Pak. J. Statist.
2009 Vol. 25(2), 93-106

GENERALIZED REGRESSION-CUM-RATIO ESTIMATORS FOR TWO-PHASE SAMPLING USING MULTI-AUXILIARY VARIABLES

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ABSTRACT

In this paper we suggest three classes of regression-cum-ratio estimators for estimating population mean of variable of interest for two-phase sampling using multi-auxiliary variables for full information, partial information and no information cases. The expressions for mean square errors are derived. Theoretical comparison is given. Special cases of estimators are also identified.

KEY WORDS

Regression-cum-ratio estimator; two-phase sampling; auxiliary variable

1. INTRODUCTION

The use of auxiliary information is a widely discussed topic in sampling theory to obtain improved designs and precise estimates of some population parameters like mean or variance. It is well known that when the auxiliary information is utilized at the estimation stage. The ratio, product and regression methods are employed in many such situations.

The estimation of the population mean is an unrelenting issue in sampling theory and several efforts have been made to improve the precision of the estimates in the presence of multi-auxiliary variables. A variety of estimators have been proposed following different ideas linking together ratio, product or regression estimators.

Olkin (1958) was the first author to deal with the problem of estimating the mean of a survey variable when auxiliary variables are made available. He suggested the use of information on more than one auxiliary variable, positively correlated with the study variable analogously to Olkin; Singh (1967a) gave a multivariate expression of Murthy's (1964) product estimator, while Raj (1965) suggested a method for using multi-auxiliary variables through a linear combination of single difference estimators. Moreover, Singh (1967b) considered the extension of the ratio-cum-product estimators to multi-auxiliary variables Shukla (1965) suggested a multiple regression estimator while Rao and Mudholkar (1967) proposed a multivariate estimator based on a weighted sum of single ratio and product estimators.

John (1969) suggested two multivariate generalizations of ratio and product estimators which actually reduce to the Olkin’s (1958) and Singh’s (1967a) estimators. Srivastava (1971) proposed a general ratio-type estimator which generates a large class of estimators including most of the estimators up to that time proposed.

Robinson (1994) proposed a regression estimator ignoring some of the assumptions usually adopted in the literature (see, e.g., Srivastava (1971)), Tracy et al. (1996) and Perri (2004) proposing an alternative to Singh’s (1965, 1967b) ratio-cum-product estimators, when two auxiliary variables are available. Ceccon and Diana (1996) provided a multivariate extension of the Naik and Gupta (1991) univariate class of estimators. Agarwal et al. (1997), moving from Raj (1965), illustrated a new approach to form a multivariate difference estimator which does not require the knowledge of any population parameters. Abu-Dayyeh et al. (2003) introduced two estimators which are definitely members of the class proposed by Srivastava (1971), while Kadilar and Cingi (2004, 2005) analyzed combinations of regression type estimators in the case of two auxiliary variables. In the same situation, Perri (2005) proposed some new estimators obtained from Singh’s (1965, 1967b) estimators. Pradhan (2005) suggested a chain regression estimator for two-phase sampling using three auxiliary variables when the population mean of one auxiliary variable is unknown and other auxiliary population means are known.

In practical surveys, the problem is to estimate population means of variables of interest. For example, in a typical socio-economic survey conducted in rural areas in Indo-Pak subcontinent, the multiple variables of interests may be size of household, monthly income and expenditure of the household, number of unemployed persons, number of illiterates, number of persons engaged in agriculture, amount of land owned, leased and leased out, number of cattle owned etc. In some situations the auxiliary information may be available through the past census data or conveniently collected. For example in a village land survey, the information on the variables such as area of the village, cultivable area, grazing grounds etc. may be easily obtained through the past census data and may be used to estimate the means of variables of interest.

If we have information on multi-auxiliary variables practically sometimes either information for all these auxiliary is available from population or available for some variables or not available for all auxiliary variables. By considering these practical situations, we suggest general classes of regression-cum-ratio estimators for estimating the population mean of study variable for two-phase sampling using multi-auxiliary variables by considering the following three cases (see Samiuddin and Hanif (2007).

1. Estimators when information on all auxiliary variables is known for population (Full Information Case).
2. Estimators when information on some auxiliary variables is known for population (Partial Information Case).
3. Estimators when information on all auxiliary variables is unknown for population (No Information Case).

Before suggesting the estimators we provide two-phase sampling scheme and some useful notations and results in the following section.

