

**A GROUP ACCEPTANCE SAMPLING PLANS FOR TRUNCATED
LIFE TESTS BASED ON THE INVERSE RAYLEIGH
AND LOG-LOGISTIC DISTRIBUTIONS**

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ABSTRACT

A group acceptance sampling plan from a truncated life test is designed when the lifetime of an item follows either an inverse Rayleigh or a log-logistic distribution, in which a multiple number of items as a group can be tested simultaneously in a tester. The minimum number of groups required for a given group size and the acceptance number is determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are found and the minimum ratios of the true average life to the specified life at the specified producer's risk are obtained. Some comparisons are made between the results for the two distributions. The results are explained with tables.

KEY WORDS

Consumer's risk; life test; operating characteristics; producer's risk; truncated life test.

1. INTRODUCTION

In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. However, testers accommodating a multiple number of items at a time are used in practice because testing time and cost can be saved by testing those items simultaneously. Sudden death testing is frequently adopted by using this type of testers (Pascual and Meeker, 1998; Vlcek et al. 2003; Jun et al. 2006). For this type of testers the number of items to be equipped in a tester is given by the specification. The sampling plan under this type of testers will be called a group sampling plan. When designing a group sampling plan, determining the sample size is equivalent to determining the number of groups as the group size is already given. The items in a group are tested independently, identically and simultaneously on the different testers for a pre-assigned time. The experiment is truncated if more than the acceptable number of failures occurred in any group during the experiment time.

The parameters of a sampling plan can be determined differently according to an underlying distribution. The choice of the underlying distribution depends on the past experience or observations of failure data. The purpose of this study is to find the

minimum number of groups required for the life test demonstrating the true mean life greater than the specified lifetime under inverse Rayleigh and log-logistic distributions. These two distributions are popularly used in the area of reliability and survival analysis.

The cumulative distribution function (cdf) of an inverse Rayleigh distribution is given by

$$F_{IR}(t) = \exp\left(-\sigma_{IR}^2/t^2\right), \quad t > 0 \quad (1.1)$$

where σ_{IR} (>0) is the scale parameter. The mean of this distribution is given by

$$\mu_{IR} = \sqrt{\pi}\sigma_{IR}. \quad (1.2)$$

Mukerjee and Saran (1984) studied the failure rate of an inverse Rayleigh distribution. According to them the failure rate of a single parameter inverse Rayleigh distribution is increasing for $t < 1.069\sqrt{\sigma_{IR}}$ and decreasing for $t > 1.069\sqrt{\sigma_{IR}}$. Voda (1972) studied the properties of the maximum likelihood estimator of σ_{IR} . Mukerjee and Maiti (1997) considered the percentile estimation in this distribution. Rosaiah and Kantam (2005) used the inverse Rayleigh distribution for the usual acceptance sampling plan. More recently, Rosaiah et al. (2008) developed the reliability plans under the assumption that the life time of a product follows the inverse Rayleigh distribution. They showed that inverse Rayleigh is well fitted to real data given by Wood (1996) using the probability plot.

The log-logistic distribution has been studied by Shah and Dave (1963) and Tadikamalla and Johnson (1982). The cdf of the log-logistic distribution is given by

$$F_{LL}(t) = \frac{(t/\sigma_{LL})^\gamma}{1+(t/\sigma_{LL})^\gamma}, \quad t > 0 \quad (1.3)$$

where γ (>1) is the shape parameter and σ_{LL} (>0) is the scale parameter. The median of this distribution is just σ_{LL} and the mean is given by

$$\mu_{LL} = \frac{\pi\sigma_{LL}/\gamma}{\sin(\pi/\gamma)}. \quad (1.4)$$

We will particularly consider the case of $\gamma = 2$ when establishing tables because of convenience, in which case the mean is obtained by

$$\mu_{LL} = 1.5708\sigma_{LL} \quad (1.5)$$

O'Quigely and Struthers (1982) studied the log-logistic distribution in survival analysis. The log-logistic distribution has been considered by Ragab and Green (1984) for the order statistics and by Balakrishnan and Malik (1987) for the linear unbiased estimation of its parameters. Kantam et al. (2001) and Kantam et al. (2006) studied some acceptance sampling plans based on the log-logistic distribution.

