^* = X(X^TWX)^{-1}X^TW$, the renovated hat matrix.

The following theorem suggests that if we fit a linear model to the scatter-plot of the entries of $R^*_{X_2}$ against the corresponding entries of X_2, then the slope estimate is $\hat{\beta}_2$, the Buckley-James estimate of β_2.

Theorem 1. *(The slop estimate). When a linear model is fitted to the scatter-plot of $R^*_{X_2}$ verses X_2 the slope estimate is $\hat{\beta}_2$.*

Proof. Suppose that we fit the linear model $R^*_{X_2} = a\mathbf{1} + bX_2 + \varepsilon$ to the scatter-plot of $R^*_{X_2}$ versus X_2. In this case the design matrix has ones in the first column, and we use the representation $X = (\mathbf{1}, X_2)$ and since $W\mathbf{1} = \mathbf{1}$ (see property (ii) of W hence $H_1^* = \mathbf{1}(\mathbf{1}^TW\mathbf{1})^{-1}\mathbf{1}^TW = \frac{1}{n}\mathbf{1}\mathbf{1}^TW$, the the Buckley-James estimator $\hat{b}$ of b as

$$\begin{aligned}
\hat{b} &= [X_2^T W(I - H_1^*)X_2]^{-1}[X_2^T W(I - H_1^*)]R^{**}_{X_2} \\
&= [X_2^T W(I - H_1^*)X_2]^{-1}X_2^T W(I - \frac{1}{n}\mathbf{1}\mathbf{1}^TW)R^{**}_{X_2}
\end{aligned}$$

where $R^{**}_{X_2}$ is the renovated version of $R^*_{X_2}$ obtained using the renovation equation (2.3). Using the fact that $W^2 = W$ and $WR^{**}_{X_2} = WR^*_{X_2}$, and with the simplifying label $c = [X_2^T W(I - H_1^*)X_2]^{-1}$ this simplifies to

$$\begin{aligned}
\hat{b} &= cX_2^* W(I - H_1^*)R^*_{X_2} \\
&= cX_2^* W(I - H_1^*)(R^* + X_2\hat{\beta}_2) \\
&= cX_2^* W(I - H_1^*)R^* + \hat{\beta}_2.
\end{aligned}$$

Now since, $R^* = (I - H^*)Y^*$, we can use (3.1) to show that the first term on the left-side of this equation vanishes:

$$\begin{aligned}
& cX_2^* W(I - H_1^*)R^* \\
= \;& cX_2^T W(I - H_1^*)(I - H^*)Y^* \\
= \;& cX_2^* W(I - H_1^*)\left[(I - H_1^*)Y^* - \frac{(I - H_1^*)X_2 X_2^T W(I - H_1^*)Y^*}{X_2^T W(I - H_1^*)X_2}\right] \\
= \;& c\left[X_2^T W(I - H_1^*)Y^* - \frac{X_2^T W(I - H_1^*)X_2 X_2^T W(I - H_1^*)Y^*}{X_2^T W(I - H_1^*)X_2}\right] \\
= \;& c[X_2^T W(I - H_1^*)Y^* - X_2^T W(I - H_1^*)Y^*] = 0.
\end{aligned}$$

Hence $\hat{b} = \hat{\beta}_2$.

Theorem 2 (Property of renovated leverage matrix). The renovated leverage of an observation

$$h_{ii}^* = x_i^T (X^T W X)^{-1} X^T w_i$$

in censored regression can be presented as follows

$$h_{ii}^{**} = w_i^T X (X^T W X)^{-1} X^T w_i.$$

and we have $h_{ii}^{**} = h_{ii}^*$, for $i = 1, 2, \ldots, n$.