2. TWO-PHASE SAMPLING USING MULTI-AUXILIARY VARIABLES

Consider a population of N units. Let Y be the variable for which we want to estimate the population mean and $X_1, X_2, ...X_q$ are q auxiliary variables. For two-phase sampling design let n_1 and $n_2\left(n_2 < n_1\right)$ are sample sizes for first and second phase respectively. $x_{(1)i}$ and $x_{(2)i}$ denote the i^{th} auxiliary variables form first and second phase samples respectively and y_2 denote the variable of interest from second phase. $\bar{X}_i$ and C_{x_i} denote the population means and coefficient of variation of i^{th} auxiliary variables respectively and ρ_{yx_i} denotes the population correlation coefficient of Y and X_i. Further let $\theta_1 = \frac{1}{n_1} - \frac{1}{N}$, $\theta_2 = \frac{1}{n_2} - \frac{1}{N}$, $y_{(2)} = Y + e_{y_{(2)}}$, $x_{(1)i} = X_i + e_{x_{(1)i}}$ and $x_{(2)i} = X_i + e_{x_{(2)i}}$; $\left(i = 1, 2, ..., k\right)$ where $e_{y_{(2)}}$, $e_{x_{(1)i}}$ and $e_{x_{(2)i}}$ are sampling errors and are of very small quantities. We assume that $E_2\left(e_{y_{(2)}}\right) = E_1\left(e_{x_{(1)i}}\right) = E_2\left(e_{x_{(2)i}}\right) = 0$. Then for simple random sampling without replacement for both first and second phases we write by using phase wise operation of expectations as:

$$E_2\left(e_{y_2}\right)^2 = \left(1 - \frac{n_2}{N}\right)S_Y^2,\ E_2\left(\bar{e}_{y_2}\right)^2 = \left(1 - \frac{n_2}{N}\right)\frac{S_Y^2}{n_2} = \theta_2\bar{Y}^2C_y^2,$$

$$E_2\left(e_{y_2}e_{x_{(2)i}}\right) = \left(1 - \frac{n_2}{N}\right)S_{YX_i} = \theta_2\overline{YX}_iC_yC_{x_i}\rho_{yx_i},$$

$$E_2\left(\bar{e}_{y_2}\bar{e}_{x_{(2)i}}\right) = \left(1 - \frac{n_2}{N}\right)\frac{S_{YX_i}}{n_2} = \theta_2\overline{YX}_iC_yC_{x_i}\rho_{yx_i},$$

$$E_1E_{2|1}\left[e_{y_2}\left(e_{x_{(1)i}} - e_{x_{(2)i}}\right)\right] = E_1E_{2|1}\left(e_{y_2}e_{x_{(1)i}}\right) - E_2\left(e_{y_2}e_{x_{(2)i}}\right)$$

$$= \left(1 - \frac{n_1}{N}\right)S_{YX_i} - \left(1 - \frac{n_2}{N}\right)S_{YX_i} = \frac{1}{N}\left(n_2 - n_1\right)S_{YX_i},$$

$$E_1E_{2|1}\left[\bar{e}_{y_2}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right] = \left(1 - \frac{n_1}{N}\right)\frac{S_{YX_i}}{n_1} - \left(1 - \frac{n_2}{N}\right)\frac{S_{YX_i}}{n_2} = \left(\theta_1 - \theta_2\right)\overline{YX}_iC_yC_{x_i}\rho_{yx_i}.$$

Similarly

$$E_1E_{2|1}\left[\bar{e}_{x_{(2)i}}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right] = \left(\theta_1 - \theta_2\right)\sigma_{x_i}^2 = \left(\theta_1 - \theta_2\right)\bar{X}_i^2C_{x_i}^2,$$

$$E_1E_{2|1}\left[\bar{e}_{x_{(1)i}}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right] = 0,$$

$$E_1E_{2/1}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)^2 = \left(\theta_2 - \theta_1\right)\sigma_{x_i}^2 = \left(\theta_2 - \theta_1\right)\bar{X}_i^2C_{x_i}^2,$$

$$E_1E_{2|1}\left[\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)\left(\bar{e}_{x_{(1)j}}-\bar{e}_{x_{(2)j}}\right)\right]=\left(\theta_2-\theta_1\right)\sigma_{x_ix_j}$$
$$=\left(\theta_2-\theta_1\right)\bar{X}_i\bar{X}_jC_{x_i}C_{x_j}\rho_{x_ix_j}\ ;\left(i\neq j\right),$$

and

$$E_1E_{2|1}\left[\left(\bar{e}_{x_{(2)i}}\right)\left(\bar{e}_{x_{(1)j}}-\bar{e}_{x_{(2)j}}\right)\right]=\left(\theta_1-\theta_2\right)\sigma_{x_ix_j}\ =\left(\theta_1-\theta_2\right)\bar{X}_i\bar{X}_jC_{x_i}C_{x_j}\rho_{x_ix_j}\ ,\left(i\neq j\right).$$

The following notations will be used in deriving the mean square errors of proposed estimators