Truncated life tests without considering groups have been studied by many authors in the literature using different statistical models. The acceptance sampling plan was

considered by Epstein (1954) for exponential distribution. Goode and Kao (1961) and Gupta and Groll (1961) developed the acceptance sampling plan using the Weibull distribution and gamma distributions respectively, as lifetime distribution. Gupta (1962) proposed the life test sampling plans for normal and log-normal distributions. Kantam and Rosaiah (1998) used the half logistic distribution for this purpose. Baklizi (2003) developed the acceptance sampling plans using the Pareto distribution of the second kind. Tsai and Wu (2006) and Rosaiah et al. (2007) considered the generalized Rayleigh distribution and the exponentiated log-logistic distributions, respectively. Rosaiah et al. (2006) studied the reliability plans using the exponentiated log-logistic distribution. Balakrishnan et al. (2007) considered the generalized Birnbaum-Saunders distribution to develop the acceptance sampling based on truncated life test. Aslam (2007) developed the double acceptance sampling plans assuming that the lifetime follows a Rayleigh distribution.

The proposed sampling plan is given in Section 2. The description of tables with some examples is given in Section 3.

2. DESIGN OF THE PROPOSED SAMPLING PLAN

We are interested in designing a group sampling plan in order to assure that the mean life of an item in a lot (μ , say) is greater than the specified life μ_0 , say under the assumption that the life time of an item follows either an inverse Rayleigh or a log-logistic distribution with known shape parameter. A lot of products or items is considered to be “good” if the true average life μ is greater than the specified life μ_0 . We will accept the lot if $\mu \geq \mu_0$ at a certain level of consumer’s risk. Otherwise, we have to reject the lot. The following group acceptance sampling plan based on the truncated life test is proposed:

- 1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = gr$.
- 2) Select the acceptance number c for a group and specify the experiment time t_0 .
- 3) Perform the experiment for the g groups simultaneously and record the number of failures for each group.
- 4) Accept the lot if at most c failures occur in each of all groups by the experiment time.
- 5) Terminate the experiment as soon as more than c failures occur in any group and reject the lot.

The proposed sampling plan is an extension of the ordinary sampling plan available in literature such as in Kantam et al. (2001) and Rosaiah and Kantam (2005), for which $r=1$. We are interested in determining the number of groups g required for each of two distributions under study, whereas the various values of acceptance number c and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified life μ_0 , we will consider $t_0 = a\mu_0$ for a specified constant a (termination ratio).

The probability of rejecting a good lot is called the producer's risk, whereas the probability of accepting a bad lot is known as the consumer's risk. When determining the parameters of the proposed sampling plan, we will use the consumer's risk. Often, the consumer's risk is expressed by the consumer's confidence level. If the confidence level is P^* , then the consumer's risk will be $\beta = 1 - P^*$. We will determine the number of groups in the proposed sampling plan so that the consumer's risk does not exceed β . The lot of products is accepted only if there were at most c failures occurred in each of g groups. So, the lot acceptance probability will be

$$L(p) = \left[\sum_{i=0}^c \binom{r}{i} p^i (1-p)^{r-i} \right]^g, \quad (2.1)$$

where p is the probability that an item in a group fails before the termination time. The probability p for the inverse Rayleigh distribution is given by

$$p = \exp \left(- \left(\frac{\sigma_{IR}}{t_0} \right)^2 \right) = \exp \left(- \frac{1}{a^2 \pi} \left(\frac{\mu_{IR}}{\mu_0} \right)^2 \right), \quad (2.2)$$

and for the log-logistic distribution with $\gamma = 2$ it is calculated by

$$p = \frac{(1.5708a)^2}{(\mu_{LL} / \mu_0)^2 + (1.5708a)^2}. \quad (2.3)$$

The minimum number of groups required can be determined by considering the consumer's risk when the true mean equals the specified life ($\mu = \mu_0$) through the following inequality:

$$L(p_0) \leq \beta, \quad (2.4)$$

where p_0 is the failure probability at $\mu = \mu_0$, so it is given by

$$p_0 = \exp \left(-1 / (a^2 \pi) \right), \quad (2.5)$$

for the inverse Rayleigh distribution and

$$p_0 = \frac{(1.5708a)^2}{1 + (1.5708a)^2}, \quad (2.6)$$

for the log-logistic distribution with $\gamma = 2$. Particularly for $c=0$ (so-called zero failure test), g can be determined by the minimum integer satisfying the following inequality:

$$g \geq \frac{\ln \beta}{r \ln(1 - p_0)}. \quad (2.7)$$

Since p_0 for the inverse Rayleigh distribution in (2.5) is smaller than that for the log-logistic distribution for $a \leq 0.825$, the number of groups required will be larger under the inverse Rayleigh distribution than under log-logistic distribution when the test time is shorter than $0.825\mu_0$ and the converse is true when the test time is longer than $0.825\mu_0$. Table 1 shows the minimum number of groups required for the proposed sampling plan according to various values of confidence level ($\beta = 0.25, 0.10, 0.05, 0.01$), group size (r), acceptance number (c) and the test termination time multiplier ($a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$) under the inverse Rayleigh distribution (first row) and log-logistic distribution with $\gamma = 2$ (second row). It can be seen from this table that the number of groups required for the inverse Rayleigh distribution is quite similar to that for the log-logistic distribution although the former is sometimes smaller than the latter when the test time is shorter than the specified average life.

Table 1
Number of groups required for the proposed plan for the
Inverse Rayleigh and Log-logistic distributions

β	r	c	A						
			0.7	0.8	1.0	1.2	1.5	2.0	
0.25	2	0	1	1	1	1	1	1	
			1	1	1	1	1	1	
	3	1	2	2	1	1	1	1	
			2	1	1	1	1	1	
	4	2	4	3	2	1	1	1	
			3	3	2	1	1	1	
	5	3	6	4	2	2	1	1	
			5	4	2	2	1	1	
	6	4	10	5	3	2	1	1	
			8	5	3	2	1	1	
	7	5	17	8	3	2	1	1	
			14	8	4	2	2	1	
	0.10	4	0	1	1	1	1	1	1
				1	1	1	1	1	1
5		1	2	1	1	1	1	1	
			2	1	1	1	1	1	
6		2	2	2	1	1	1	1	
			2	2	1	1	1	1	
7		3	3	2	1	1	1	1	
			3	2	2	1	1	1	
8		4	5	3	2	1	1	1	
			4	3	2	1	1	1	
9		5	7	4	2	1	1	1	
			6	4	2	2	1	1	

β	r	c	A					
			0.7	0.8	1.0	1.2	1.5	2.0
0.05	5	0	1	1	1	1	1	1
			1	1	1	1	1	1
	6	1	2	1	1	1	1	1
			2	1	1	1	1	1
	7	2	2	2	1	1	1	1
			2	2	1	1	1	1
	8	3	3	2	1	1	1	1
			3	2	1	1	1	1
	9	4	4	3	2	1	1	1
			4	3	2	1	1	1
	10	5	6	3	2	1	1	1
			5	3	2	1	1	1
0.01	7	0	1	1	1	1	1	1
			1	1	1	1	1	1
	8	1	1	1	1	1	1	1
			2	1	1	1	1	1
	9	2	2	2	1	1	1	1
			2	1	1	1	1	1
	10	3	3	2	1	1	1	1
			3	2	1	1	1	1
	11	4	4	2	2	1	1	1
			3	2	1	1	1	1
	12	5	5	3	2	1	1	1
			4	3	2	1	1	1

(Note) For given values of β , r and c , the first row represents the number of groups under the inverse Rayleigh distribution and the second row does that under the log-logistic distribution.

3. OPERATING CHARACTERISTICS

Once the minimum number of groups is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is good enough. As mentioned earlier, the product is considered to be good if $\mu > \mu_0$ or $\mu/\mu_0 > 1$. The probabilities of acceptance based on (2.1) for various mean lifetimes ($\mu/\mu_0 = 2, 4, 6, 8, 10, 12$) under the plan parameters chosen before are reported in Table 2 for the log-logistic and inverse Rayleigh distributions. Here again, the shape parameter in a log-logistic distribution is assumed as $\gamma = 2$.

Table 2
Operating characteristics values of the group sampling plan with
c=2 for inverse Rayleigh and log-logistic distributions