Proof. From equation (3.1)

$$H^* = H_1^* + (I - H_1^*)(X_2 M X_2^T W)(I - H_1^*)$$

where $H_1^* = X_1(X_1^T W X_1)^{-1} X_1^T W$, and based on the Lemma 2.1 in Chartajee and Hadi (1988) we find

$$\begin{aligned}
H^{**} &= WX(X^T W X)^{-1} X^T W \\
&= \begin{pmatrix} WX_1 : & WX_2 \end{pmatrix} \begin{pmatrix} X_1^T W X_1 & X_1^T W X_2 \\ X_2^T W X_1 & X_2^T W X_2 \end{pmatrix}^{-1} \begin{pmatrix} X_1^T W \\ X_2^T W \end{pmatrix} \\
&= \begin{pmatrix} WX_1 : & WX_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & M \end{pmatrix} \begin{pmatrix} X_1^T W \\ X_2^T W \end{pmatrix} \\
&= WX_1(X_1^T W X_1)^{-1} X_1^T W + (I - H_1^*)(X_2 M X_2^T W W)(I - H_1^*) \\
&= X_1(X_1^T W X_1)^{-1} X_1^T W W + (I - H_1^*)(X_2 M X_2^T W W)(I - H_1^*) \\
&= X_1(X_1^T W X_1)^{-1} X_1^T W^2 + (I - H_1^*)(X_2 M X_2^T W^2)(I - H_1^*),
\end{aligned}$$

where $g_{11}=(X_1^TWX_1)^{-1}+(X_1^TWX_1)^{-1}(X_1^TWX_2)M(X_2^TWX_1)(X_1^TWX_1)^{-1}$;
$g_{12}=-(X_1^TWX_1)^{-1}(X_1^TWX_2)M$;
$g_{21}=-M(X_2^TWX_1)(X_1^TWX_1)^{-1}$ and $M=[X_2^TW(I-H_1^*)X_2]^{-1}$.
From the properties of the weight matrix, we know $W^2=W$, idempotence. Hence

$$\begin{aligned} H^{**} &= X_1(X_1^TWX_1)^{-1}X_1^TW^2+(I-H_1^*)(X_2MX_2^TW^2)(I-H_1^*) \\ &= X_1(X_1^TWX_1)^{-1}X_1^TW+(I-H_1^*)(X_2MX_2^TW)(I-H_1^*) \\ &= H_1^*+(I-H_1^*)(X_2MX_2^TW)(I-H_1^*) \\ &= H^*. \end{aligned}$$

Since the renovated leverage, h_{ii}^*, comprises the diagonal entries of H^*, therefore $h_{ii}^{**}=h_{ii}^*$ for $i=1,2,...,n$.

4 RENOVATED PARTIAL RESIDUAL PLOTS

In order to illustrate the renovated partial residual plots and their properties in the censored regression model, we consider two examples from the Stanford heart transplantation and the Kidney data which were used by Miller and Halpern (1982) and McGilchrist and Aisbett (2001) respectively.

We here consider renovate partial residual plots for X_1 and X_2 respectively using log Y as responses variable with log-survival times to the base 10 in the censored regression model. Let X_1 be the age of patient as transplant while the variable X_2 be t5 for Stanford heart data. We now develop the renovated partial residuals $\{R_{X_i}^*\}'s$ scatter plots based on results of sections 2 and 3.

Firstly, the regression coefficient $\hat{\beta}_{bj}=[X^TW(\hat{\beta}_{bj})X]^{-1}X^TW(\hat{\beta}_{bj})Z$ is obtained based on Buckley-James method in the censored regression for 157 patients with complete records for the Stanford heart transplantation and 76 patients for the Kidney data sets respectively.

The second we compute the renovated responses $\{Y^*\}$ based on the censored regression equation (2.3) with Buckley James estimator for both data sets. There are two options to find renovated responses $\{Y^*\}$: (i) using regression line (2.3) with intercept and (ii) without intercept respectively.

The third we calculate the renovate partial residuals $R_{X_i}^*=R^*+X_i\hat{\beta}_i=(I-H^*)Y^*+X_i\hat{\beta}_i$, and plot the renovated partial residuals $R_{X_i}^*$ vs X_i.

Finally a linear model is fitted the the scatter-plot $R_{X_i}^*$ verses X_i for both data sets. The renovated partial residual scatter plots $\{R_{X_i}^*\}$ verses $\{X_i\}$ are given to show the results of the sections 2 and 3.

Example 4.1. The Stanford Heart Transplantation

The scatter plots of $\{(X_1, R^*_{X_1})\}$ are exhibited in Figure 1 whereas for $\{(X_2, R^*_{X_2})\}$, the plots are displayed in Figure 2. The two figures display renovate partial residual plots which are develop from the renovated responses Y^*. The procedure of developing both plots is similar except the renovated responses $\{Y^*\}$ which are calculated with intercept and without intercept respectively. Both plots in each figures display the regression line. The regression line shows the slope of partial regression, and it will be compared to the corresponding regression coefficients $\{\hat{\beta}_i\}$ for full variables. These coefficients of the fitted regression line are given in the Table 1.