$\lvert \mathrm{R}\rvert_{y\underset{\sim}{x}_q}$	Determinant of population correlation matrix of variables $y,\ x_1,\ x_2,...,x_{q-1}$ and x_q.
$\left\lvert R_{yx_i}\right\rvert_{y\underset{\sim}{x}_q}$	Determinant of i^{th} minor of $\lvert \mathrm{R}\rvert_{y\underset{\sim}{x}_q}$ corresponding to the i^{th} element of ρ_{yx_i}.
$\rho^2_{y.\underset{\sim}{x}_s}$	Denotes the multiple coefficient of determination of y on $x_1,\ x_2,...,x_{r-1}$ and x_r.
$\rho^2_{y.\underset{\sim}{x}_q}$	Denotes the multiple coefficient of determination of y on $x_1,\ x_2,...,x_{q-1}$ and x_q.
$\lvert R\rvert_{\underset{\sim}{x}_s}$	Determinant of population correlation matrix of variables $x_1,\ x_2,...,x_{r-1}$ and x_r.
$\lvert R\rvert_{\underset{\sim}{x}_q}$	Determinant of population correlation matrix of variables $x_1,\ x_2,...,x_{q-1}$ and x_q.
$\lvert R\rvert_{y_i\underset{\sim}{x}_s}$	Determinant of the correlation matrix of $y_i,\ x_1,\ x_2,...,x_{r-1}$ and x_r.
$\lvert R\rvert_{y_i\underset{\sim}{x}_q}$	Determinant of the correlation matrix of $y_i,\ x_1,\ x_2,...,x_{q-1}$ and x_q.
$\lvert R\rvert_{y_iy_j\underset{\sim}{x}_s}$	Determinant of the minor corresponding to $\rho_{y_iy_j}$ of the correlation matrix of $y_i,\ y_j,x_1,\ x_2,...,x_{r-1}$ and x_r, for $\left(i\neq j\right)$.
$\lvert R\rvert_{y_iy_j\underset{\sim}{x}_q}$	Determinant of the minor corresponding to $\rho_{y_iy_j}$ of the correlation matrix of $y_i,\ y_j,x_1,\ x_2,...,x_{q-1}$ and x_q, for $\left(i\neq j\right)$.

2.1 Result: 1

The following result will help us in deriving the mean square errors of suggested estimators

$$\frac{\lvert \mathrm{R}\rvert_{y\underset{\sim}{x}_q}}{\lvert R\rvert_{\underset{\sim}{x}_q}}=\left(1-\rho^2_{y.\underset{\sim}{x}_q}\right). \qquad \text{[Arora and Lal (1989)]}$$

2.2 Regressions-Cum-Ratio Estimator (Full Information Case)

If we estimate a study variable when information on all auxiliary variables is available from population, it is utilized in the form of their means. By taking the advantage of Regression-cum-Ratio technique for two-phase sampling, a generalized estimator for estimating population mean of study variable $\bar{Y}$ with the use of multi-auxiliary variables are suggested as:

$$t_1 = \left[\bar{y}_2 + \sum_{i=1}^{r}\alpha_i\left(\bar{X}_i - \bar{x}_{(2)i}\right)\right]\prod_{i=r+1}^{r+s=q}\left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\gamma_i}$$

$$= \left[\bar{Y} + \bar{e}_{y_2} - \sum_{i=1}^{r}\alpha_i\bar{e}_{x_{(2)i}}\right]\prod_{i=r+1}^{r+s=q}\left(1+\frac{\bar{e}_{x_{(2)i}}}{\bar{X}_i}\right)^{-\gamma_i} .$$

Ignoring second and higher terms for each expansion of product and after simplification, we write $t_1 = \left[\bar{Y} + \bar{e}_{y_2} - \sum_{i=1}^{r}\alpha_i\bar{e}_{x_{(2)i}} - \sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i\bar{e}_{x_{(2)i}}\right]$.

The mean square error is

$$MSE\left(t_1\right) = E_2\left[\bar{e}_{y_2} - \sum_{i=1}^{r}\alpha_i\bar{e}_{x_{(2)i}} - \sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i\bar{e}_{x_{(2)i}}\right]^2 . \qquad (3.2.1)$$

The optimum values of α_i and γ_i for which the mean square error of estimator t_1 is minimum to term of o(1/n) are

$$\alpha_i = \left(-1\right)^{i+1}\frac{\bar{Y}}{\bar{X}_i}\frac{C_y}{C_{x_i}}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}} = \left(-1\right)^{i+1}\beta_{y\underset{\sim}{x}_i\cdot\underset{\sim}{x}_q}\ \left(i = 1, 2, ...r \text{ and } r+s=q\right)$$

and

$$\gamma_i = \left(-1\right)^{i+1}\frac{C_y}{C_{x_i}}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}} = \left(-1\right)^{i+1}\frac{\bar{X}_i}{\bar{Y}}\beta_{y\underset{\sim}{x}_i\cdot\underset{\sim}{x}_q}\ \left(i = r+1, r+2, ...r+s \text{ and } r+s=q\right) .$$

The unknown constants are related to the partial regression coefficients of study variable and auxiliary variables. If these partial regression coefficients are not known, these will be estimated from second phase because the estimator t_1 utilizes the information on q auxiliary variables and study variable from second phase sample.