β	μ / μ_0									
	r	g	a	2	4	6	8	10	12	
0.25	4	4	0.7	0.9938	1.0000	1.0000	1.0000	1.0000	1.0000	
		3		0.8812	0.9961	0.9996	0.9999	1.0000	1.0000	
	4	3	0.8	0.9727	1.0000	1.0000	1.0000	1.0000	1.0000	
		3		0.8007	0.9919	0.9991	0.9998	1.0000	1.0000	
	4	2	1.0	0.8662	1.0000	1.0000	1.0000	1.0000	1.0000	
		2		0.7081	0.9829	0.9980	0.9996	0.9999	1.0000	
	4	1	1.2	0.8055	0.9999	1.0000	1.0000	1.0000	1.0000	
		1		0.7306	0.9793	0.9973	0.9994	0.9998	0.9999	
	4	1	1.5	0.5795	0.9959	1.0000	1.0000	1.0000	1.0000	
		1		0.5570	0.9449	0.9914	0.9981	0.9994	0.9998	
	4	1	2.0	0.3004	0.9307	0.9993	1.0000	1.0000	1.0000	
		1		0.3279	0.8415	0.9666	0.9914	0.9973	0.9990	
	0.10	6	2	0.7	0.9862	1.0000	1.0000	1.0000	1.0000	1.0000
			2		0.7359	0.9882	0.9987	0.9998	0.9999	1.0000
6		2	0.8	0.9272	1.0000	1.0000	1.0000	1.0000	1.0000	
		2		0.6008	0.9766	0.9973	0.9995	0.9999	1.0000	
6		1	1.0	0.7806	1.0000	1.0000	1.0000	1.0000	1.0000	
		1		0.5827	0.9652	0.9954	0.9991	0.9997	0.9999	
6		1	1.2	0.5173	0.9995	1.0000	1.0000	1.0000	1.0000	
		1		0.4008	0.9223	0.9882	0.9974	0.9993	0.9997	
6		1	1.5	0.2266	0.9824	1.0000	1.0000	1.0000	1.0000	
		1		0.2062	0.8184	0.9652	0.9915	0.9974	0.9991	
6		1	2.0	0.0508	0.7806	0.9968	1.0000	1.0000	1.0000	
		1		0.0617	0.5827	0.8816	0.9652	0.9882	0.9954	
0.05		7	2	0.7	0.9772	1.0000	1.0000	1.0000	1.0000	1.0000
			2		0.6283	0.9805	0.9978	0.9996	0.9999	1.0000
	7	2	0.8	0.8866	1.0000	1.0000	1.0000	1.0000	1.0000	
		2		0.4695	0.9619	0.9954	0.9991	0.9997	0.9999	
	7	1	1.0	0.6921	1.0000	1.0000	1.0000	1.0000	1.0000	
		1		0.4608	0.9450	0.9924	0.9984	0.9995	0.9998	
	7	1	1.2	0.3918	0.9992	1.0000	1.0000	1.0000	1.0000	
		1		0.2780	0.8820	0.9808	0.9957	0.9987	0.9995	
	7	1	1.5	0.1308	0.9715	1.0000	1.0000	1.0000	1.0000	
		1		0.1156	0.7405	0.9450	0.9861	0.9957	0.9984	
	7	1	2.0	0.0189	0.6921	0.9946	1.0000	1.0000	1.0000	
		1		0.0243	0.4608	0.8250	0.9450	0.9808	0.9924	

β	μ / μ_0								
	r	g	a	2	4	6	8	10	12
0.01	9	2	0.7	0.9514	1.0000	1.0000	1.0000	1.0000	1.0000
		2		0.4235	0.9582	0.9950	0.9990	0.9997	0.9999
	9	2	0.8	0.7854	1.0000	1.0000	1.0000	1.0000	1.0000
		1		0.5088	0.9597	0.9949	0.9989	0.9997	0.9999
	9	1	1.0	0.5173	1.0000	1.0000	1.0000	1.0000	1.0000
		1		0.2682	0.8923	0.9835	0.9964	0.9989	0.9996
	9	1	1.2	0.2081	0.9982	1.0000	1.0000	1.0000	1.0000
		1		0.1226	0.7853	0.9597	0.9904	0.9971	0.9989
	9	1	1.5	0.0394	0.9416	1.0000	1.0000	1.0000	1.0000
		1		0.0328	0.5795	0.8923	0.9704	0.9904	0.9964
	9	1	2.0	0.0023	0.5173	0.9880	1.0000	1.0000	1.0000
		1		0.0033	0.2682	0.6975	0.8923	0.9597	0.9835

(Note) For given values of β , r and a the first row represents the OC values under the inverse Rayleigh distribution and the second row does that under the log-logistic distribution.

We see from this table that OC values increase more quickly under the inverse Rayleigh distribution than under the log-logistic distribution as the quality increases. For example, when $\beta=0.10$, $r=6$, $c=2$ and $a=0.7$, the number of groups required is $g=2$ for both distributions. However, the OC value goes to 1.0 when the true mean becomes four times the specified average life under the inverse Rayleigh distribution, whereas it requires almost ten times under the log-logistic distribution.