In the Table 1, the slope of renovate partial residuals of X_1 which calculated with intercept is equal to 1.501. This finding is almost similar to the regression coefficient, $\hat{\beta}_1 = 1.136$ for full regression model which develop from Y^* that computed using intercept. Next we compared the slope of renovate partial residual, X_1 that calculated using Y^* developed without intercept with the corresponding regression coefficient value, $\hat{\beta}_1$. Again the result is parallel with the theoretical proving. The slope for renovate partial residual, X_1 that developed using Y^* which calculated without intercept is equal to 1.578 whereas the $\hat{\beta}_1$ is equal to 1.211 for full regression model. It is clearly display that these two values did not show any large difference.

Then, we do a comparison for the X_2 renovate partial residuals slope with the corresponding regression coefficient, $\hat{\beta}_2$. From Table 1, the slope of renovate partial residuals calculated using intercept is equal to 64.55 and the $\hat{\beta}_2 = 62.20$, from the full regression model develop from Y^* with intercept. The first plot shows no large difference in these two values. Then we refer to the second plot in Figure 2, the slope for the second partial plot is 64.90 and the $\hat{\beta}_2 = 62.40$, from regression model computed without intercept. These two values also show the same sign and almost similar magnitude.Details partial regression line for each plot and the full model regression can be refer to Table 1.

Example 4.2. The Kidney Data

There are seven variables which are id, time, censored status, age, sex, disease and frail in Kidney data set which had been used by McGilchrist and Aisbett in 2001. Renovate partial residual plots for X_1 using the Kidney data are exhibited in Figure 3 whereas for X_2, the plots are displayed in Figure 4.

Figure 3 displays renovate partial residual plots using Kidney data for X_1, where each plot represent the one using $\log Y^*$ which computed with intercept, and the other one is developed from $\log Y^*$ that calculated without intercept. The renovate partial residual plot gives the slope value equal to -1.173 and this value is almost similar to the corresponding regression coefficient $\hat{\beta}_1 = -1.048$ for full model. Next we look at the second plot, and again the values for these two is not much different both -69.740 and -70.600.

In the Figure 4, the plots are corresponding to renovate partial residuals for X_2. The findings for this plot also not disappointed, and we can show that the slope for renovate partial residual plot for coefficient of X_i is similar to the corresponding regression coefficient $\hat{\beta}_i$ for full model in Table 2.

Now we refer to the Table 2 which is exhibited the similar results as Table 1. When we look at the value of the regression coefficients in Table 2, we can see that the values of coefficient display almost the same except intercept $\hat{\beta}_0$. This is another proof that can support the theorem 1 that the slope of renovate partial residual plot is equal to the corresponding regression coefficient for full variables.

5 CONCLUSION

Both examples produce satisfactory findings. Table 1 and Table 2, present a strong evidence that the partial slope of X_i for renovate partial plots and the corresponding regression coefficient for full model, $\hat{\beta}_i$ displays almost the same value.

Each figure in section 4 produce plots for a partial residuals which develop from Y^* that computed using intercept and without intercept. As we can see from the plot there is no different in the pattern and the location of the observations on the plot. The axis scale for both plots also similar. The values of regression coefficient in Table 1 and Table 2 exhibit the same sign and almost similar magnitude. Therefore the new Y^* does not effect the findings for the two cases of with intercept and without intercept.

Both renovated partial residual plots and properties are useful in the diagnostic and local influence analysis for censored regression models. These problems will be discussed elsewhere (see Aziz and Wang, 2009).