Using normal equations that are used to find the optimum values of α_i and γ_i, (3.2.1) can be written in simplified form as:

$$
\begin{aligned}
MSE(t_1) &= E_2\left[\bar{e}_{y_2}\left(\bar{e}_{y_2} - \sum_{i=1}^{r}\alpha_i \bar{e}_{x_{(2)i}} - \sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i \bar{e}_{x_{(2)i}}\right)\right] \\
&= E_2\left(\bar{e}_{y_2}^2\right) - \sum_{i=1}^{r}\alpha_i E_2\left(\bar{e}_{y_2}\bar{e}_{x_{(2)i}}\right) - \sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i E_2\left(\bar{e}_{y_2}\bar{e}_{x_{(2)i}}\right) \\
&= \theta_2 \bar{Y}^2 C_y^2 - \sum_{i=1}^{r}\alpha_i \theta_2 \overline{YX}_i C_y C_{x_i}\rho_{yx_i} - \sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i \theta_2 \overline{YX}_i C_y C_{x_i}\rho_{yx_i}\,.
\end{aligned}
$$

Using the values of α_i and γ_i and after simplification, we get:

$$
\begin{aligned}
MSE(t_1) &= \bar{Y}^2 C_y^2\left[\theta_2 - \theta_2\sum_{i=1}^{r}(-1)^{i+1}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}}\rho_{yx_i} - \theta_2\sum_{i=r+1}^{r+s=q}(-1)^{i+1}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}}\rho_{yx_i}\right] \\
&= \bar{Y}^2 C_y^2\left[\theta_2 + \theta_2\sum_{i=1}^{q}(-1)^{i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}}\rho_{yx_i}\right] \\
&= \frac{\theta_2 \bar{Y}^2 C_y^2}{|R|_{\underset{\sim}{x}_q}}\Big[|R|_{\underset{\sim}{x}_q} - \rho_{yx_1}\left|R_{yx_1}\right|_{y\underset{\sim}{x}_q} + \rho_{yx_2}\left|R_{yx_2}\right|_{y\underset{\sim}{x}_q} \\
&\qquad -\rho_{yx_3}\left|R_{yx_3}\right|_{y\underset{\sim}{x}_q} + \ldots\ldots + (-1)^q\rho_{yx_q}\left|R_{yx_q}\right|_{y\underset{\sim}{x}_q}\Big]
\end{aligned}
$$

or

$$
MSE(t_1) = \theta_2 \bar{Y}^2 C_y^2\frac{|\mathrm{R}|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}}\,.
$$

Using Result 1, we get:

$$
MSE(t_1) = \theta_2 \bar{Y}^2 C_y^2\left(1 - \rho_{y.\underset{\sim}{x}_q}^2\right).
$$

2.3 Regressions-Cum-Ratio Estimator (Partial Information Case)

In this case suppose we have no information on all q auxiliary variables but only for r auxiliary variables from population. Considering Regression-Cum-Ratio technique of estimating technique, the population mean of study variable $\bar{Y}$ can be estimated for two-phase sampling using multi-auxiliary variables as:

$$
t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r}\alpha_i^{"}\left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right]\prod_{i=r+1}^{r+s=q}\left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{"}}\prod_{i=r+1}^{r+s=q}\left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{"}}
$$

$$=\left[\bar{Y}+\bar{e}_{y_2}+\sum_{i=1}^{r}\alpha_i^{"}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)\right]\prod_{i=r+1}^{r+s=q}\left(1+\frac{\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}}{\bar{X}_i}\right)^{\gamma_i^{"}}\prod_{i=r+1}^{r+s=q}\left(1+\frac{\bar{e}_{x_{(2)i}}}{\bar{X}_i}\right)^{-\delta_i^{"}}.$$

Ignoring second and higher terms for each expansion of products and after simplification we write

$$t_2=\left[\bar{Y}+\bar{e}_{y_2}+\sum_{i=1}^{r}\alpha_i^{"}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)+\sum_{i=r+1}^{r+s}\gamma_i^{"}\frac{\bar{Y}}{\bar{X}_i}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)-\sum_{i=r+1}^{r+s}\delta_i^{"}\frac{\bar{Y}}{\bar{X}_i}\bar{e}_{x_{(2)i}}\right].$$

The mean square error of t_2 is written as

$$MSE\left(t_2\right)=E_1E_{2/1}\left[\bar{e}_{y_2}+\sum_{i=1}^{r}\alpha_i^{"}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)\right.$$
$$\left.+\sum_{i=r+1}^{r+s}\gamma_i^{"}\frac{\bar{Y}}{\bar{X}_i}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)-\sum_{i=r+1}^{r+s}\delta_i^{"}\frac{\bar{Y}}{\bar{X}_i}\bar{e}_{x_{(2)i}}\right]^2 \qquad (3.3.1)$$

The optimum values of $\alpha_i^{"},\gamma_i^{"}$ and $\delta_i^{"}$ for which the mean square error of t_2 is minimum to term of o (1/n) are:

$$\alpha_i^{''}=\left(-1\right)^{i+1}\frac{\bar{Y}}{\bar{X}_i}\frac{C_y}{C_{x_i}}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}=\left(-1\right)^{i+1}\beta_{y\underset{\sim}{x}_i\cdot\underset{\sim}{x}_q},\ \left(i=1,2,....r\right).$$

$$\gamma_i^{"}=\left(-1\right)^{i+1}\frac{C_y}{C_{x_i}}\left[\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}-\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}\right]$$

$$=\left(-1\right)^{i+1}\frac{\bar{X}_i}{\bar{Y}}\left(\beta_{yx_i\cdot\underset{\sim}{x}_q}-\beta_{yx_i\cdot\underset{\sim}{x}_s}\right),\ \left(i=r+1,r+2,....r+s\right)$$

and

$$\delta_i^{''}=\left(-1\right)^{i+1}\frac{C_y}{C_{x_i}}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}=\left(-1\right)^{i+1}\beta_{y\underset{\sim}{x}_i\cdot\underset{\sim}{x}_s},\ i=r+1,r+2,.......r+s.$$