Further, the producer may be interested in enhancing the quality level of the product so that the acceptance probability should be greater than a specified level. At the producer's risk α the minimum ratio μ / μ_0 can be obtained by satisfying the following inequality:

$$\left[\sum_{i=1}^c \binom{r}{i} p^i (1-p)^{r-i} \right]^g \geq 1 - \alpha, \tag{3.1}$$

where p is given by equation (2.2) for the inverse Rayleigh distribution or equation (2.3) for the log-logistic distribution and g is chosen at the consumer's risk β when $\mu / \mu_0=1$.

Table 3 shows the minimum ratio of μ / μ_0 for inverse Rayleigh and log-logistic distributions at the producer's risk of $\alpha=0.05$ under the plan parameters chosen before.

Table 3
Minimum ratio of true average life to specified life for the
producer's risk of 0.05 under two distributions

β	c	r	a						
			0.700	0.800	1.0	1.20	1.50	2.0	
0.25	0	2	2.38	2.72	3.40	4.08	5.10	6.80	
			6.82	7.80	9.75	11.69	14.62	19.49	
	1	3	1.90	2.17	2.51	3.01	3.76	5.02	
			3.38	3.18	3.98	4.77	5.96	7.95	
	2	4	1.70	1.89	2.26	2.51	3.14	4.19	
			2.44	2.78	3.18	3.29	4.11	5.48	
	3	5	1.54	1.70	1.99	2.38	2.75	3.67	
			1.88	2.25	2.49	2.99	3.27	4.36	
	4	6	1.43	1.56	1.87	2.15	2.48	3.31	
			1.80	1.92	2.25	2.51	2.78	3.71	
	5	7	1.36	1.48	1.71	1.97	2.28	3.05	
			1.67	1.77	2.03	2.20	2.75	3.28	
	0.10	0	4	2.60	2.97	3.71	4.45	5.57	7.42
				9.77	11.17	13.96	16.75	20.94	27.92
1		5	2.12	2.27	2.84	3.41	4.26	5.68	
			4.61	4.37	5.46	6.55	8.19	10.92	
2		6	1.81	2.07	2.43	2.92	3.64	4.86	
			2.98	3.41	3.70	4.44	5.55	7.40	
3		7	1.67	1.84	2.17	2.60	3.25	4.33	
			2.48	2.63	3.29	3.50	4.37	5.83	
4		8	1.57	1.74	2.10	2.37	2.96	3.95	
			2.13	2.34	2.75	2.96	3.70	4.93	
5		9	1.49	1.64	1.92	2.19	2.74	3.66	
			1.93	2.10	2.35	2.82	3.25	4.33	
0.05		0	5	2.66	3.04	3.80	4.55	5.69	7.59
				10.83	12.38	15.47	18.57	23.21	30.95
	1	6	2.19	2.36	2.95	3.54	4.43	5.91	
			5.12	4.88	6.10	7.32	9.15	12.20	
	2	7	1.88	2.15	2.54	3.05	3.81	5.08	
			3.29	3.76	4.09	4.90	6.13	8.17	
	3	8	1.74	1.92	2.27	2.73	3.41	4.55	
			2.72	2.89	3.21	3.86	4.82	6.43	
	4	9	1.62	1.82	2.20	2.50	3.12	4.17	
			2.34	2.59	3.01	3.25	4.07	5.42	
	5	10	1.54	1.69	2.04	2.32	2.90	3.87	
			2.06	2.22	2.62	2.86	3.57	4.76	

β	c	a						
		r	0.700	0.800	1.0	1.20	1.50	2.0
0.01	0	7	2.75	3.15	3.93	4.72	5.90	7.86
			12.83	14.66	18.33	21.99	27.49	36.66
	1	8	2.17	2.49	3.11	3.73	4.66	6.21
			6.00	5.70	7.13	8.55	10.69	14.25
	2	9	1.99	2.28	2.70	3.24	4.05	5.41
			3.84	3.82	4.77	5.73	7.16	9.55
	3	10	1.85	2.05	2.44	2.93	3.66	4.88
			3.15	3.36	3.74	4.49	5.61	7.48
	4	11	1.74	1.89	2.36	2.70	3.38	4.50
			2.59	2.79	3.15	3.78	4.72	6.29
	5	12	1.64	1.81	2.21	2.52	3.15	4.20
			2.32	2.56	3.03	3.31	4.13	5.51

(note) For given values of β and c the first row represents the minimum ratio of the true average life to the specified life under the inverse Rayleigh distribution and the second row does that under the log-logistic distribution.