5.1 Adding Tables

Table 1: Regression Line for Stanford Heart Transplantation

Renovated Y^*	Using Variable	Estimate $\hat{\beta}_0$	of $\hat{\beta}_1$	Coefficient $\hat{\beta}_2$
Calculated	X_1 and X_2	182.750	1.136	62.197
with	X_1	236.974	1.501	—
intercept	X_2	228.950	—	64.550
Calculated	X_1 and x_2	176.785	1.211	62.394
without	X_1	231.171	1.578	—
intercept	X_2	226.000	—	64.902

Table 2: Regression Line for Kidney Data Set

Renovated Y^*	Using Variable	Estimate $\hat{\beta}_0$	of $\hat{\beta}_1$	Coefficient $\hat{\beta}_2$
Calculated	X_1 and X_2	220.094	-1.048	-69.740
with	X_1	144.179	-1.173	—
intercept	X_2	175.400	—	-70.600
Calculated	X_1 and X_2	218.346	-1.089	-68.979
without	X_1	143.271	-1.182	—
intercept	X_2	173.22	—	69.860

5.2 Adding Figures

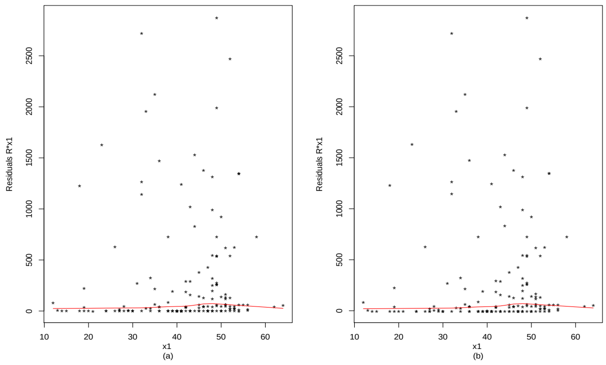

Figure 1: Renovated Partial Residuals Plot for X_1 Using Stanford Heart Data (a) with intercept (b) without intercept

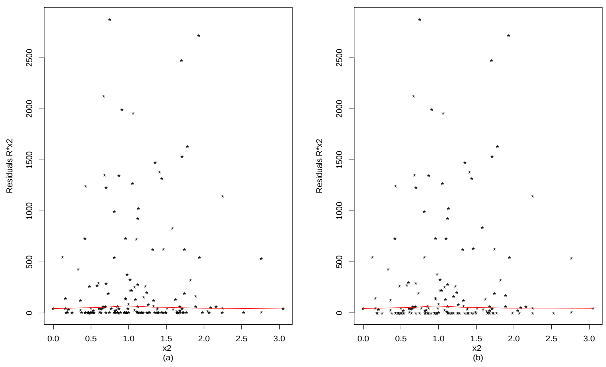

Figure 2: Renovated Partial Residuals Plot for X_2 Using Stanford Heart Data (a) with intercept (b) without intercept

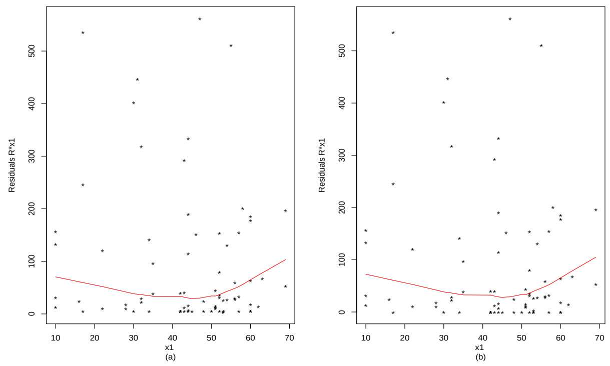

Figure 3: Renovated Partial Residuals Plot for X_1 Using Kidney Data (a) with intercept (b) without intercept

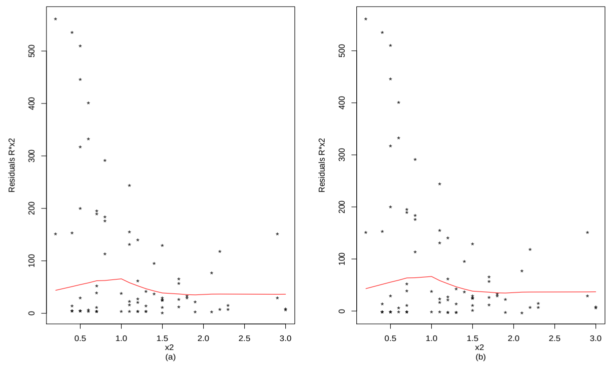

Figure 4: Renovated Partial Residuals Plot for X_2 Using Kidney Data (a) with intercept (b) without intercept

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