The optimum values are related to the partial regression coefficients of variable of interest and auxiliary variables. Usually these partial regression coefficients are unknown then these can be estimated from sample data. The estimator t_2 utilizes the information on *q* auxiliary variables from both first and second phase. Keenly observing the estimators t_2 the optimum values of unknown constants $\alpha_i^{''}$ and $\gamma_i^{''}$ will be estimated from first phase sample and $\delta_i^{''}$ from the second phase. Using normal equations that are used to fined the optimum values of α_i'', γ_i'' and δ_i'', (3.3.1) can be written in a simplified form as:

$$MSE(t_2) = E_1 E_{2/1}\left[\bar{e}_{y_2}\left(\bar{e}_{y_2} + \sum_{i=1}^{r}\alpha_i''\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right.\right.$$
$$\left.\left. + \sum_{i=r+1}^{r+s}\gamma_i''\frac{\bar{Y}}{\bar{X}_i}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right) - \sum_{i=r+1}^{r+s}\delta_i''\frac{\bar{Y}}{\bar{X}_i}\bar{e}_{x_{(2)i}}\right)\right]$$
$$= E_1 E_{2/1}\left(\bar{e}_{y_2}^2\right) + \sum_{i=1}^{r}\alpha_i'' E_1 E_{2/1}\left\{\bar{e}_{y_2}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right\}$$
$$+ \sum_{i=r+1}^{r+s}\gamma_i''\frac{\bar{Y}}{\bar{X}_i}E_1 E_{2/1}\left\{\bar{e}_{y_2}\left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}\right)\right\} - \sum_{i=r+1}^{r+s}\delta_i''\frac{\bar{Y}}{\bar{X}_i}E_1 E_{2/1}\left(\bar{e}_{y_2}\bar{e}_{x_{(2)i}}\right)$$
$$= \theta_2\bar{Y}^2 C_y^2 + (\theta_1 - \theta_2)\sum_{i=1}^{r}\alpha_i''\overline{YX}_i C_y C_{x_i}\rho_{yx_i}$$
$$+ (\theta_1 - \theta_2)\sum_{i=r+1}^{r+s}\gamma_i''\frac{\bar{Y}}{\bar{X}_i}\overline{YX}_i C_y C_{x_i}\rho_{yx_i} - \theta_2\sum_{i=r+1}^{r+s}\delta_i''\frac{\bar{Y}}{\bar{X}_i}\overline{YX}_i C_y C_{x_i}\rho_{yx_i}.$$

Using the values of α_i'', γ_i'' and δ_i'' and after simplification we get:

$$MSE(t_2) = \bar{Y}^2 C_y^2\left[\theta_2 - (\theta_1 - \theta_2)\sum_{i=1}^{r}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\right.$$
$$\left. - (\theta_1 - \theta_2)\sum_{i=r+1}^{r+s}(-1)^i\rho_{yx_i}\left\{\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}} - \frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}\right\} + \theta_2\sum_{i=r+1}^{r+s}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}\right]$$
$$= \bar{Y}^2 C_y^2\left[(\theta_2 - \theta_1) + (\theta_2 - \theta_1)\sum_{i=1}^{r}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\right.$$
$$\left. + (\theta_2 - \theta_1)\sum_{i=r+1}^{r+s}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}} + \theta_1 + \theta_1\sum_{i=r+1}^{r+s}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}\right]$$

or

$$= \bar{Y}^2 C_y^2\left[(\theta_2 - \theta_1)\left(1 + \sum_{i=1}^{q}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\right) + \theta_1\left(1 + \sum_{i=r+1}^{r+s}(-1)^i\rho_{yx_i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_s}}{\left|R\right|_{\underset{\sim}{x}_s}}\right)\right],$$

or

$$MSE(t_2) = \bar{Y}^2 C_y^2 \left[\theta_2 \frac{|\mathrm{R}|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}} + \theta_1 \left(\frac{|\mathrm{R}|_{y\underset{\sim}{x}_s}}{|R|_{\underset{\sim}{x}_s}} - \frac{|\mathrm{R}|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}} \right) \right].$$

Using Result 1 we get:

$$MSE(t_2) = \bar{Y}^2 C_y^2 \left[\theta_2 \left(1 - \rho^2_{y.\underset{\sim}{x}_q} \right) + \theta_1 \left(\rho^2_{y.\underset{\sim}{x}_q} - \rho^2_{y.\underset{\sim}{x}_s} \right) \right].$$