It can be also seen from this table that the effect of improving the quality on the lot acceptance probability is quicker for the inverse Rayleigh case than for the log-logistic case. For example, when $\beta=0.10$, $r=4$, $g=1$, $c=0$ and $a=0.7$, the manufacturer requires to increase the true mean 2.60 times the specified life under the inverse Rayleigh distribution in order to keep the producer's risk at 5 percent, whereas it requires to increase the true mean 9.77 times under the log-logistic distribution. Table 1-3 can be generated for any other values of γ . An Excel program preparing these tables is available from the authors upon request.

4. DESCRIPTION OF TABLES AND EXAMPLES

Let us consider an example. Suppose bulb manufacturers would like to know if the mean life of their product is greater than the specified average life, $\mu_0=1000$ hours. Suppose that they want to run an experiment 700 hours by using testers equipped with 12 items each. It is assumed that $c=5$ and $\beta=0.01$. This leads to the termination multiplier $a=0.700$ and from Table 1 the minimum group required is $g=5$ for the inverse Rayleigh distribution and $g=4$ for the log-logistic distribution. If the underlying distribution is the inverse Rayleigh, then we will draw a random sample of size 60 items and allocate 12 items to 5 groups to put on test for 700 hours. Suppose now that we observed only one failure from Group 1 before the termination time and three failures from Group 2, two failures from Group 3, no failures from Group 4 and six failures from Group 5. Then, we reject the lot and declare that a bulb product in this lot has the mean life smaller than 1000 hours at the consumer's risk of 1 percent. On the other hand, if the underlying distribution is the log-logistic distribution, then we have to draw 48 items and allocate them into 4 groups to put on test. If the numbers of failures from Group 1 to Group 4 are same as the above, then we accept the lot and declare that a bulb product in this lot has the mean life at least 1000 hours at the consumer's risk of 1 percent.

Suppose that the lifetime of a product under consideration is known to follow the inverse Rayleigh or the log-logistic distribution with $\gamma = 2$. Also it is required to demonstrate that through the proposed sampling plan the mean life of the product under consideration is at least 1000 hours at consumer's risk of 5 percent. We want to run this experiment 700 hours using the tester to be equipped with seven items each. When the acceptance number is $c=2$, we will accept the lot if at most two failures occur before 700 hours in each of two groups. We truncate the experiment as soon as the 3rd failure occurs before the 700th hours. The minimum number of groups for the inverse Rayleigh distribution and the log-logistic distribution can be found as $g=2$ from Table 1. This means that a total of 14 products are needed and that 7 items will be allocated to each of 2 testers. For this proposed sampling plan under the two distributions the operating characteristics can be seen from Table 2 as follows:

μ / μ_0	1	2	4	6	8	10	12
Inverse Rayleigh	0.0368	0.9772	1.0000	1.0000	1.0000	1.0000	1.0000
Log-logistic	0.0245	0.6283	0.9805	0.9978	0.9996	0.9999	1.0000

This shows that, if the true average life is 4 times of 1000 hours, the producer's risk is almost zero either for the inverse Rayleigh distribution, whereas for the log-logistic distribution it requires almost 10 times to have zero risk. It can be also seen that the OC values for the inverse Rayleigh distribution is more rapidly increasing as the quality increases than for the log-logistic distribution. If we need the ratio corresponding to the producer's risk of 0.05, we can obtain it from Table 3. For example, when $r=7$, $g=2$, $c=2$, $a=0.700$, the ratios of μ / μ_0 for the inverse Rayleigh and the log-logistic distributions are 1.88 and 3.29, respectively.

5. CONCLUDING REMARKS

In this paper, a group acceptance sampling plan from the truncated life test was proposed and the number of groups was determined for the inverse Rayleigh and log-logistic distributions when the consumer's risk and the other plan parameters are specified. It can be observed that the number of groups required is a little larger for the inverse Rayleigh than for the log-logistic distribution when the test time is shorter than the specified life. However, the operating characteristics for the inverse Rayleigh distribution is more desirable than the log-logistic distribution in a sense that the former increases more rapidly than the latter as the quality improves.

The major two parameters in an acceptance sampling plan based on a truncated life test are the sample size and the test termination time. However, most studies have been focused on determining the sample size while the test termination time is assumed to be specified. Obviously, the termination time can be determined by a similar approach when the sample size is specified. The study can also extended to develop the group sampling plan using many other distributions including Weibull and log-normal distributions. It may be interesting to study on the determination of these two parameters simultaneously as a future research.

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