2.4 Regressions-Cum-Ratio Estimator (No Information Case)

We consider the following regression-cum-ratio estimator for two-phase sampling using multi-auxiliary variables for estimating the population mean $\bar{Y}$ when information on all auxiliary variables is not available from a population as:

$$t_3 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i' \left(\bar{x}_{(1)i} - \bar{x}_{(2)i} \right) \right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}} \right)^{\gamma_i'}$$

$$= \left[\bar{Y} + \bar{e}_{y_2} + \sum_{i=1}^{r} \alpha_i' \left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}} \right) \right] \prod_{i=r+1}^{r+s=q} \left(1 + \frac{\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}}}{\bar{X}_i} \right)^{\gamma_i'} .$$

Ignoring second and higher terms for each expansion of product and after simplification we can write

$$t_3 = \left[\bar{Y} + \bar{e}_{y_2} + \sum_{i=1}^{r} \alpha_i' \left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}} \right) + \sum_{i=r+1}^{r+s=q} \frac{\bar{Y}}{\bar{X}_i} \gamma_i' \left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}} \right) \right].$$

The mean square error is

$$MSE(t_3) = E_2 \left[\bar{e}_{y_2} + \sum_{i=1}^{r} \alpha_i' \left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}} \right) + \sum_{i=r+1}^{r+s=q} \frac{\bar{Y}}{\bar{X}_i} \gamma_i' \left(\bar{e}_{x_{(1)i}} - \bar{e}_{x_{(2)i}} \right) \right]^2 . \quad (3.4.1)$$

The optimum values of α_i' and γ_i' for which the mean square error of t_3 is minimum to the order o(1/n) are:

$$\alpha_i' = (-1)^{i+1} \frac{\bar{Y}}{\bar{X}_i} \frac{C_y}{C_{x_i}} \frac{\left| R_{yx_i} \right|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}} = (-1)^{i+1} \beta_{y\underset{\sim}{x}_i \cdot \underset{\sim}{x}_q} \quad (i = 1, 2, ...r)$$

and

$$\gamma_i' = (-1)^{i+1} \frac{C_y}{C_{x_i}} \frac{\left| R_{yx_i} \right|_{y\underset{\sim}{x}_q}}{|R|_{\underset{\sim}{x}_q}} = (-1)^{i+1} \frac{\bar{X}_i}{\bar{Y}} \beta_{y\underset{\sim}{x}_i \cdot \underset{\sim}{x}_q} , \quad (i = r+1, r+2, ...r+s).$$

In this case the optimum values are also related to the partial regression coefficients of study variable and auxiliary variables. Mostly these partial regression coefficients are unknown but these can be estimated from sample data. The estimator t_3 utilizes the information on q auxiliary variables from both first and second phase. Analyzing the estimators t_3 the optimum values of unknown constants $\alpha_i^{'}$ and $\gamma_i^{'}$ will be estimated from first phase sample in the form of sample regression coefficients.

(3.4.1) is also written in a simplified form as:

$$MSE\left(t_3\right)=E_2\left(\bar{e}_{y_2}^2\right)+\sum_{i=1}^{r}\alpha_i^{'}E_2\left(\bar{e}_{y_2}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)\right)-\sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i^{'}E_2\left(\bar{e}_{y_2}\left(\bar{e}_{x_{(1)i}}-\bar{e}_{x_{(2)i}}\right)\right)$$

$$=\theta_2\bar{Y}^2C_y^2+\left(\theta_1-\theta_2\right)\sum_{i=1}^{r}\alpha_i^{'}\overline{YX}_iC_yC_{x_i}\rho_{yx_i}+\left(\theta_1-\theta_2\right)\sum_{i=r+1}^{r+s=q}\frac{\bar{Y}}{\bar{X}_i}\gamma_i^{'}\overline{YX}_iC_yC_{x_i}\rho_{yx_i}.$$

Using the values of $\alpha_i^{'}$ and $\gamma_i^{'}$ and after simplification we get:

$$MSE\left(t_3\right)=\bar{Y}^2C_y^2\left[\theta_2+\left(\theta_1-\theta_2\right)\sum_{i=1}^{r}(-1)^{i+1}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\rho_{yx_i}\right.$$
$$\left.+\left(\theta_1-\theta_2\right)\sum_{i=r+1}^{r+s=q}(-1)^{i+1}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\rho_{yx_i}\right]$$
$$=\bar{Y}^2C_y^2\left[\theta_2+\left(\theta_2-\theta_1\right)\sum_{i=1}^{q}(-1)^{i}\frac{\left|R_{yx_i}\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}\rho_{yx_i}\right]$$
$$=\bar{Y}^2C_y^2\left[\left(\theta_2-\theta_1\right)\frac{\left|R\right|_{y\underset{\sim}{x}_q}}{\left|R\right|_{\underset{\sim}{x}_q}}+\theta_1\right].$$

Using Result1, we get:

$$MSE\left(t_3\right)=\bar{Y}^2C_y^2\left[\left(\theta_2-\theta_1\right)\left(1-\rho_{y.\underset{\sim}{x}_q}^2\right)+\theta_1\right]=\bar{Y}^2C_y^2\left[\theta_2\left(1-\rho_{y.\underset{\sim}{x}_q}^2\right)+\theta_1\rho_{y.\underset{\sim}{x}_q}^2\right].$$

3. THEORETICAL COMPARISON OF NEW ESTIMATORS

The MSE criterion is most common for comparing various estimators [Lee and Peddada (1987) and Cox and Hinkley (1974)]. We suggest three estimators in this paper in which first one is for full information case, second one is for partial information case and last one is for no information case. The estimator for full information case is more efficient than the estimator for partial information case and the estimator for partial information case is more efficient than for the no information case. It can be checked by comparing their MSE's as:

$$MSE(t_1) - MSE(t_2) = -\theta_1 \bar{Y}^2 C_y^2 \left(\rho^2_{y.\underset{\sim}{x}_q} - \rho^2_{y.\underset{\sim}{x}_s}\right) < 0; \text{ as } q>s$$

and $$MSE(t_2) - MSE(t_3) = -\theta_1 \bar{Y}^2 C_y^2 \rho^2_{y.\underset{\sim}{x}_s} < 0.$$

4. SPECIAL CASES

We summarize the results for the three cases as follows:

Proposed Estimators	**Unknown Constants**			**Special Cases**
I. Full Information Case	α_i		γ_i	
$t_1 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i \left(\bar{X}_i - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\gamma_i}$	0		γ_i	A class of ratio estimators with s auxiliary variables for Full Information Case
$t_1 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i \left(\bar{X}_i - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\gamma_i}$	α_i		0	A class of regression estimators with r auxiliary variables for Full Information Case
II. Partial Information Case	$\alpha_i^{''}$	$\gamma_i^{''}$	$\delta_i^{''}$	
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	0	$\gamma_i^{''}$	$\delta_i^{''}$	A class of ratio estimators with s auxiliary variables for Partial Information Case
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	$\alpha_i^{''}$	0	$\delta_i^{''}$	A class of ratio-cum-regression estimators with q auxiliary variables for Partial Information Case
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	$\alpha_i^{''}$	$\gamma_i^{''}$	0	A class of ratio-cum-regression estimators with q auxiliary variables for No Information Case
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q} \left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	$\alpha_i^{''}$	0	0	A class of regression estimators with r auxiliary variables for No Information Case

Proposed Estimators	Unknown Constants			Special Cases
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''}\left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	0	$\gamma_i^{''}$	0	A class of ratio estimators with s auxiliary variables for No Information Case
$t_2 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i^{''}\left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i^{''}} \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{X}_i}{\bar{x}_{(2)i}}\right)^{\delta_i^{''}}$	0	0	$\alpha_i^{''}$	A class of ratio estimators with s auxiliary variables for Partial Information Case
III. No Information Case	$\alpha_i^{'}$		$\gamma_i^{'}$	
$t_3 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i\left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i}$	0		$\gamma_i^{'}$	A class of ratio estimators with s auxiliary variables for No Information Case
$t_3 = \left[\bar{y}_2 + \sum_{i=1}^{r} \alpha_i\left(\bar{x}_{(1)i} - \bar{x}_{(2)i}\right)\right] \prod_{i=r+1}^{r+s=q}\left(\frac{\bar{x}_{(1)i}}{\bar{x}_{(2)i}}\right)^{\gamma_i}$	$\alpha_i^{'}$		0	A class of regression estimators with r auxiliary variables for No Information Case

Obviously the classes of special cases are not efficient than suggested classes of estimators.

5. NUMERICAL ILLUSTRATION

Description of populations is given in Table 1 and mean square errors of suggested estimators are given in Table 2.

Table 1:
Description of Population

Source	"Measurement of four characters of: Flucus Religiousament" by Pradhan (2000)						
y	Length of petiole						
x_1	Length of lamina (blade) of the leaf						
x_2	Width of the leaf at its widest paint						
x_3	Width of leaf half way along the blade						
	N	ρ_{yx_1}	ρ_{yx_2}	ρ_{yx_3}	$\rho_{x_1x_2}$	$\rho_{x_1x_3}$	$\rho_{x_2x_3}$
	160	0.5423	0.6166	0.2704	0.8568	0.7424	0.8027

Table 2:
Mean Square Errors of Estimators

Estimator		Auxiliary Variables for which Information is Known for the Population	Auxiliary Variables for which Information is unknown for the Population	Relative Efficiency when N = 160, n_1 = 50, n_2 = 20
t_3 (NIC)		-	x_1, x_2, x_3	100
t_2	t_{21}	x_2, x_3	x_1	129.34
PIC	t_{22}	x_1, x_3	x_2	123.46
	t_{23}	x_1, x_2	x_3	156.65
	t_{24}	x_1,	x_2, x_3	168.07
	t_{25}	x_2	x_1, x_3	149.43
	t_{26}	x_3	x_1, x_2	164.12
t_1(FIC)		x_1, x_2, x_3	-	135.73

In Table 2 we provide MSE's of eight estimators, first estimator is for full information case, last estimator is for no information case and other six estimators are for partial information case with all possible combinations of auxiliary variables with known and unknown information from population. In theoretical comparisons t_1 is more efficient than t_2 and t_2 is more efficient than t_3. But in empirical comparison, we see that a special case of partial information i.e. t_{24} performs better than all others. It means that population characteristics like mean, coefficient of variation, variances, sample sizes of both phases and especially correlation coefficients of study variable with auxiliary variables and correlation coefficients within auxiliary variables count a lot for suggesting an estimator for use in real life situations. This can be adequately judged by considering at least ten different types of natural populations and MSE's should be calculated for study variable in the presence of at least five auxiliary variables.

REFERENCES

1. Abu-Dayyeh, W.A., Ahmed, M.S., Ahmed, R.A. and Muttlak, H.A. (2003). Some estimators of finite population mean using auxiliary information. *App. Math. and Compu.*, 139, 287-298.
2. Agarwal, S.K., Sharma, U.K. and Kashyap, S. (1997). A new approach to use multivariate auxiliary information in sample surveys. *J. Statist. Plann. and Infer.*, 60, 261-267.
3. Arora, S. and Lal, Bansi (1989). *New Mathematical Statistics*. Satya Prakashan, New Delhi.
4. Ceccon, C. and Diana, G. (1996). Classi di stimatori di tipo rapporto in presenza di pi´u variabili ausiliarie, *Proceedings of Italian Statistical Society*, XXXVIII Scientific Meeting, Rimini, 9-13 April 1996, 119-126.
5. Cox, D.R. and Hinkley, D.V. (1974). *Theoretical Statistics*. Champman and Hall, London.

6. John, S. (1969). On multivariate ratio and product estimators. *Biometrika*, 56, 533-536.
7. Kadilar, C. and Cingi, H. (2004). Estimator of a population mean using two auxiliary variables in simple random sampling. *Int. Math. J.*, 5, 357-367.
8. Kadilar, C. and Cingi, H. (2005). A new estimator using two auxiliary variables, *App. Math. and Compu*., 162, 901-908.
9. Lee, Carl M.S. and Peddada, S.D. (1987). Comparisons of some common ratio estimators using Pitman measure of nearess. *Comm. Statist. Theo Math.*, 16(7), 2017-2027.
10. Murthy, M.N. (1964). Product method of estimation. *Sankhya*, 26, 294-307.
11. Naik, V.D. and Gupta, P.C. (1991). A general class of estimators for estimating population means using auxiliary information. *Metrika*, 38, 11-17.
12. Olkin, I. (1958). Multivariate ratio estimation for finite populations. *Biometrika*, 45, 154-165.
13. Perri, P.F. (2004). Alcune considerazioni sull'efficienza degli stimatori rapporto-cum-prodotto. *Statistica and Applicazioni*, 2(2), 59-75.
14. Perri, P.F. (2005). Combining two auxiliary variables in ratio-cum-product type estimators. *Proceedings of Italian Statistical Society*, *Intermediate Meeting on Statistics and Environment*, Messina, 21-23 September 2005, 193-196.
15. Pradhan, B.K. (2000). *Some problems of estimation in multi-phase sampling*. Ph.D. Dissertation (unpublished), Utkal University, Orissa, India.
16. Pradhan, B.K. (2005). A Chain Regression Estimator in Two Phase Sampling Using Multi-auxiliary Information. *Bull. Malays. Math. Sci. Soc.*, (2) 28(1), 81-86.
17. Raj, D. (1965). On a method of using multi-auxiliary information in sample surveys. *J. Amer. Statist. Assoc.*, 60, 309, 270-277.
18. Rao, P.S.R.S. and Mudholkar, G.S. (1967). Generalized multivariate estimators for the mean of a finite population. *J. Amer. Statist. Assoc.*, 62, 1009-1012.
19. Robinson, P.M. (1994). A class of estimators for the mean of a finite population using auxiliary information. *Sankhya*, B, 56, 389-399.
20. Samiuddin, M. and Hanif, M. (2007). Estimation of population mean in single and two phase sampling with or without additional information. *Pak. J. Statist.*, 23(2), 99-118.
21. Shukla, G.K. (1965). Multivariate regression estimate. *J. Indian Statist. Assoc.*, 3, 202-211.
22. Singh, M.P. (1965). On the estimation of ratio and product of population parameters. *Sankhya*, Ser. C, 27, 321-328.
23. Singh, M.P. (1967a). Multivariate product method of estimation for finite populations. *J. Ind. Soc. of Agricul. Statist*., 31, 375-378.
24. Singh, M.P. (1967b). Ratio cum product method of estimation. *Metrika*, 12, 34-43.
25. Srivastava, S.K. (1971). A generalized estimator for the mean of a finite population using multi-auxiliary information. *J. Amer. Statist. Assoc.*, 66, 404-407.
26. Tracy, D.S., Singh, H.P. and Singh, R. (1996). An alternative to the ratio-cum-product estimator in sample surveys. *J. Statist. Plann. and Infer.*, 53, 375-387.